CONTROL SYSTEMS - UNIT -3

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Stability Analysis (Part – I)

Short Questions with answers (2 marks)

1. The number of roots of $S^3 + 5s^2 + 7s + 3 = 0$ in the left half of the s – plane is
   (a) Zero
   (b) One
   (c) Two
   (d) Three

Soln. Using R – H criterion

s^3	1	7
s^2	5	3
s^1	6.4	0
s^0	3	

There is no sign change in the first column of R – H array, so no roots lie in RHS of s – plane. All the three roots lie in the left half of s – plane

Option (d)

2. The open loop transfer function of an unity feedback open loop system $\frac{2s^2+6s+5}{(s+1)^2(s+2)}$. The characteristic equation of the closed loop system is
   (a) $2s^2 + 6s + 5 = 0$
   (b) $(s + 1)^2(s + 2) = 0$
   (c) $2s^2 + 6s + 5 + (s + 1)^2(s + 2) = 0$
   (d) $2s^2 + 6s + 5 - (s + 1)^2(s + 2) = 0$

Soln. Characteristic equation

$$1 + GH = 0$$

$$1 + \frac{2\,s^2+6s+5}{(s+1)^2(s+2)} = 0$$

$$(s + 1)^2(s + 2) + 2s^2 + 6s + 5 = 0$$

Option (c)

3. The gain margin (in dB) of a system having the loop transfer function

$$G(s)\,H(s) = \frac{\sqrt{2}}{s(s+1)} \quad \text{is}$$

(a) 0
(b) 3
(c) 6
(d) ∞

Soln. Loop transfer of function $G(s)H(s) = \frac{\sqrt{2}}{s(s+1)}$

It is a second order function, so its gain margin is infinity

Option (d)

4. The phase margin (in degrees) of a system having the loop transfer function

$$G(s)\,H(s) = \frac{2\sqrt{3}}{s(s+1)} \quad \text{is}$$

(a) 45^0
(b) -30^0
(c) 60^0
(d) 30^0

Soln. Loop transfer function

$$G(s)\,H(s) = \frac{2\sqrt{3}}{s(s+1)}$$

Phase margin $\gamma = 180 + \phi gc$

ϕgc is the phase angle ϕ of loop transfer function at the gain cross over frequency where $\left|G(j\omega_g)\,H(j\omega_g)\right| = 1$ where ω_g is the gain cross over frequency

$$\left|\frac{2\sqrt{3}}{j\omega_g(j\omega_g+1)}\right| = 1$$

$$\frac{2\sqrt{3}}{\omega_g\left(\sqrt{1+\omega_g^2}\right)} = 1$$

$$2\sqrt{3} = \omega_g\sqrt{1+\omega_g^2}$$

$$\omega_g = \sqrt{3}$$

$$\angle G(j\omega_g)H(j\omega_g) = -90^0 - tan^{-1}\,\omega_g$$

$$= -90^0 - tan^{-1}\sqrt{3}$$

$$= -150^0$$

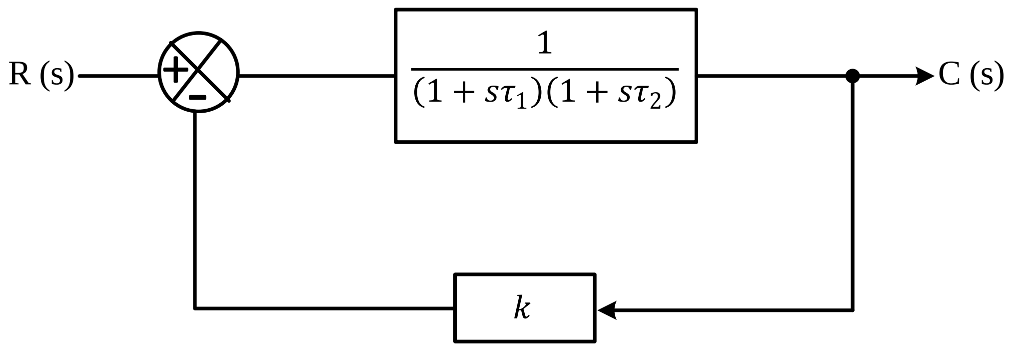
Phase margin $\gamma = 180 + \phi_{gc} = 180 - 150 = 30^0$

