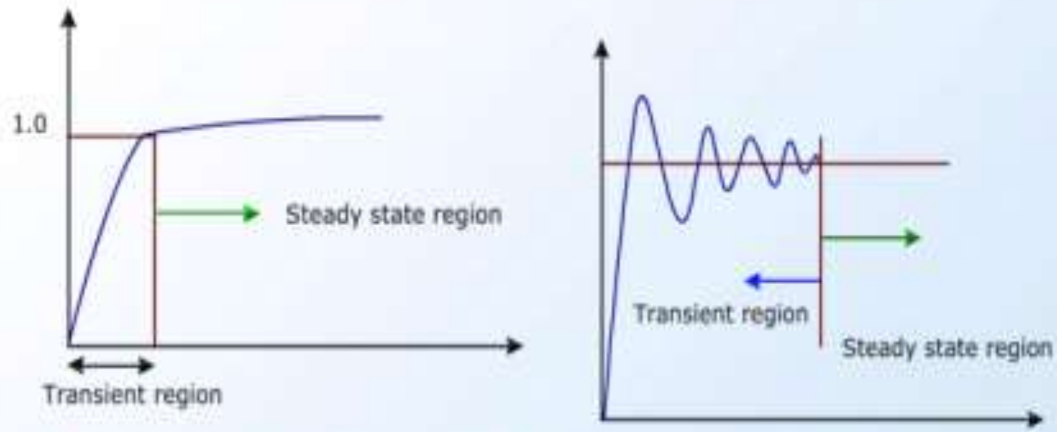
CONTROL SYSTEMS

UNIT-II

TIME RESPONSE ANALYSIS

Introduction

- The time response of a dynamic system provides information about how the system responds to certain inputs and it determines the stability of the system and the performance of the controller
- The Time Response of a control system consists of two parts:
  - Transient Response
  - Steady State Response



Transient Response and Steady State Response

Standard Test Signals

- The following are the standard test signals:
  - Step Signal
  - Ramp Signal
  - Parabolic Signal
  - Impulse Signal

Standard Test Signals

Step Signal

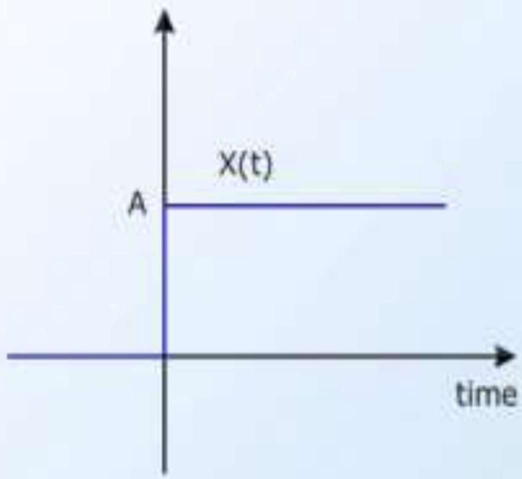
- Signal whose value changes from one level to another level (A) in zero time

$$x(t) = Au(t)$$

$$x(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

- In Laplace Transform

$$X(s) = \frac{A}{s}$$



Step Signal

Standard Test Signals

Ramp Signal

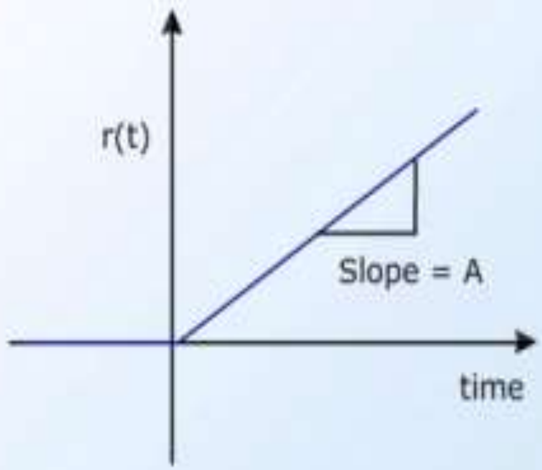
- Signal which starts at zero value and increases linearly with time

