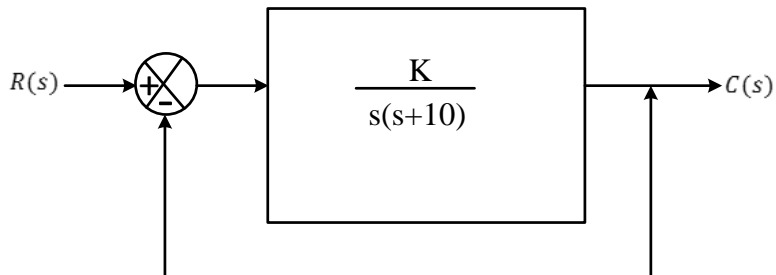


CONTROL SYSTEMS UNIT-2

Time Response Analysis

Questions with answers(2 marks)

1. The unity feedback system shown in figure has:



- (a) Zero steady state position error
- (b) Zero steady state velocity error
- (c) Steady state position error $K/10$ units
- (d) Steady state velocity error $K/10$ units

Soln.
$$G(s) = \frac{k}{s(s+10)}$$

$$H(s) = 1$$

Position error coefficient
$$k_p = \lim_{s \rightarrow 0} G(s) H(s)$$

$$k_p = \lim_{s \rightarrow 0} \frac{k}{s(s+10)} = \infty$$

Steady state error
$$e_{ss} = \frac{1}{1+k_p} = 0$$

Option (a)

2. The steady state error of a stable 'type 0' unity feedback system for a unit step function is

- (a) 0
- (b) $\frac{1}{1+K_P}$
- (c) ∞
- (d) $\frac{1}{K_P}$

4. The step error coefficient of a system $G(s) = \frac{1}{(s+6)(s+1)}$ with unity feedback is
- (a) 1/6 (c) 0
(b) ∞ (d) 1

Soln. Step error coefficient $k_p = \lim_{s \rightarrow 0} G(s) H(s)$

$$= \lim_{s \rightarrow 0} \frac{1}{(s+6)(s+1)} = \frac{1}{6}$$

Option (a)

5. Consider a unity feedback control system with open-loop transfer function $G(s) = \frac{K}{s(s+1)}$ the steady state error of the system due to a unit step input is
- (a) zero (c) 1/K
(b) K (d) infinite

Soln. Step error coefficient $k_p = \lim_{s \rightarrow 0} G(s) H(s)$

$$= \lim_{s \rightarrow 0} \frac{k}{s(s+1)}$$
$$= \infty$$

$$\text{Steady state error} = \frac{A}{1+k_p} = \frac{A}{1+\infty} = 0$$

Option (a)

6. For a second-order system with the closed-loop transfer function $T(s) = \frac{9}{s^2+4s+9}$ the settling time for 2-percent band, in seconds, is

- (a) 1.5
- (b) 2.0

- (c) 3.0
- (d) 4.0

Soln. Closed loop transfer function $T(s) = \frac{9}{s^2+4s+9}$ comparing with 2nd order equation $s^2 + 2\xi \omega_n s + \omega_n^2$