Option (d)

5. An amplifier with resistive negative feedback has two left half plane poles in its open – loop transfer function. The amplifier

(a) Will always be unstable at high frequency
(b) Will be stable for all frequency
(c) May be unstable, depending on the feedback factor
(d) Will oscillate at low frequency

Soln. The block diagram of the system is shown below



For resistive negative feedback, the feedback factor is always less then unity. The system is stable for all frequencies

Option (b)

6. The phase margin of a system with the open – loop transfer function

$$G(s)H(s) = \frac{(1-s)}{(1+2)(2+s)}$$

(a) 0^0 (c) 90^0
(b) 63.4^0 (d) ∞

Soln. $$G(s)\,H(s) = \frac{1-s}{(1+s)(2+s)}$$

Let ω_g be the gain cross over frequency where $|G(s)\,H(s)| = 1$

$$\left|\frac{1-s}{(1+s)(2+s)}\right| = \left|\frac{1-j\omega_g}{(1+j\omega_g)(2+j\omega_g)}\right| = 1$$

$$or, \quad \frac{\sqrt{1+\omega_g^2}}{\sqrt{1+\omega_g^2}\quad\sqrt{4+\omega_g^2}} = 1$$

$$or, \quad \sqrt{4+\omega_g^2} = 1$$

$$\omega_g^2 = -3$$

ω_g is imaginary so no gain cross over frequency

Phase margin $\gamma = \infty$

Option (d)

7. The gain margin for the system with open – loop transfer function

$$G(s)H(s) = \frac{2(1+s)}{s^2} \quad \text{is}$$

(a) ∞ (c) 1
(b) 0 (d) $-\infty$

Soln. $$G(s)H(s) = \frac{2(1+s)}{s^2}$$

The gain margin GM is the value of gain to be added to the system to bring the system to the verge of instability

$$GM = \frac{1}{|G(j\omega_{pc})\,H(j\omega_{pc})|} = \frac{1}{M}$$

ω_{pc} is the phase cross over frequency where $\angle G(s)H(s) = -180^0$

$$G(j\omega)\,H(j\omega) = \frac{2(1+j\omega)}{j\omega\ \ j\omega}$$

$$\angle G(j\omega)\,H(j\omega) = -90^0 - 90^0 + tan^{-1}\omega$$

$$= -180^0 + tan^{-1}\omega$$

$$at\quad \omega = \omega_{pc}, \qquad \phi = -180^0$$

$$-180 = -180 + tan^{-1}\omega_{pc}$$

$$tan^{-1}\omega_{pc} = 0$$

$$\omega_{pc} = 0$$

$$M = \left|\frac{2\sqrt{1+\omega^2}}{\omega^2}\right|_{\omega_{pc}} = \infty$$

$$G.M = \frac{1}{M} = \frac{1}{\infty} = 0$$

Option (b)

8. The open – loop transfer function of a unity – gain feedback control system is given by

$$G(s) = \frac{K}{(s+1)(s+2)}$$

The gain margin of the system in dB is given by

(a) 0 (c) 20

(b) 1 (d) ∞

Soln.

$$G(s) = \frac{K}{(s+1)(s+2)}$$

It is a second order system, so it’s gain margin is ∞

Option (d)

9. If the closed – loop transfer function of a control system is given as $T(s) = \frac{s-5}{(s+2)(s+3)}$, then it is
   (a) an unstable system
   (b) an uncontrollable system
   (c) a minimum phase system
   (d) a non – minimum phase system

Soln. The system in which one or more zeros lie in the right half of s – plane and remaining poles and zeros in the left half of s – plane is called non minimum phase system

Option (d)

10. Consider a characteristic equation given by
   $s^2 + 3s^3 + 5s^2 + 6s + K + 10$
   The condition for stability is
   (a) K > 5
   (b) – 10 < K
   (c) K > – 4
   (d) – 10 < K < – 4