$$r(t) = \begin{cases} 0 & t < 0 \\ At & t \geq 0 \end{cases}$$

- In Laplace transform

$$R(s) = \frac{A}{s^2}$$

- Integration of step signal results in a ramp signal



Ramp Signal

Standard Test Signals

Parabolic Signal

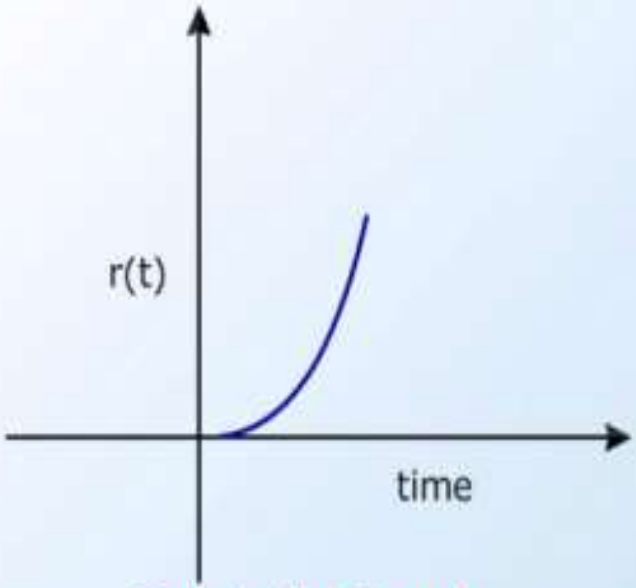
- The instantaneous value of a parabolic signal varies as square of time from an initial value of zero at t=0

$$r(t) = \begin{cases} 0 & t < 0 \\ At^2/2 & t > 0 \end{cases}$$

- In the Laplace transform

$$R(s) = \frac{A}{s^3}$$

- Integration of ramp signal results in a parabolic signal



Parabolic Signal

Standard Test Signals

Impulse Signal

- Impulse signal is otherwise called shock input that occurs for a small interval of time. Mathematically it is expressed as

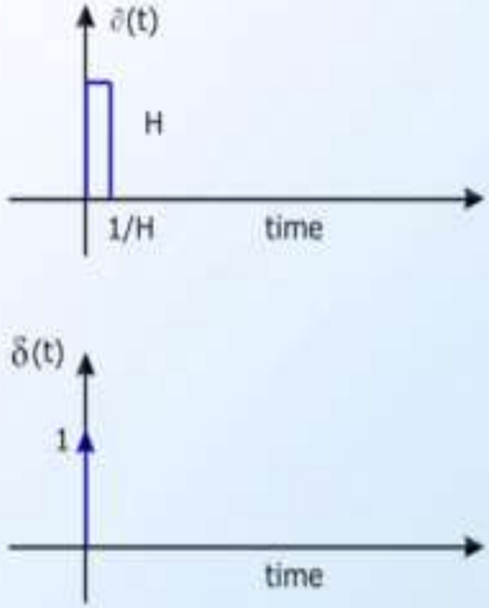
$$x(t) = \delta(t) \;\; for\, 0 < t < \frac{1}{H} \; where\; H \to \infty$$

- For Unit impulse

$$\delta(t) = 1 ; t = 0$$
$$= 0 ; t \neq 0$$

- The Laplace transform of a Unit Impulse is:

$$L[\delta(t)] = 1 = R(s)$$



Impulse Signal

Time Response of First Order Systems

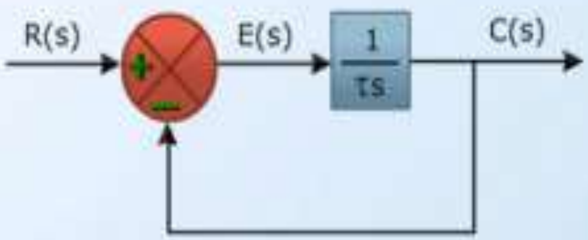
Step Response of a First Order System

- Transfer function of a first order system without zeros can be represented as:

$$\frac{C(s)}{R(s)} = \frac{1}{\tau s + 1}$$

- Given a step input i.e., R(s) = 1/s, then the system output (called step response in this case) is

