# Power System Analysis

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# Course Objectives:

1. To understand and develop Ybus and Zbus matrices

2. To know the importance of load flow studies and its importance

3. To analyze various types of short circuits

4. To know rotor angle stability of power systems

### **Course Outcomes:**

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1. Develop the Ybus and Zbus matrices

2. Analyze load flow for various requirements of the power system

3. Analyze short circuit studies for the protection of power system

4. Estimate stability and instability in power systems

# Importance of subject in competitive Exams

- 1. This is very important subject in Competitive point of view
- 2. GATE weightage is 2-4 question (3 to 5 marks)
- 3. In GATE exam many questions have appeared from each and every topic of this subject
- 4. In IES exam 30 to 35% power system question are belongs to this subject
- 5. Mostly in every PSU there will be questions from this subject
- 6. In PSU interviews also mostly tricky questions will be asked from PSA topics only
- 7. In on campus core company placement also there is more probability to get written test questions from the PSA

# Unit-1 Power System Network Matrices

#### **Syllabus**

Graph Theory: Definitions and Relevant concepts in Graph Theory, Network Matrices.

Transmission Network Representations: Bus Admittance frame and Bus Impedance frame.

Formation of Ybus: Direct and Singular Transformation Methods, Numerical Problems.

Formation of Zbus: Modification of existing Zbus Matrix for addition of a new branch, & complete Zbus building algorithm Numerical Problems.

### 1.NETWORK TOPOLOGY

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- ✓ The solution of a given linear network problem requires the formation of a set of equations describing the response of the network.
- ✓ The mathematical model so derived, must describe the characteristics of the individual network components, as well as the relationship which governs the interconnection of the individual components.
- ✓ In the bus frame of reference the variables are the node voltages and node currents. The independent variables in any reference frame can be either currents or voltages.
- ✓ Correspondingly, the coefficient matrix relating the dependent variables and the independent variables will be either an impedance or admittance matrix.

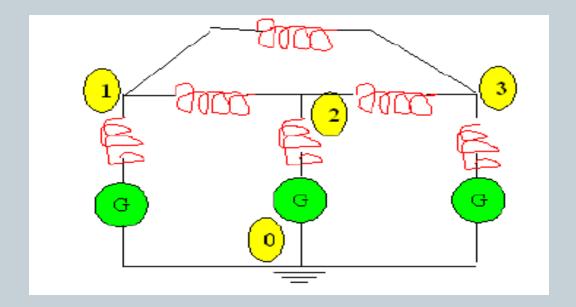
# 1.1 ELEMENTARY LINEAR GRAPH THEORY

- ➤ The geometrical interconnection of the various branches of a network is called the topology of the network.
- ➤ The connection of the network topology, shown by replacing all its elements by lines is called a graph.
- A linear graph consists of a set of objects called nodes and another set called elements such that each element is identified with an ordered pair of nodes.
- An element is defined as any line segment of the graph irrespective of the characteristics of the components involved.

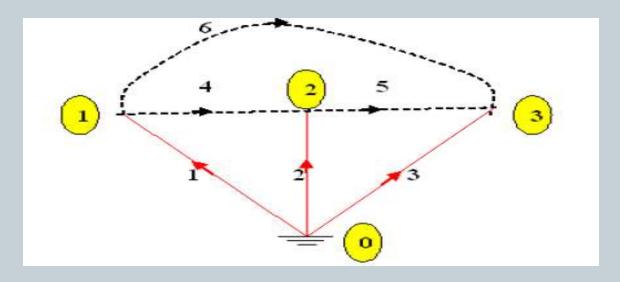
- > A graph in which a direction is assigned to each element is called an oriented graph or a directed graph.
- It is to be noted that the directions of currents in various elements are arbitrarily assigned and the network equations are derived, consistent with the assigned directions.
- Elements are indicated by numbers and the nodes by encircled numbers.
- The ground node is taken as the reference node. In electric networks the convention is to use associated directions for the voltage drops.
- > This means the voltage drop in a branch is taken to be in the direction of the current through the branch. Hence, we need not mark the voltage polarities in the oriented graph.