Soln. The characteristic equation is

$$s^4 + 3s^3 + 5s^2 + 6s + k + 10 = 0$$

Using R – H criterion

s^4	1	5	$k+10$
s^3	3	6	
s^1	3	$k+10$	0
s	$\frac{-12-3k}{3}$	0	0
s^0	$k+10$		

For stable system, all coefficients of 1st column should be positive

$\frac{-12-3k}{3} > 0 \quad or \quad -12-3k > 0$

$-12 > 3k$

$-4 > k$

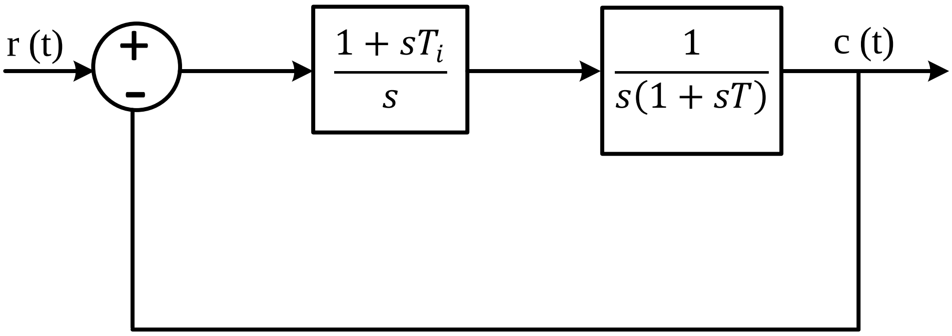
$k + 10 > 0$

$k > -10$

$-10 < k < -4$

Option (d)

11. In order to stabilize the system shown in figure. Ti should satisfy



(a) $T_i = -T$

(b) $T_i = T$

(c) $T_i < T$

(d) $T_i > T$

Soln. Characteristic equation

$1 + GH = 0$

$1 + \frac{(1+sT_i)}{s} \times \frac{1}{s(1+sT)} \times 1 = 0$

$s^2(1+sT) + (1+sT_i) = 0$

$s^3T + s^2 + sT_i + 1 = 0$

Using R – H criteria

s^3	T	T_i
s^2	1	1
s^1	$T_i - T$	0
s^0	1	

For stability 1st column should be positive

$T_i - T > 0$

$T_i > T$

Option (d)

12. An electromechanical closed-loop control system has the following characteristic equation; $s^3 + 6K\,s^2 + (K+2)s + 8 = 0$. Where K is the forward gain of the system. The condition for closed loop stability is:
(a) K = 0.528 (c) K = 0
(b) K = 2 (d) K = -2.258

Soln. $$s^3 + 6ks^2 + (k+2)s + 8 = 0$$

Using R – H criteria

s^3	1	$k+2$
s^2	$6k$	8
s	$\frac{6k^2+12k-8}{6k}$	0
s^0	8	

For stability, 1st column should be positive

$$\frac{6k^2+12k-8}{6k} > 0$$

$$6k^2 + 12k - 8 > 0$$

$$3k^2 + 6k - 4 > 0$$

$$k = \frac{-6 \pm \sqrt{36+48}}{6}$$

$$= \frac{-6 \pm \sqrt{84}}{6}$$

$$= \frac{-6 \pm 9.165}{6} = 0.528, -2.528$$

$$k > 0.528$$

$$k > -2.528$$

so, form the given option k = 2

Option (b)

Stability Analysis

1. If $s^3 + 3s^2 + 4s + A = 0$, then all the roots of this equation are in the left half plane provided that
   (a) A > 12
   (b) – 3 < A < 4
   (c) 0 < A < 12
   (d) 5 < A < 12

Soln. $\boldsymbol{s^3 + 3s^2 + 4s + A = 0}$

Using R – H criterion

S^3	1	4
S^2	3	A
S	$\frac{12 - A}{3}$	0
S^0	A	

For stable system, 1st column should be positive.