$$2\xi\omega_n = 4$$

$$\xi\omega_n = 2$$

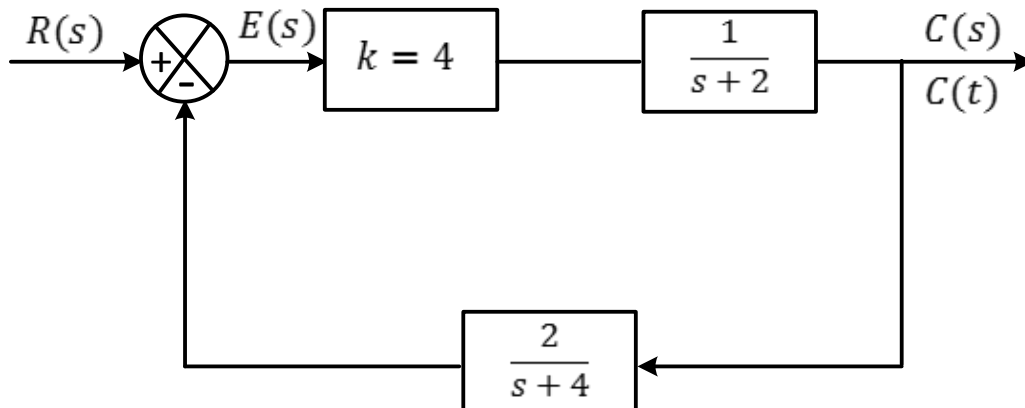
Time constant $T = \frac{1}{\xi\omega_n}$

Settling time for 2% band is $t_s = 4T = \frac{4}{\xi\omega_n} = \frac{4}{2} = 2$

Option (b)

7. The steady state error of the system shown in the figure of a unit step input is _____

Soln.



$$G(s) = \frac{4}{s+2} \quad , \quad H(s) = \frac{2}{s+4}$$

$$\begin{aligned} \text{For unit step input } k_p &= \lim_{s \rightarrow 0} \left(\frac{4}{s+2} \right) \left(\frac{2}{s+4} \right) \\ &= \frac{4}{2} \times \frac{2}{4} = \mathbf{1} \end{aligned}$$

$$\text{Steady state error } e_{ss} = \frac{1}{1+k_p} = \frac{1}{2} = \mathbf{0.50}$$

Time Response Analysis (Part – II)

1. A critically damped, continuous-time, second order system, when sampled, will have (in Z domain)
- (a) A simple pole
 - (b) Double pole on real axis
 - (c) Double pole on imaginary axis
 - (d) A pair of complex conjugate poles

Soln. For critically damped continuous time second order system roots of denominator are: $s^2 + 2 \xi \omega_n s + \omega_n^2 = 0$ when $\xi = 1, s^2 + 2\omega_n s + \omega_n^2 = 0$

$$(s + \omega_n)^2 = 0$$

Root are $s = -\omega_n, -\omega_n$

Double pole on real axis

Option (b)

2. A second-order system has transfer function given by $G(s) = \frac{25}{s^2+8s+25}$. If the system, initially at rest, is subjected to a unit step input at $t = 0$, the second peak in the response will occur at
- (a) π sec
 - (b) $\pi/3$ sec
 - (c) $2\pi/3$ sec
 - (d) $\pi/2$ sec

Soln. $G(s) = \frac{25}{s^2+8s+25}$

$$\omega_n^2 = 25$$

$$\omega_n = 5$$

$$2 \xi \omega_n = 8$$

$$\xi = \frac{8}{10} = 0.8$$

$$\begin{aligned}\omega_d &= \omega_n \sqrt{1 - \xi^2} \\ &= 5 \sqrt{1 - (0.8)^2} = 3\end{aligned}$$

for 2nd peak $n = 3$

$$t_p = \frac{n\pi}{\omega_d} = \frac{3\pi}{3} = \pi \text{ sec}$$

Option (a)

3. The poles of a continuous time oscillator are

Soln. The poles of a continuous time oscillator are imaginary

4. The response of an LCR circuit to a step input is

- (a) Over damped
- (b) Critically damped
- (c) Oscillatory

If the transfer function has

- (i) poles on the negative real axis
- (ii) poles on the imaginary axis
- (iii) multiple poles on the positive real axis
- (iv) poles on the positive real axis
- (v) multiple poles on the negative real axis

Soln. The response of an LCR circuit to a step input

(a) over damped case $\xi > (1)$ *poles on the negative real axis*

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

Roots of denominator are $s = -\xi \omega_n \pm \omega_n \sqrt{\xi^2 - 1}$

(b) critically damped $\xi = 1$ — — — — — (5)

$$\begin{aligned}\frac{C(s)}{R(s)} &= \frac{\omega_n^2}{s^2 + 2\omega_n s + \omega_n^2} \\ &= \frac{\omega_n^2}{(s + \omega_n)^2}, \quad s = -\omega_n, -\omega_n\end{aligned}$$