## 1.1.1. Connected Graph

This is a graph where at least one path (disregarding orientation) exists between any two nodes of the graph. A representative power system and its oriented graph are as shown in Fig below, with:







e = number of elements = 6

n = number of nodes = 4

b = number of branches = n-1 = 3

l = number of links = e-b = 3

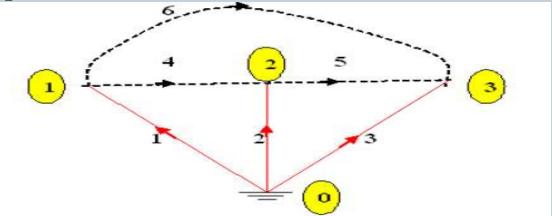
Tree = T(1,2,3) and

Co-tree = T(4,5,6)

## 1.1.2. Sub-graph



- > sG is a sub-graph of G if the following conditions are satisfied:
- > sG is itself a graph
- > Every node of sG is also a node of G
- > Every branch of sG is a branch of G
- For eg., sG(1,2,3), sG(1,4,6), sG(2), sG(4,5,6), sG(3,4),... are all valid sub-graphs of
- the oriented graph



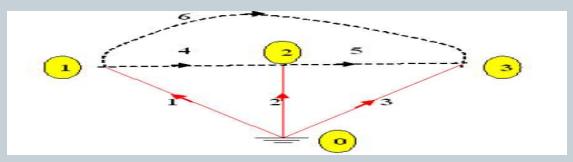
## 1.1.3. Loop



#### A sub-graph L of a graph G is a loop if

- ✓ L is a connected sub-graph of G
- ✓ Precisely two and not more/less than two branches are incident on each node in L

In Fig below the set{1,2,4} forms a loop, while the set{1,2,3,4,5} is not a valid, although the set(1,3,4,5) is a valid loop. The KVL (Kirchhoff's Voltage Law) for the loop is stated as follows: *In any lumped network, the algebraic sum of the branch voltages around any of the loops is zero.* 

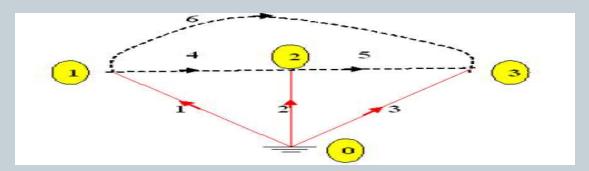


## **1.1.4 Cutset**



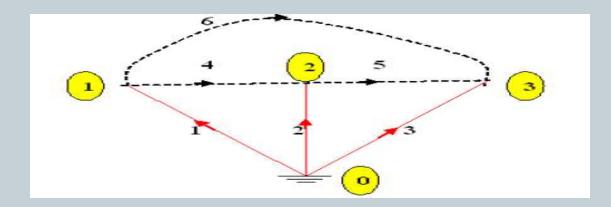
# It is a set of branches of a connected graph G which satisfies the following conditions:

- ✓ The removal of all branches of the cutset causes the remaining graph to have two separate unconnected sub-graphs.
- ✓ The removal of all but one of the branches of the set, leaves the remaining graph connected.
  - Referring to Fig , the set {3,5,6} constitutes a cutset since removal of them isolates node 3 from rest of the network, thus dividing the graph into two unconnected sub graphs. However, the set(2,4,6) is not a valid cutset! The KCL (Kirchhoff's Current Law) for the cutset is stated as follows: In any lumped network, the algebraic sum of all the branch currents traversing through the given cutset branches is zero.



## 1.1.5. Tree

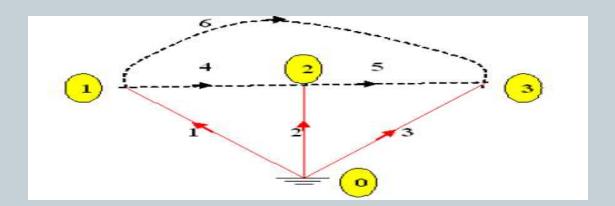
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- ✓ It is a connected sub-graph containing all the nodes of the graph G, but without any closed paths (loops). There is one and only one path between every pair of nodes in a tree. The elements of the tree are called twigs or branches. In a graph with *n nodes*,
- $\checkmark$  The number of branches: b = n-1 (1)
- For the graph of Fig 1c, some of the possible trees could be T(1,2,3), T(1,4,6), T(2,4,5), T(2,5,6), etc.