$\boldsymbol{A > 0}$

$\boldsymbol{\frac{12-A}{3} > 0}$

or, $\boldsymbol{12 - A > 0}$

$\boldsymbol{12 > A}$

$\boldsymbol{0 < A < 12}$

Option (c)

2. If G(s) is a stable transfer function, then $F(s) = \frac{1}{G(s)}$ is always a stable transfer function. (T/F)

Soln. **G(s) is a stable function** $= \frac{(s+Z_1)(s+Z_2)}{(s+P_1)(s+P_2)}$

$$F(s) = \frac{1}{G(s)} = \frac{(s+P_1)(s+P_2)}{(s+Z_1)(s+Z_2)}$$

The condition for stability is that none of the poles of G(s) should be on the right half of s-plane, but zeros become pole of F(s). Therefore F(s) need not be stable

3. A system described by the transfer function $H(s) = \frac{1}{s^3+\alpha s^2+Ks+3}$ is stable. The constraints on α and k are,

(a) $\alpha > 0, \alpha K < 3$
(b) $\alpha > 0, \alpha K > 3$
(c) $\alpha < 0, \alpha K > 3$
(d) $\alpha < 0, \alpha K < 3$

Soln. **Transfer function** $H(s) = \frac{1}{s^3+\alpha s^2+ks+3}$

Using R – H criterion

S^3	1	k
S^2	α	3
S	$\frac{\alpha k - 3}{\alpha}$	0
S^0	3	

For system to be stable $\alpha > 0$

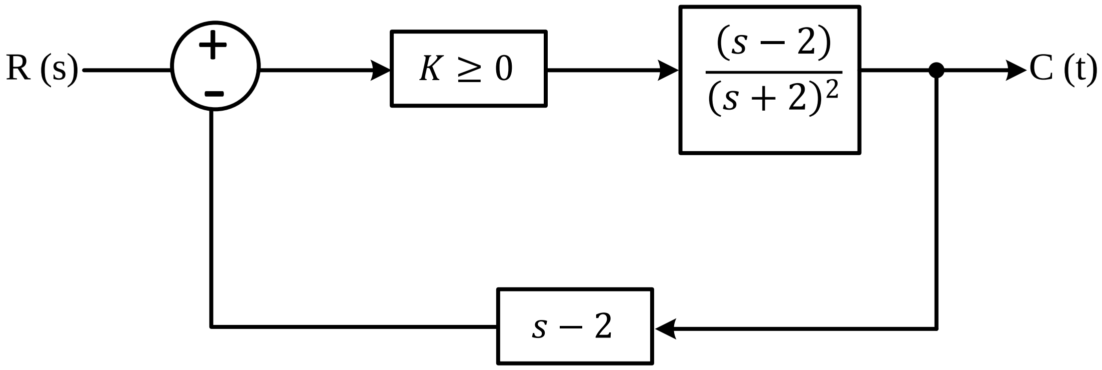
$$\frac{\alpha k-3}{\alpha} > 0$$

$$\alpha k - 3 > 0$$

$$\alpha k > 3$$

Option (b)

4. The feedback control system in the figure is stable



(a) For all $K \geq 0$
(b) only is $K \geq 0$
(c) only if $0 \leq K < 1$
(d) only if $0 \leq K \leq 1$

Soln.

$$\boldsymbol{T.F = \frac{G_1 G_2}{1 + G_1 G_2\, H}}$$

$$= \frac{\boldsymbol{k(s-2)/(s+2)^2}}{\boldsymbol{1 + \frac{k\,(s-2)\,(s-2)}{(s+2)^2}}}$$