It has multiple poles on the negative real axis

(c) oscillatory – (2) poles on the imaginary axis

$$\begin{aligned}\xi &= 0 \\ s &= \pm j\omega_n\end{aligned}$$

Option a – 1, b – 5, c – 2

5. Match the following codes with List-I with List-II:

List – I

- (a) Very low response at very high frequencies
- (b) Over shoot
- (c) Synchro-control transformer output

List – II

- | | |
|-------------------------|---------------------------------|
| (i) Low pass systems | (iv) Phase-sensitive modulation |
| (ii) Velocity damping | (v) Damping ratio |
| (iii) Natural frequency | |

Soln. (a) very low response at very high frequencies → low pass system

(b) over sheet → Damping ratio

(c) synchro – control transformer output → phase sensitive modulation

Option: a – 1, b – 5, c – 4

8. The final value theorem is used to find the
- (a) steady state value of the system output
 - (b) initial value of the system output
 - (c) transient behavior of the system output
 - (d) none of these

Soln. The final value theorem is used to find the steady state value of the system.

Option (a)

9. If $F(s) = \frac{\omega}{s^2 + \omega^2}$, then the value of $\lim_{t \rightarrow \infty} f(t)$,
{where $F(s)$ is the $L[f(t)]$ }
- (a) cannot be determined
 - (b) is zero
 - (c) is unity
 - (d) is infinite

Soln. $F(s) = \frac{\omega}{s^2 + \omega^2}$

$F(s) = \frac{\omega}{s^2 + \omega^2}$ has poles $s \pm j\omega$ (pure imaginary) it is oscillatory function hence final value $\lim_{t \rightarrow \infty} f(t)$ can not be determined

10. Consider a feedback control system with loop transfer function

$$G(s)H(s) = \frac{K(1 + 0.5s)}{s(1 + s)(1 + 2s)}$$

The type of the closed loop system is

- (a) zero
- (b) one
- (c) two
- (d) three

Soln. $G(s)H(s) = \frac{k(1+0.5s)}{s(1+s)(1+2s)}$

The number of poles at the origin of open loop transfer function given the type of the system. It is a type one system

Option (b)

11. The unit impulse response of a linear time invariant system is the unit step function $u(t)$. For $t > 0$, the response of the system to an excitation

$e^{-at}u(t)$, $a > 0$ will be

(a) ae^{-at}

(c) $a(1 - e^{-at})$

(b) $(1/a)(1 - e^{-at})$

(d) $1 - e^{-at}$

Soln. $H(s) = \frac{1}{s}$

System excitation $r(t) = e^{-at} u(t)$

$$R(s) = \frac{1}{s+a}$$

Response of the system $C(s) = R(s) H(s) = \frac{1}{s(s+a)}$

$$C(s) = \frac{1}{a} \left[\frac{1}{s} - \frac{1}{(s+a)} \right]$$

$$C(t) = \frac{1}{a} [1 - e^{-at}]$$

Option (b)

12. If the characteristic equation of a closed-loop system is $s^2 + 2s + 2 = 0$, then the system is

(a) Overdamped

(c) Under damped

(b) Critically damped

(d) undamped

Soln. Characteristic equation of closed loop system is $s^2 + 2s + 2 = 0$

Comparing with 2nd order equation

$$s^2 + 2 \xi \omega_n s + \omega_n^2 = 0$$

$$2 \xi \omega_n = 2$$

$$\xi = \frac{1}{\omega_n}$$

$$\omega_n^2 = 2, \quad \omega_n = \sqrt{2}, \quad \xi = \frac{1}{\sqrt{2}} \text{ which is less than 1}$$