#### 1.1.6. Co-Tree



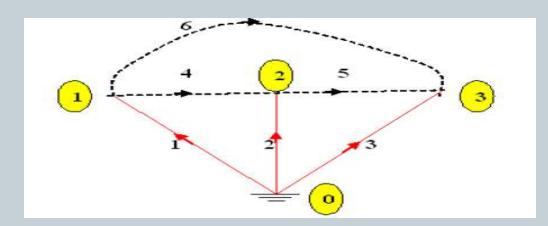
- ✓ The set of branches of the original graph G, not included in the tree is called the co-tree. The co-tree could be connected or non-connected, closed or open. The branches of the co-tree are called links. By convention, the tree elements are shown as solid lines while the co-tree elements are shown by dotted lines as shown in Fig for tree T(1,2,3). With e as the total number of elements,
- ✓ The number of links: l = e b = e n + 1



(16)

✓ For the graph of Fig 1c, the co-tree graphs corresponding to the various tree graphs are as shown in the table below:

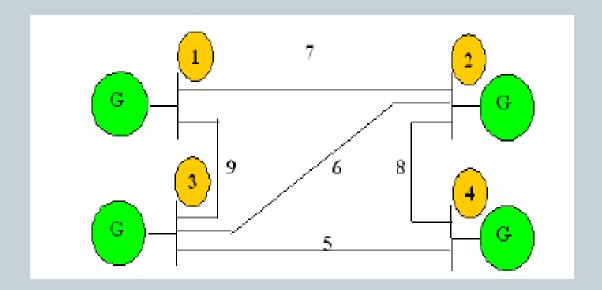
Tree	T(1,2,3)	T(1,4,6)	T(2,4,5)	T(2,5,6)
Co-Tree	T(4,5,6)	T(2,3,5)	T(1,3,6)	T(1,3,4)