$$C(s) = \frac{1}{s(\tau s + 1)}$$



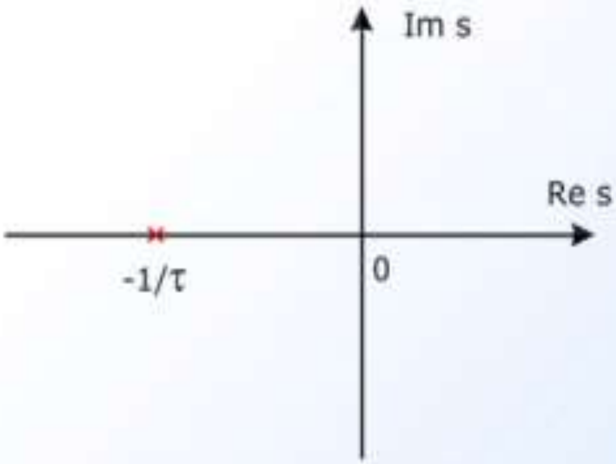
First Order System

Time Response of First Order Systems

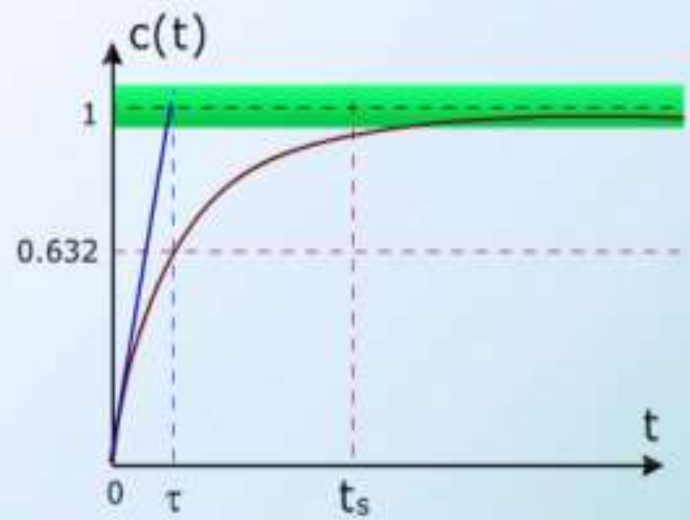
Step Response of a First Order System

- At $t = \tau$, the step response is

  $C(t) = 1 - 0.37 = 0.632$



Pole-Zero Plot of First Order System

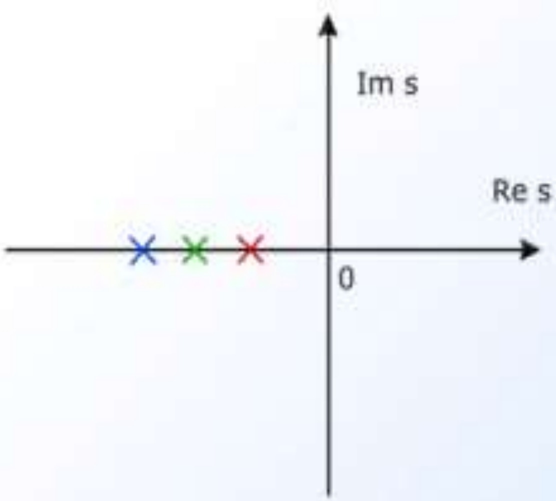


Time Response of First Order System

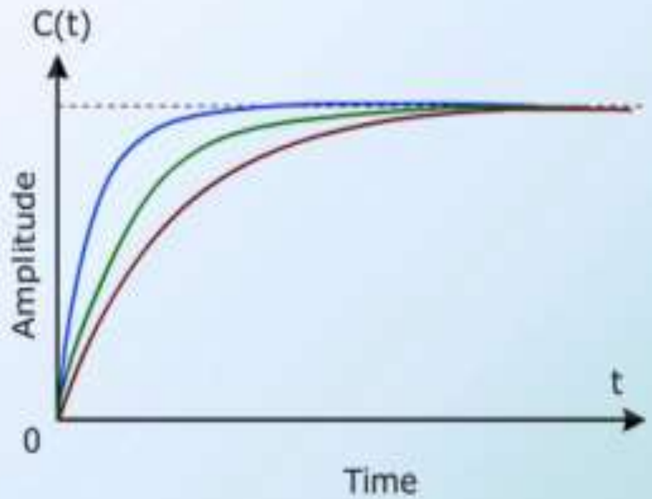
Time Response of First Order Systems

Step Response of a First Order System

- Response from any First Order System is exponential
- As the pole moves away from the imaginary axis, response becomes fast