It is case of under damped

Option (c)

13. Consider a system with the transfer function $G(s) = \frac{s+6}{Ks^2+s+6}$. Its damping ratio will be 0.5 when the value of K is

(a) 2/6

(c) 1/6

(b) 3

(d) 6

Soln. Transfer function $G(s) = \frac{s+6}{ks^2+s+6}$

Damping ratio $\xi = 0.5$

Comparing with 2nd order equation $s^2 + 2 \xi \omega_n s + \omega_n^2$

$$G(s) = \frac{s+6}{k[s^2 + \frac{s}{k} + \frac{6}{k}]}$$

$$\omega_n^2 = \frac{6}{k}, \quad \omega_n = \sqrt{\frac{6}{k}}$$

$$2 \xi \omega_n = \frac{1}{k}$$

$$5[1 - e^{-2t}] = \frac{5 \times 99}{100}$$

$$5(1 - e^{-2t}) = 5 \times 0.99$$

$$1 - e^{-2t} = 0.99$$

$$\text{Or } e^{-2t} = 0.1$$

$$-2t = \ln(0.1)$$

$$t = 2.3 \text{ sec}$$

Option (c)

15. The transfer function of a plant is $T(s) = \frac{5}{(s+5)(s^2+s+1)}$. The second order approximation of $T(s)$ using dominant pole concept is

(a) $\frac{1}{(s+5)(s+1)}$

(c) $\frac{5}{(s^2+s+1)}$

(b) $\frac{5}{(s+5)(s+1)}$

(d) $\frac{1}{(s^2+s+1)}$

Soln. Transfer function $= \frac{5}{(s+5)(s^2+s+1)}$

$$= \frac{5}{5\left(\frac{s}{5}+1\right)(s^2+s+1)}$$

The poles nearer to imaginary axis dominates nature of time response and are called dominant poles. The factor that has to be eliminated should be in time constant form so, $T(s) = \frac{1}{s^2+s+1}$

Option (d)

16. Group I lists a set of four transfer functions.

Group II gives a list of possible step responses $y(t)$. Match the step responses with the corresponding transfer functions.

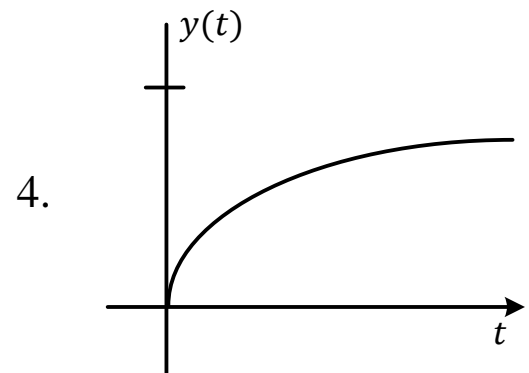
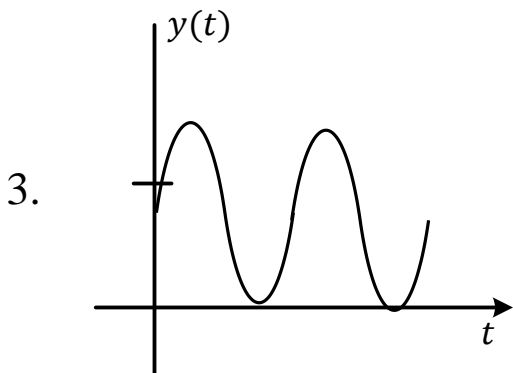
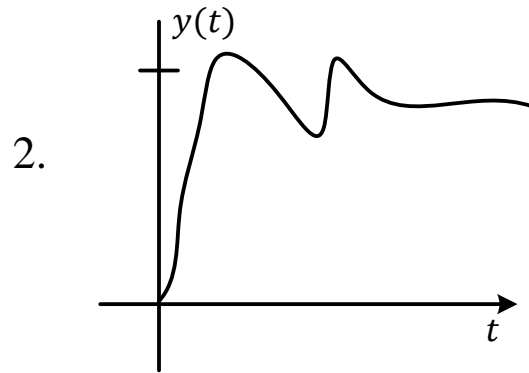
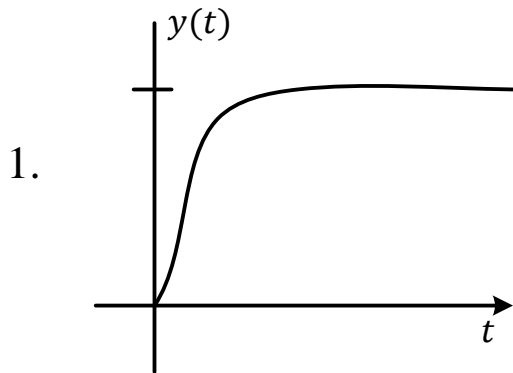
Group I