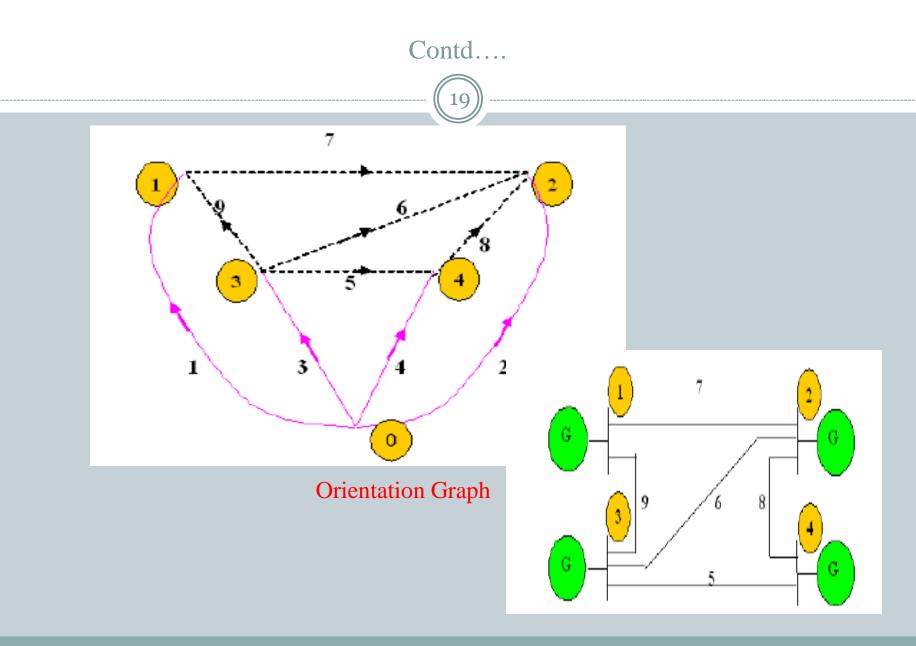


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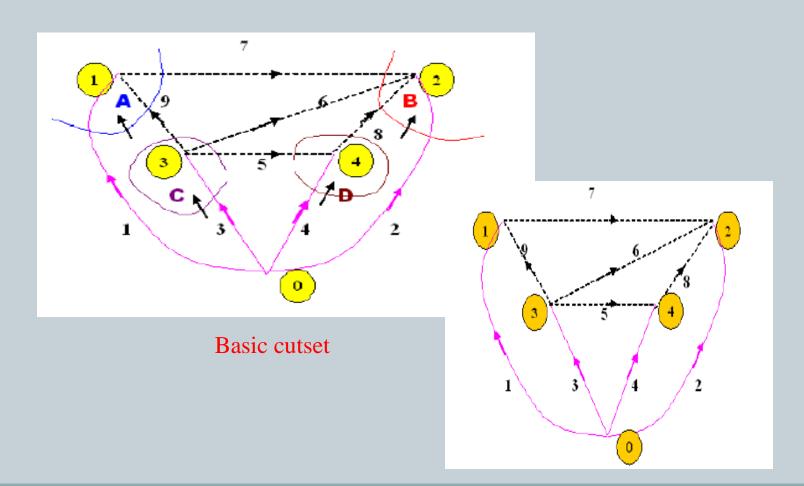
- ✓ 1.1.7. Basic loops: When a link is added to a tree it forms a closed path or a loop. Addition of each subsequent link forms the corresponding loop. A loop containing only one link and remaining branches is called a basic loop or a fundamental loop. These loops are defined for a particular tree. Since each link is associated with a basic loop, the number of basic loops is equal to the number of links.
- ✓ 1.1.8. Basic cut-sets: Cut-sets which contain only one branch and remaining links are called basic cut sets or fundamental cut-sets. The basic cut-sets are defined for a particular tree. Since each branch is associated with a basic cut-set, the number of basic cut-sets is equal to the number of branches.

Example-1: Obtain the oriented graph for the system shown in Fig. E1. Select any four possible trees. For a selected tree show the basic loops and basic cutsets.

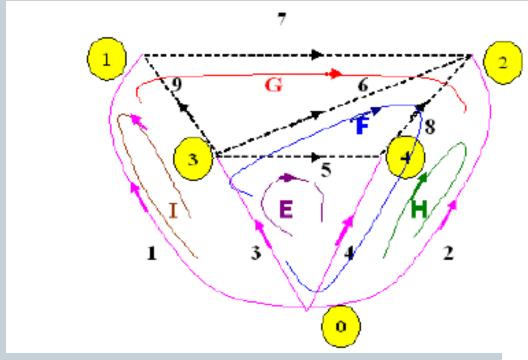




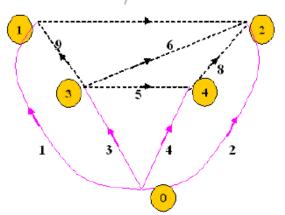








**Basic Loops** 



#### 1.2 INCIDENCE MATRICES



#### 1.2.1 Element-node incidence matrix: A<sup>^</sup>

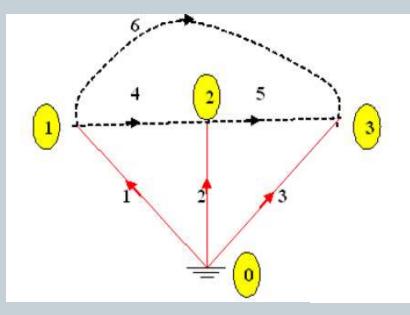
The incidence of branches to nodes in a connected graph is given by the element-node incidence matrix, A<sup>^</sup>.

An element aij of A^ is defined as under:

- $\checkmark$  aij = 1 if the branch-i is incident to and oriented away from the node-j.
  - = -1 if the branch-i is incident to and oriented towards the node-j.
  - = 0 if the branch-i is not at all incident on the node-j.

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✓ Thus the dimension of Aˆ is eXn, where e is the number of elements and n is the number of nodes in the network. For example, consider again the sample system with its oriented graph as in fig. the corresponding element-node incidence matrix, is obtained as under:



Nodes	0	1	2	3
Elements	U	1	2	
1	1	-1		
2	1		-1	
3	1			-1
4		1	-1	
5			1	-1
6		1		-1



✓ The sum of every row is found to be equal to zero always. Hence, the rank of the matrix is less than n. Thus in general, the matrix Aˆ satisfies the identity:

$$\sum_{j=1}^{n} a_{ij} = 0 \quad \forall \quad i = 1, 2, \dots e.$$

#### 1.2.2 Bus incidence matrix: A

By selecting any one of the nodes of the connected graph as the reference node, the corresponding column is deleted from A<sup>\*</sup> to obtain the bus incidence matrix, A. The dimensions of A are e (n-1) and the rank is n-1. In the above example, selecting node-0 as reference node, the matrix A is obtained by deleting the column corresponding to node-0



	Buses	_		_			
	Elements	1	2	3			
<b>A</b> =	1	-1				$\mathbf{A}_{\mathbf{b}}$	Branches
	2		-1				
	3			-1	=		
	4	1	-1			$\mathbf{A}_1$	Links
	5		1	-1			
	6	1		-1			

It may be observed that for a selected tree, say, T(1,2,3), the bus incidence matrix can be so arranged that the branch elements occupy the top portion of the A-matrix followed by the link elements. Then, the matrix-A can be partitioned into two sub matrices Ab and Al as shown, where,

- (i) Ab is of dimension (bxb) corresponding to the branches and
- (ii) Al is of dimension (lxb) corresponding to links.