$$= \frac{\boldsymbol{k(s-2)}}{\boldsymbol{(s+2)^2 + k(s-2)^2}}$$

Characteristic equation $= \boldsymbol{s^2 + 4 + 4s + ks^2 - 4ks + 4k}$

$\boldsymbol{(1+k)s^2 + 4s - 4ks + (4+4k) = 0}$

Or, $\boldsymbol{s^2(1+k) + s(4-4k) + (4+4k) = 0}$

Using R – H criterion

S^2	$(1+k)$	$4+4k$
S	$(4-4k)$	0
S^0	$4+4k$	

For the system to stable $\boldsymbol{4 - 4k > 0}$

Or, $\boldsymbol{1 - k > 0}$

$$\mathbf{1 > k}$$

$$\textbf{Or,} \quad \mathbf{k < 1}$$

$$\mathbf{1 + k > 0}$$

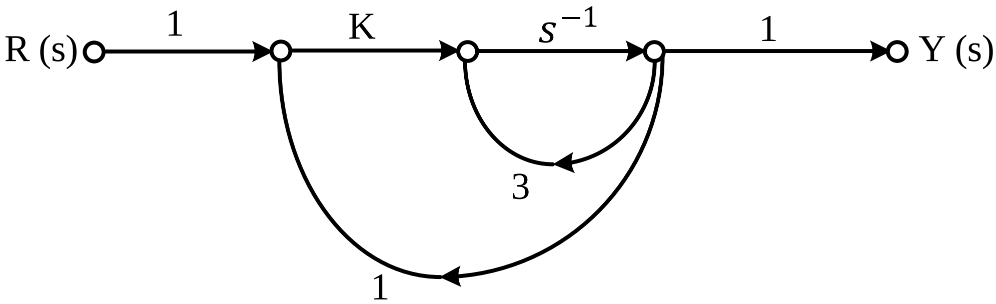
$$\mathbf{k > -1}$$

Since it is given that $k \geq 0$ hence range of k for stability is

$$\mathbf{0 \leq k < 1}$$

Option (c)

5. The system shown in the figure remains stable when



(a) $K < -1$
(b) $-1 < K < 1$
(c) $1 < K < 3$
(d) $K < -3$

Soln.

$$\frac{Y(s)}{R(s)} = \frac{\frac{k}{s}}{1 - \left(\frac{3}{s} + \frac{k}{s}\right)}$$

$$= \frac{k}{s - (3 + k)}$$

For system to stable

$$\mathbf{3 + k < 0}$$

$$\mathbf{k < -3}$$

Option (d)

6. The characteristic polynomial of system is $q(s) = 2s^5 + s^4 + 4s^3 + 2s^2 + 2s + 1$. The system is
(a) stable (c) unstable
(b) marginally stable (d) oscillatory

Soln. $\boldsymbol{q(s) = 2s^5 + s^4 + 4s^3 + 2s^2 + 2s + 1}$

Routh table is

S^5	2	4	2
S^4	1	2	1
S^3	0	0	
S^2			
S^2			
S^1			
S^0			

The row with all zeros indicate the possibility of roots on imaginary axis.

The auxiliary polynomial is

$$\boldsymbol{s^4 + 2s^2 + 1 = 0}$$

$$\boldsymbol{(s^2 + 1)^2 = 0}$$

$$\boldsymbol{s = \pm j, s = \pm j}$$

$$\boldsymbol{\frac{d}{ds}(s^4 + 2s^2 + 1) = 0}$$

$$\boldsymbol{4s^3 + 4s = 0}$$

$$\boldsymbol{s(4s^2 + 4) = 0}$$

$$\boldsymbol{s = 0,\ s = \pm j}$$

The roots are $\boldsymbol{s = 0,\ s = \pm j,\ s = \pm j}$

The system is unstable, because of repeated roots on imaginary axis

Option (c)

7. The system with the open loop transfer function $G(s)H(s) = \frac{1}{s(s^2+s+1)}$ has a gain margin of

(a) – 6 dB
(b) 0 dB
(c) 3.5 dB
(d) 6 dB

Soln. Open loop transfer function

$$G(s)H(s) = \frac{1}{s(s^2+s+1)}$$

$$\angle G(j\omega)\, H(j\omega) = -90^0 - \tan\frac{-\omega}{1-\omega^2}$$

$$at \quad \omega = \omega_{pc}, \qquad \varphi = -180^0$$

$$so, \quad -180^0 = -90^0 - \tan\frac{-\omega_{pc}}{1-\omega_{pc}^2}$$

$$90^0 = \tan\frac{-\omega}{1-\omega_{pc}^2}$$

$$1 - \omega_{pc}^2 = 0$$

$$\omega_{pc} = 1$$

Value of gain at $\omega_{pc} = 1$

$$|G(s)H(s)| = M = \frac{1}{|\omega_{pc}|\sqrt{\left(1-\omega_{pc}^2\right)^2 + \omega_{pc}^2}}$$

= 1

$$G.M = \frac{1}{M} = 1$$

$$G.M_{db} = 20log_{10}^{1} = 0$$

Option (b)

8. The open-loop transfer function of a unity feedback system is $G(s) = \frac{K}{s(s^2+s+2)(s+3)}$. The range of K for which the system is stable is

(a) $\frac{21}{4} > K > 0$
(b) $13 > K > 0$
(c) $\frac{21}{4} < K < \infty$
(d) $-6 < K < \infty$