$$P = \frac{25}{s^2 + 25}$$

$$R = \frac{36}{s^2 + 12s + 36}$$

$$Q = \frac{36}{s^2 + 20s + 36}$$

$$S = \frac{36}{s^2 + 7s + 49}$$



- (a) P – 3, Q – 1, R – 4, S – 2
- (b) P – 3, Q – 2, R – 4, S – 1
- (c) P – 2, Q – 1, R – 4, S – 3
- (d) P – 3, Q – 4, R – 1, S – 2

Soln. The figure 1 is a case of critically damped, Figure 2 underdamped, Figure 3 undamped, figure 4 is overdamped

The given transfer function

$$P = \frac{25}{s^2 + 25} \qquad R = \frac{36}{s^2 + 12s + 36}$$

$$Q = \frac{36}{s^2 + 20s + 36} \qquad S = \frac{36}{s^2 + 7s + 49}$$

Comparing the given transfer function with

$$\frac{\omega_n^2}{s^2 + 2 \xi \omega_n s + \omega_n^2}, \quad P = \frac{25}{s^2 + 25}$$

$$\omega_n = 5, \quad \xi = 0$$

P is undamped, matched to figure 3 so P – 3

$$Q = \frac{36}{s^2 + 20s + 36}, \quad \omega_n = 6$$

$$2 \xi \omega_n = 20$$

$$\xi = \frac{20}{2 \times 6} = 1.67$$

Q is overdamped matched to fig. 4 Q – 4

$$R = \frac{36}{s^2 + 12s + 36}, \quad \omega_n = 6$$

$$2 \xi \omega_n = 12$$

$$\xi = \frac{12}{2 \times 6}$$

$$= 1$$

R is critically damped matched to fig 1, R – 1

$$S = \frac{49}{s^2+7s+49}, \quad \omega_n = 7$$

$$2 \xi \omega_n = 7$$

$$\xi = \frac{7}{2 \times 7} = 0.5$$

$$= 1$$

S is underdamped matched to fig 2

Option (d) P – 3, Q – 4, R – 1, S – 2

17. The unit step response of an under-damped second order system has steady state value of -2 . Which one of the following transfer functions has these properties?

(a) $\frac{-2.24}{s^2+2.59s+1.12}$

(c) $\frac{-2.24}{s^2-2.59s+1.12}$

(b) $\frac{-3.82}{s^2+1.91s+1.91}$

(d) $\frac{-3.82}{s^2-1.91s+1.91}$

Soln. Steady state value = - 2

$$\text{The 2}^{\text{nd}} \text{ order } T/F = \frac{\omega_n^2}{s^2+2 \xi \omega_n s+\omega_n^2}$$