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## Course outcome of the Topic:

- Basics of Graph Theory
- 2. Development of Incident Matrix, Basic loop Matrix and Cut set Matrix..

3. Solution of linear equation.

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https://www.youtube.com/watch?v=ZHqQDA3be-k

# Previous GATE, IES and PSU questions



- 1. The graph of a network has 8 nodes and 5 independent loops. The number of branches of the graph is -GATE 2018
  - A) 11

B) 12

C) 13

D) 14

Answer B

individual loops =branches-nodes+1

$$5 = b+1-8$$

$$b = 7+5$$

$$b = 12$$



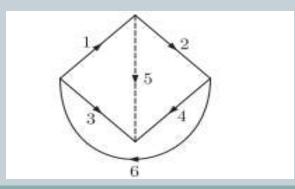
- Q)The graph associated with an electrical network has 7 branches and 5 nodes. The number of independent KCL equations and the number of independent KVL equations, respectively, are (GATE 2016)
  - A) 2 and 5 (B) 5 and 2 (C) 3 and 4 (D) 4 and 3

Answer D

KCL equation = 
$$N-1 = 5-1 = 4$$

$$KVL equations = B-N+1 = 7-5+1 = 3$$

Q) Consider the graph and tree (dotted) of the given fig. (IES)



The fundamental loops include the set of lines

$$(C)(1,5,3),(5,4,2),(3,4,6)$$
 and  $(2,4,3,1)$ 

## 1.4 PRIMITIVE NETWORKS

- ✓ So far, the matrices of the interconnected network have been defined. These matrices contain complete information about the network connectivity, the orientation of current, the loops and cut sets.
- ✓ However, these matrices contain no information on the nature of the elements which form the interconnected network.
- ✓ The complete behavior of the network can be obtained from the knowledge of the behavior of the individual elements which make the network, along with the incidence matrices. An element in an electrical network is completely characterized by the relationship between the current through the element and the voltage across it.

General representation of a network element: In general, a network element may contain active or passive components. Figure below represents the alternative impedance and admittance forms of representation of a general network component.

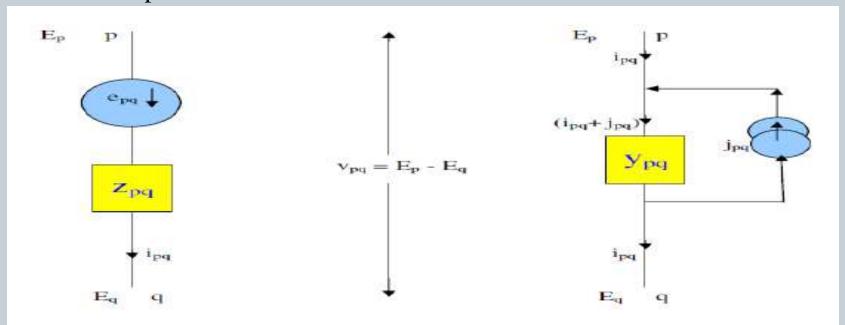


Fig. Representation of a primitive network element (a) Impedance form (b) Admittance form



The network performance can be represented by using either the impedance or the admittance form of representation. With respect to the element, p-q, let,

vpq = voltage across the element p-q,

epq = source voltage in series with the element p-q,

ipq= current through the element p-q,

jpq= source current in shunt with the element p-q,

zpq= self impedance of the element p-q and

ypq= self admittance of the element p-q.

# 1.4.1. Performance equation:

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Each element p-q has two variables, Vpq and ipq. The performance of the given element p-q can be expressed by the performance equations as under:

vpq + epq = zpqipq (in its impedance form)

ipq + jpq = ypqvpq (in its admittance form)

Thus the parallel source current jpq in admittance form can be related to the series source voltage, epq in impedance form as per the identity:

$$jpq = -ypq epq$$



A set of non-connected elements of a given system is defined as a primitive Network and an element in it is a fundamental element that is not connected to any other element.