Soln.

$$G(s) = \frac{k}{s(s^2+s+2)(s+3)}$$

$$H(s) = 1$$

$$1 + G(s)H(s) = 1 + \frac{k}{s(s^2+s+2)(s+3)}$$

$$= 1 + \frac{k}{s(s^3+3s^2+s^2+3s+2s+6)}$$

$$= \frac{s^4+4s^3+5s^2+6s+k}{s^4+4s^3+5s^2+6s} = 0$$

Or $s^4+4s^3+5s^2+6s+k = 0$

S^4	1	5	k
S^3	4	6	0
S^2	$\frac{7}{2}$	k	0
S	$\frac{21-4k}{7/2}$	0	
S^0	k		

for the system to be stable $k > 0$

$$(21-4k)\frac{2}{7} > 0$$

$$\frac{\mathbf{24}}{\mathbf{4}} > \boldsymbol{k} \rightarrow \boldsymbol{k} < \frac{\mathbf{21}}{\mathbf{4}}$$

$$\frac{\mathbf{21}}{\mathbf{4}} > \boldsymbol{k} > \mathbf{0}$$

Option (a)

9. For the polynomial $P(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15$, the number of roots which lie in the right half of the s-plane is

(a) 4 (c) 3
(b) 2 (d) 1

Soln. $\boldsymbol{P(s) = s^5 + s^4 + 2s^3 + 2s^2 + 3s + 15}$

S^5	1	2	3
S^4	1	2	15
S^3	0	-12	0
S^2	$\frac{2\in +12}{\in}$	15	
S	$\frac{-12\left(\frac{2\in +12}{\in}\right) - 15\in}{\left(\frac{2\in +12}{\in}\right)}$		
S^0	15		

$\in$ **be a small positive number**

$$\frac{\mathbf{2\in +12}}{\in} = \pm$$

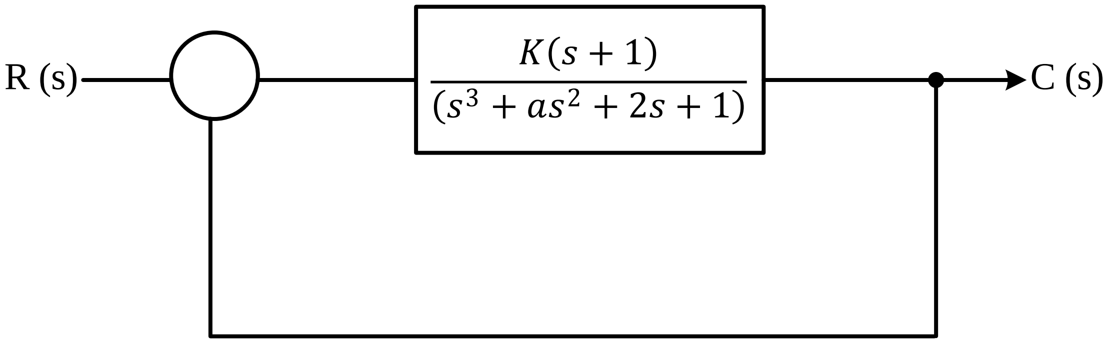
$$s = -12 - \frac{15 \in}{t}$$

$$s^0 = 15$$

The number of sing changes in first column from $s^2 to\ s$ and from $s\ to\ s^2$ are two. Two roots on right half of s – plane

Option (b)

10. The positive values of “K” and “a” so that the system shown in the figure below oscillates at a frequency of 2 rad/sec respectively are



(a) 1, 0.75
(b) 2, 0.75
(c) 1, 1
(d) 2, 2

Soln.

$$1 + G(s)H(s) = 1 + \frac{k(s+1)}{s^3 + as^2 + 2s + 1} = 0$$

$$\frac{s^3 + as^2 + 2s + 1 + ks + k}{s^3 + as^2 + 2s + 1} = 0$$

$$s^3 + as^2 + (2+k)s + k + 1 = 0$$

S^3	1	$(2+k)$
S^2	a	$(k+1)$
S	$\frac{a(2+k)-(k+1)}{a}$	0

For oscillations,

$$\frac{a(2+k)-(k+1)}{a} = 0$$

$$a = \frac{k+1}{2+k}$$

The auxiliary polynomial is

$$as^2 + k + 1 = 0$$

$$s = j\omega, \quad s^2 = -\omega^2 = -4 \qquad \text{given } \omega = 2 \text{ rad/sec}$$