For underdamped system $\xi < 1$

Option (c) and (d) are wrong because the system is unstable as ξ is negative

In option (a) $\omega_n = \sqrt{1.12} = 1.06$

$$2 \xi \omega_n = 2.59$$

$$\xi = \frac{2.59}{2 \times 1.06} = 1.22$$

$\xi > 1$ hence it is wrong

In option (b) $T.F = \frac{-3.82}{s^2+1.91s+1.91}$

$$\omega_n = \sqrt{1.91} = 1.38$$

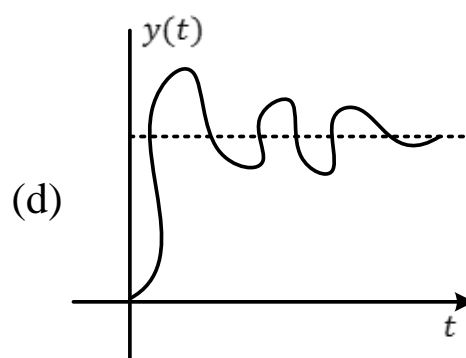
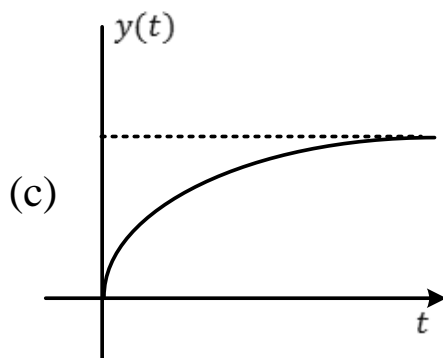
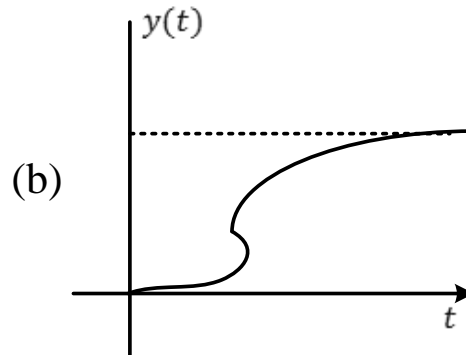
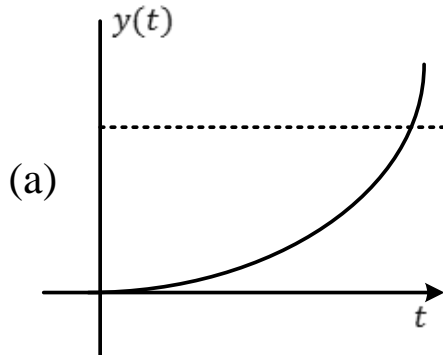
$$2 \xi \omega_n = 1.91$$

$$\xi = \frac{1.91}{2 \times 1.38} = 0.69$$

$\xi < 1$ so system is underdamped

Option (b)

18. The differential equation $100 \frac{d^2y}{dt^2} - 20 \frac{dy}{dt} + y = x(t)$ describes a system with an input $x(t)$ and an output $y(t)$. The system, which is initially relaxed, is excited, by a unit step input. The output $y(t)$ can be represented by the waveform.



Soln. $100 \frac{d^2y}{dt^2} - 20 \frac{dy}{dt} + y = x(t)$

Taking Laplace transform of both sides

$$100 s^2 y(s) - 20 s y(s) + y(s) = X(s) = \frac{1}{s}$$

$$(100s^2 - 20s + 1)y(s) = \frac{1}{s}$$

$$y(s) = \frac{1}{s(10s-1)^2} \quad \text{Poles are at } s = 0, \frac{1}{10}, \frac{1}{10}$$

Poles are on the right hand side of s plane so given system is unstable

Option (a) represents unstable system.