Pole-Zero Plot of First Order System



Time Response of First Order System

Characteristic Equation of Feedback Control Systems

- A general second order system is characterized by the following transfer function:

$$\frac{C(s)}{R(s)} = \frac{b}{s^2 + as + b}$$

- We can re-write the above transfer function in the following form:

$$\frac{C(s)}{R(s)} = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Where ω_n = Undamped natural frequency

ξ = Damping ratio

- Damping ratio determines how much the system oscillates as the response decays toward steady state or it is a measure of system's ability to oppose oscillatory response
- The denominator in the transfer function of a second order system is called the **"characteristic equation of feedback control system"**

Transient Response of Second Order Systems

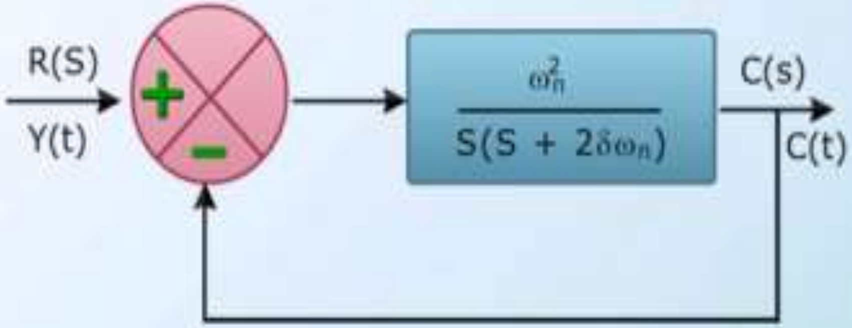
Time-response of Second-order Control System

- The order of a control system is defined as the highest derivative present in the differential equation of the system
- In the s-domain, the higher power of "s" in the characteristic equation 1 + G(s)H(s), 0 is the "order"
- Consider

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)} = \frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

$$= \frac{-2\delta\omega_n \pm \sqrt{4\delta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$S_{1,2} = -\delta\omega_n + \omega_n\sqrt{\delta^2 - 1}$$



Standard Form of Second Order System

Transient Response of Second Order Systems

Case 1: Under damped (δ <1)

- The conditional frequency the two roots are said to be "complex conjugates"

$$S_{1,2} = -\delta\omega_n \pm j\omega_n\sqrt{\delta^2 - 1}$$

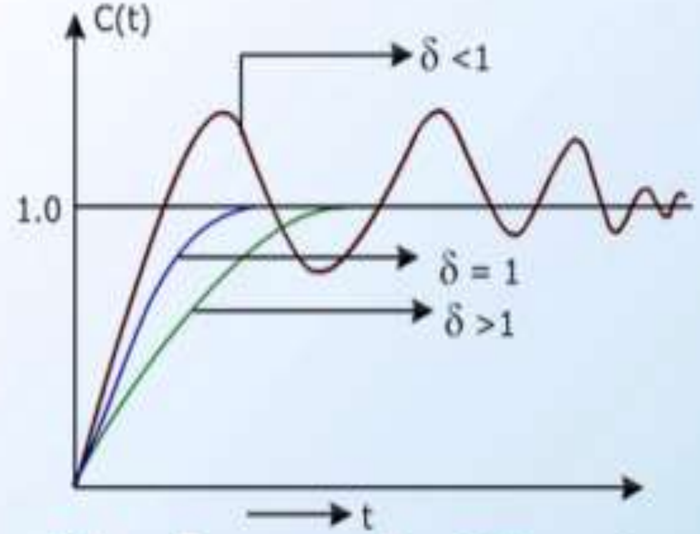
$$S_1 = -\xi\omega_n + j\omega_n\sqrt{1-\xi^2} = -\xi\omega_n + j\omega_d$$

$$S_2 = -\xi\omega_n - j\omega_n\sqrt{1-\xi^2} = -\xi\omega_n - j\omega_d$$

Case 2: Over damped (δ >1)