In the equations above, if the variables and parameters are replaced by the corresponding vectors and matrices, referring to the complete set of elements present in a given system, then, we get the performance equations of the primitive network in the form as under:

$$v + e = [z] i$$

$$i + j = [y] v$$

## 1.4.2. Primitive network matrices:



- > A diagonal element in the matrices, [z] or [y] is the self impedance zpq-pq or self admittance, ypq-pq. An off-diagonal element is the mutual impedance, zpq-rs or mutual admittance, ypq-rs, the value present as a mutual coupling between the elements p-q and r-s.
- > The primitive network admittance matrix, [y] can be obtained also by inverting the primitive impedance matrix, [z].
- > Further, if there are no mutually coupled elements in the given system, then both the matrices, [z] and [y] are diagonal.
- In such cases, the self impedances are just equal to the reciprocal of the corresponding values of self admittances, and vice-versa.

## 1.5. FORMATION OF YBUS



- The bus admittance matrix, YBUS plays a very important role in computer aided power system analysis. It can be formed in practice by either of the methods as under:
  - 1. Rule of Inspection
  - 2. Singular Transformation
  - 3. Non-Singular Transformation
  - 4.ZBUS Building Algorithms, etc.

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The performance equations of a given power system can be considered in three different frames of reference as discussed below:

#### **Frames of Reference:**

i) Bus Frame of Reference: There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

EBUS = ZBUS IBUS

IBUS = YBUS EBUS

ii) Branch Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

EBR = ZBR IBR

IBR = YBR EBR

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Loop Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

ELOOP = ZLOOP ILOOP

ILOOP = YLOOP ELOOP

Of the various network matrices refered above, the bus admittance matrix (YBUS) and the bus impedance matrix (ZBUS) are determined for a given power system by the rule of inspection as explained next.

## 1.5.1. Rule of Inspection

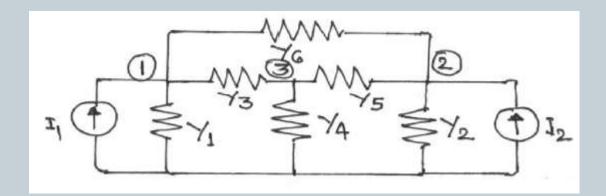
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Consider the 3-node admittance network as shown in figure. Using the basic branch relation: I = (YV), for all the elemental currents and applying Kirchhoff's Current Law principle at the nodal points, we get the relations as under:

At node 1: 
$$I1 = Y1V1 + Y3(V1-V3) + Y6(V1-V2)$$

At node 2: 
$$I2 = Y2V2 + Y5(V2-V3) + Y6(V2-V1)$$

At node 3: 
$$0 = Y3 (V3-V1) + Y4V3 + Y5 (V3 - V2)$$





These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} = \begin{vmatrix} (Y_1 + Y_3 + Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2 + Y_5 + Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3 + Y_4 + Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix}$$

Diagonal elements: A diagonal element (Yii) of the bus admittance matrix, YBUS, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,

Yii = S yij 
$$(j = 1, 2, .....n)$$



✓ Off Diagonal elements: An off-diagonal element (Yij) of the bus admittance matrix, YBUS, is equal to the negative of the admittance value of the connecting element present between the buses I and j, if any. This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

$$Yij = -yij (j = 1,2,....n)$$

For i = 1,2,....n, n = no. of buses of the given system, yij is the admittance of element connected between buses i and j and yii is the admittance of element connected between bus i and ground (reference bus).



#### **Course Outcome of Y-Bus Formation:**

- 1. Formation of Y-Bus
- 2. Modification of Y-bus if Link is added
- 3. Modification of Y-bus if Brach is added Link for Video Lecture

https://youtu.be/rWjoK28YMkk

## Previous GATE, IES and PSU questions

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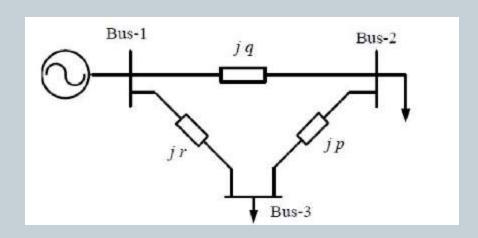
Q) A 3-bus power system is shown in the figure below, where the diagonal elements of YY-bus matrix are: Y11=-j12pu, Y22=-j15pu and Y33=-j7pu.Y33=-j7pu. The per unit values of the lines reactance p, q and r shown in the figure are (GATE-2017)