19. The open-loop transfer function of a dc motor is given as

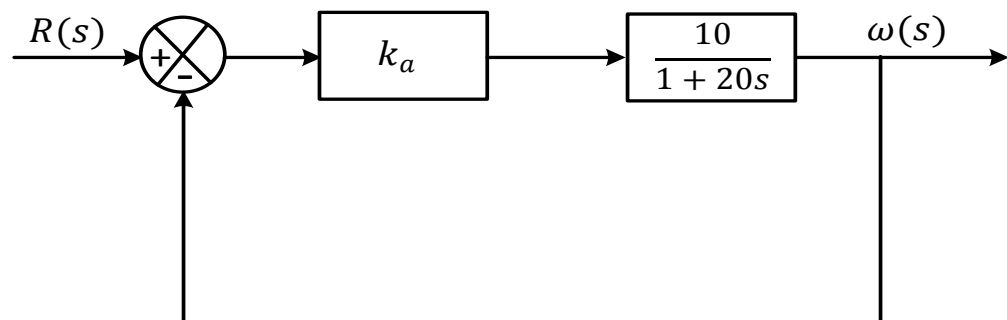
$$\frac{\omega(s)}{V_a(s)} = \frac{10}{1 + 10s}$$

when connected in feedback as shown below, the approximate value of K_a that will reduce the time constant of the closed loop system by one hundred times as compared to that of the open-loop system is

- | | |
|-------|---------|
| (a) 1 | (c) 10 |
| (b) 5 | (d) 100 |

Soln. The open loop transfer function of a dc motor is given as $\frac{\omega(s)}{V_a(s)} = \frac{10}{1+10s}$

When connected in feedback is shown below.



$$\text{Open loop transfer function} = \frac{10 k_a}{1+10 s}$$

$$G(s) = \frac{k_a}{s+\frac{1}{10}}$$

$$g(t) = e^{-\frac{t}{10}}$$

Loop time constant $\tau_{ol} = 10$

$$\text{Closed loop transfer function} = H(s) = \frac{G(s)}{1+G(s)}$$

$$H(s) = \frac{10k_a/1+10s}{1+(10k_a)/1+10s} = \frac{10k_a}{1+10s+10k_a}$$

$$H(s) = \frac{10k_a}{1+10s+10k_a} = \frac{k_a}{s+(k_a+\frac{1}{10})}$$

$$h(t) = k_a e^{-\left(\frac{1}{k_a+0.1}\right)t}$$

$$\text{Time constant of closed loop system} = \frac{1}{\left(k_a+\frac{1}{10}\right)}$$

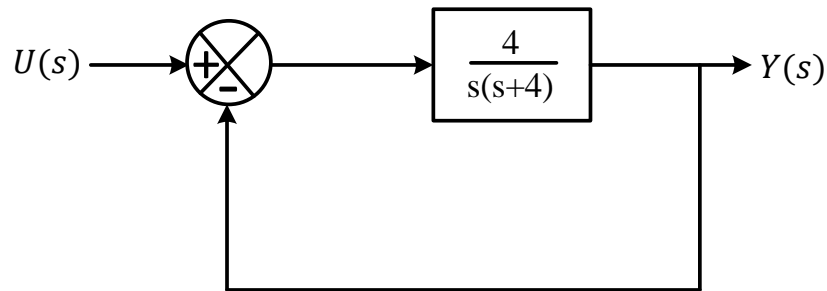
$$\tau_{cl} = \frac{\tau_{ol}}{100} = \frac{1}{k_a+0.1}$$

$$= \frac{10}{100} = \frac{1}{k_a+0.1}$$

$$\text{or } k_a = 10 - 0.1 = 9.9 \cong 10$$

Option (c)

20. For the second order closed-loop system shown in the figure, the natural frequency (in rad/s) is



- (a) 16
- (b) 4

- (c) 2
- (d) 1

Soln.

$$\frac{Y(s)}{U(s)} = \frac{4/s(s+4)}{1+4/s(s+4)}$$
$$= \frac{4}{s(s+4)+4}$$
$$= \frac{4}{s^2+4s+4}$$

Comparing with 2nd order system transfer

Function $\frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$

$$\omega_n^2 = 4, \quad \text{or} \quad \omega_n = 2$$

Option (c)