- The two roots are real and unequal
- The nature of the response is non-oscillatory

$$S_{1,2} = -\delta\omega_n \pm \omega_n\sqrt{\delta^2 - 1}$$



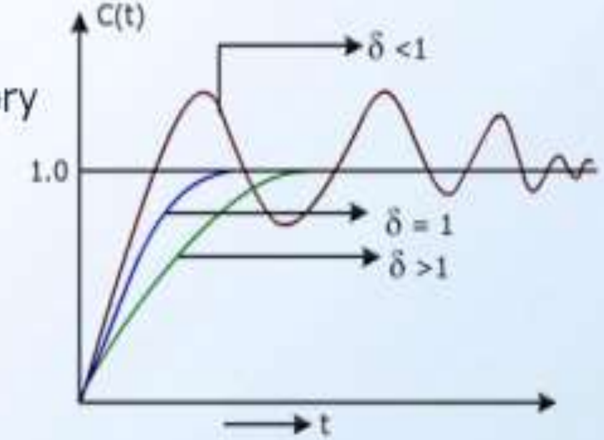
Time-Response for Different Ranges of δ

Transient Response of Second Order Systems

Case 3: Critically damped ($\delta = 1$)

- The two roots are real and equal
- The response is on the range of becoming oscillatory

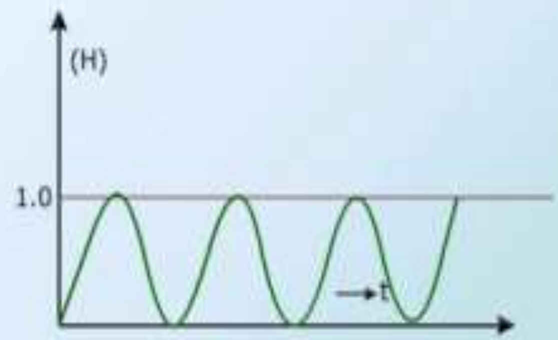
$$S_{1,2} = -\xi\omega_n$$



Time-Response for Different δ

Case 4: Undamped ($\delta = 0$)

- The response is oscillatory with a frequency of "ω_n" rad/sec
- The oscillations sustain without any change in the amplitude



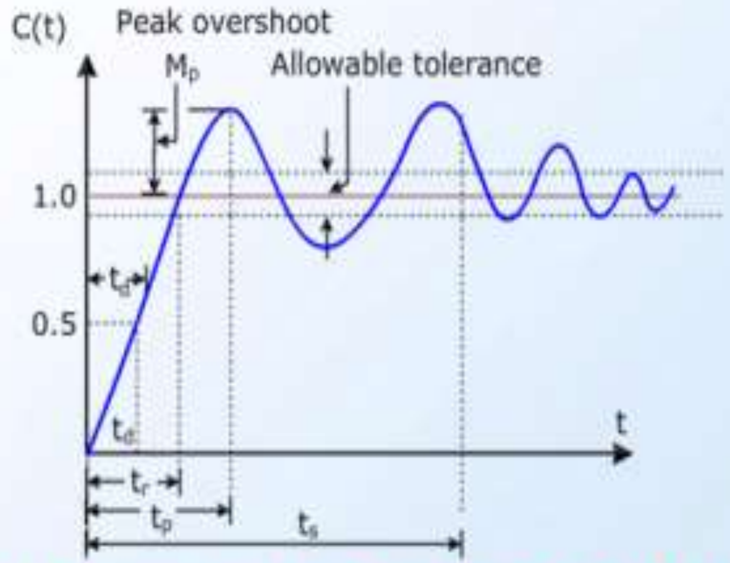
Time-Response for δ =0

Transient Response of Second Order Systems

Time-response for unit step input r(t) = u(t) and R(s)= 1/s

- Many control systems are generally under-damped in nature and their roots are "complex conjugates"