So, $-4a + k + 1 = 0$

$$a = \frac{k+1}{4} = \frac{k+1}{k+2}$$

$$k = 2, \quad a = 0.75$$

Option (b)

11. Consider a unity gain feedback control system whose open loop transfer function is $G(s) = \frac{as+1}{s^2}$. The value of “a” so that the system has a phase-margin equal to $\pi/4$ is approximately equal to

(a) 2.40 (c) 0.84
(b) 1.40 (d) 0.74

Soln.

$$G(s) = \frac{as+1}{s^2}$$

$$\angle G(j\omega) = tan^{-1} a\omega - 180^0$$

Phase margin $= \frac{\pi}{4}$

$$PM = 180 + \tan^{-} a\omega - 180^0 = \frac{\pi}{4}$$

$$tan^{-} a\omega = \frac{\pi}{4}$$

$$\tan\frac{\pi}{4} = a\omega$$

$$a\omega = 1$$

Then gain crossover frequency $\omega = \omega_{gc}$ $\quad$ *where* $|G(s)| = 1$

$$\sqrt{\frac{1+a^2\omega^2}{\omega^2}} = 1 \qquad a\omega = 1$$

$$\sqrt{\frac{1+1}{\omega^2}} = 1$$

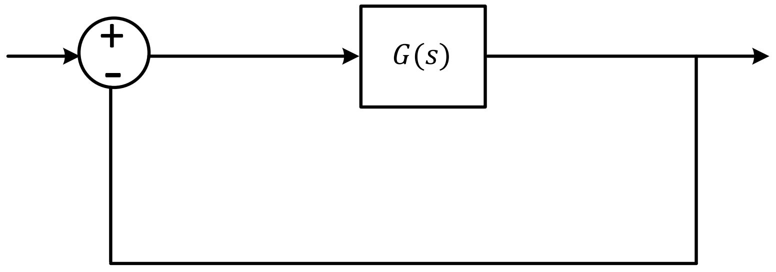
$$\omega^2 = \sqrt{2}$$

$$\omega = 2^{\frac{1}{4}}$$

$$a = \frac{1}{2^{1/4}} = 0.84$$

Option (c)

12. A certain system has transfer function $G(s) = \frac{s+8}{s^2+\alpha s-4}$, where α is parameter. Consider the standard negative unity feedback configuration as shown below



Which of the following statements is true?

(a) The closed loop system is never stable for any value of α

(b) For some positive value of α, the closed loop system is stable, but not for all positive values

(c) For all positive value of α, the closed loop system is stable

(d) The closed loop system is stable for all value of α, both positive and negative

Soln.

$$G(s) = \frac{s+8}{s^2 + \alpha s - 4}$$

Closed loop gain is $\frac{G(s)}{1+G(s)}$

$$\frac{G(s)}{1+G(s)} = \frac{s+8}{s^2+\alpha s-4+s+8}$$

$$= \frac{s+8}{s^2+(\alpha+1)s+4}$$

Characteristic equation

$$q(s) = s^2 + (\alpha + 1)s + 4$$

S^2	1	4
S	$\alpha + 1$	0
S^0	4	

The closed loop system is stable for $\alpha + 1 > 0$

$$\alpha > -1$$

For all positive value of α, the closed loop system is stable

Option (c)

13. The number of open right half plane poles of $G(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$ *is*

(a) 0
(b) 1
(c) 2
(d) 3

Soln.

$$G(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$$

Using R – H criteria

S^5	1	3	5
S^4	2	6	3
S^3	$0(\in)$	$\frac{7}{2}$	
S^2	$\frac{6\in -7}{\in} = \frac{7}{\in}$	3	
S^1	$\left\{\frac{-\frac{7}{\in}\left(\frac{7}{2}\right) - 3\in}{-7/\in}\right\} + ve$		
S^0	3		

In the first column, there are two sign changes occurs hence two poles lie in the right half of s – plane

Option (c)