$$C(s) = R(s).\frac{\omega_n^2}{s^2 + 2\delta\omega_n s + \omega_n^2}$$

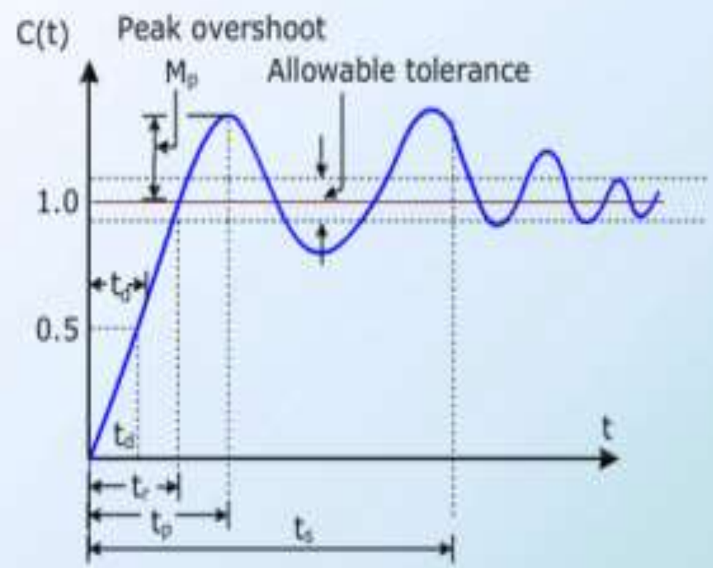


Time Response of Second-order System for the Under Damped

Time domain specifications

Definitions of Specifications

- The following are the time domain specifications:
  - Delay Time t_d
  - Rise Time t_r
  - Peak Time t_p
  - Peak Overshoot or Max Overshoot M_p
  - Settling Time t_s
  - Steady-State Error



Time domain specifications

Steady State Error

- From the block diagram:

  $E(s) = R(s) - C(s)H(s)$

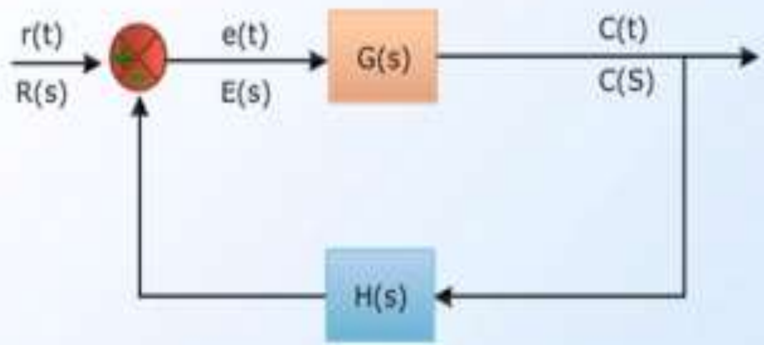
  $C(s) = E(s)\,G(s)$

  $E(s)\,[1 + G(s)H(s)] = R(s)$

  $E(s) = R(s) / [1 + G(s)H(s)]$

- E(s) is called steady state error
- Final Value Theorem:
  - The value of the time-function as $t \rightarrow \infty$ can be found readily using the final value theorem
  - It states that if f(t) is the time-function and $f(\infty)$ is its final value, it can be determined from $L\{f(t)\} = F(s)$ as below

$$\underset{t\to\infty}{Lt}\, f(t) = f(\infty) = \underset{s\to 0}{Lt}\, sF(s)$$



Simple Block Diagram

Static Error Constants

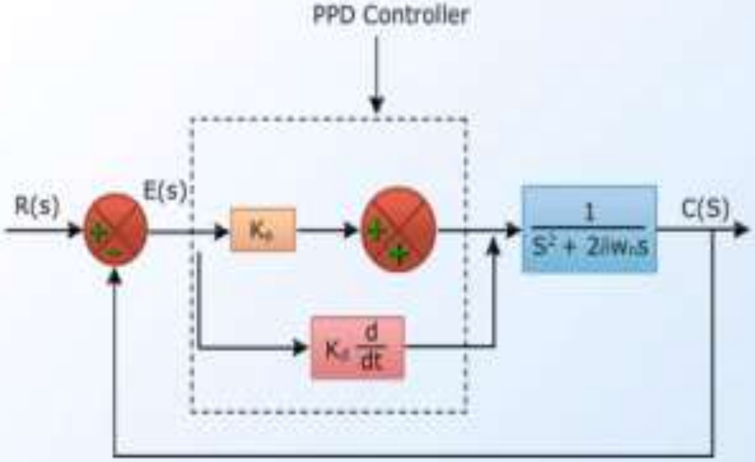
- The following table gives steady state error constants for various types and inputs:

Input signal	Type of system			
	0	1	2	3 or More
Unit step	$\frac{1}{1+k_p}$	0	0	0
Unit velocity	∞	$\frac{1}{k_v}$	0	0
Unit acceleration	∞	∞	$\frac{1}{k_a}$	0

Table: Static Error Constants

Effects of Proportional Derivative Systems

- To improve the performance of system, proportional and derivative controllers are added to the control system
- Generally, in the proportional controller "K_p", the amplifier gain is easily controllable
- But, in other cases, this control fails to satisfy both damping and ω_n.
- Then, another controller proportional plus derivative control or PD control is tried



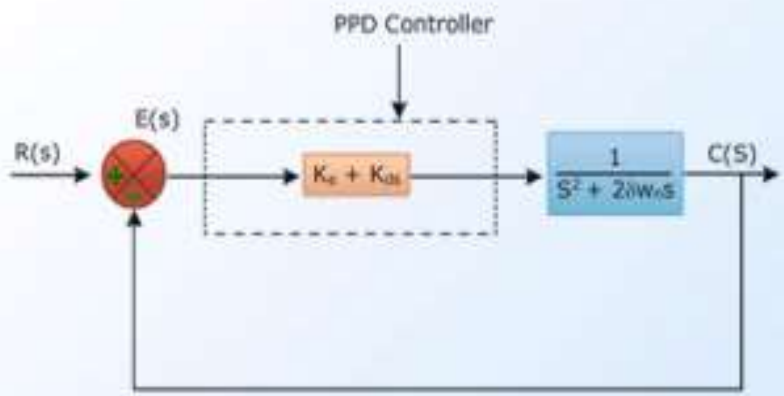
System with P and D controllers

Effects of Proportional Derivative Systems

- From the block diagram the characteristic equation is:

$$1+\frac{(k_e+k_a s)}{s^2+2\delta\omega_n s}=0$$

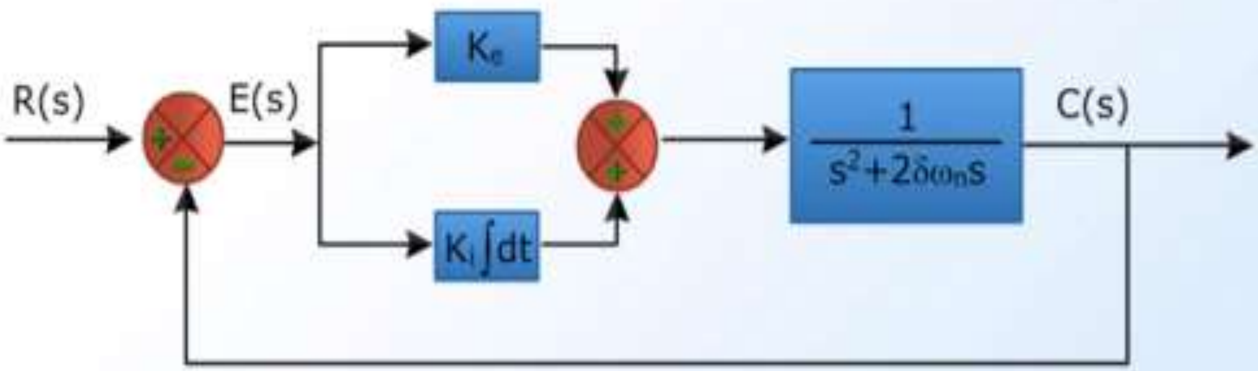
$$s^2+(2\delta\omega_n+k_d)s+k_e=0$$



Modified System with P and D controllers

Effects of Proportional Integral Systems

- Here, a signal is the sum of proportional error signal and the "Integral of the error signal". This signal is applied to the plant



System with P and I controllers

- When PD controller and PI controller are compared, the modified system gives much reduced steady state errors
- A proportional plus integral control improves the steady state response