(A) 
$$p=-0.2, q=-0.1, r=-0.5$$

(B) 
$$p=0.2, q=0.1, r=0.5$$

(C) 
$$p=-5, q=-10, r=-2$$

(D) 
$$p=5, q=10, r=2$$





q) A 3-bus power system network consists of 3 transmission lines. The bus admittance matrix of the uncompensated system is

If the shunt capacitance of all transmission line is 50% compensated, the imaginary part of the 3rd row 3rd column element (in pu) of the bus admittance matrix after compensation is (GATE 2015)

(A) -j7.0

- (B) -j8.5 (C) -j7.5 (D) -j9.0

Answer B

# 1.6. FORMATION OF BUS IMPEDANCE MATRIX

- The bus impedance matrix is the inverse of the bus admittance matrix. An alternative method is possible, based on an algorithm to form the bus impedance matrix directly from system parameters and the coded bus numbers.
- The bus impedance matrix is formed adding one element at a time to a partial network of the given system. The performance equation of the network in bus frame of reference in impedance form using the currents as independent variables is given in matrix form by

$$\overline{E}_{bus} = [Z_{bus}]\overline{I}_{bus}$$



When expanded so as to refer to a *n* bus system, will be of the form

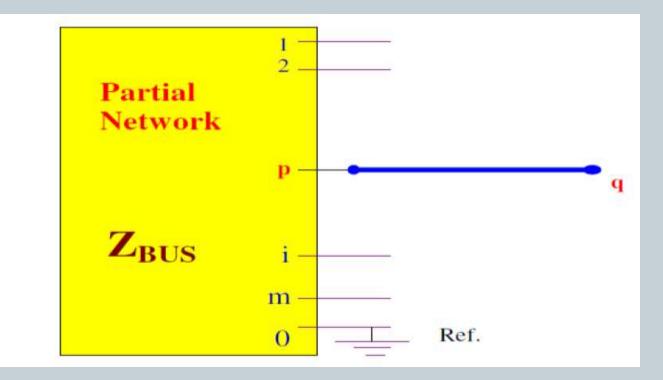
$$\begin{split} E_1 &= Z_{11}I_1 + Z_{12}I_2 + \dots + Z_{1k}I_k \dots + Z_{1n}I_n \\ \vdots \\ E_k &= Z_{k1}I_1 + Z_{k2}I_2 + \dots + Z_{kk}I_k + \dots + Z_{kn}I_n \\ \vdots \\ E_n &= Z_{n1}I_1 + Z_{n2}I_2 + \dots + Z_{nk}I_k + \dots + Z_{nn}I_n \end{split}$$

Now assume that the bus impedance matrix Zbus is known for a partial network of m buses and a known reference bus. Thus, Zbus of the partial network is of dimension mxm. If now a new element is added between buses p and q we have the following two possibilities:

(i) p is an existing bus in the partial network and q is a new bus; in this case p-q is a branch added to the p-network as shown in Fig 1a, and

(ii) both p and q are buses existing in the partial network; in this case p-q is a link added to the p-network as shown in Fig 1b.

#### 1.6.1. When New Bus is created





✓ If the added element ia a branch, p-q, then the new bus impedance matrix would be of order m+1, and the analysis is confined to finding only the elements of the new row and column (corresponding to bus-q) introduced into the original matrix.

✓ If the added element is a link, p-q, then the new bus impedance matrix will remain unaltered with regard to its order. However, all the elements of the original matrix are updated to take account of the effect of the link added.

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots \\ E_p \\ \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix}$$

- ✓ It is assumed that the added branch p-q is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have
- ✓ Vector ypq-rs is not equal to zero and Zij= Zji "i,j=1,2,...m,q

### To find vpq:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pq} \\ \overline{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pqpq} & \overline{y}_{pqrs} \\ \overline{y}_{rxpq} & \overline{y}_{rsrs} \end{bmatrix} \begin{bmatrix} v_{pq} \\ \overline{v}_{rs} \end{bmatrix}$$

## Course outcome of Z-bus

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- Development of Z-Bus
- Modification of Z-Bus if Branch is added
- Modification of Z-Bus if Link is added

#### Lecture Video link

https://youtu.be/2CYoKtyO3Yc

## Previous GATE, IES and PSU questions

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ACE

(70)

Class Room Picciles

1.1 pu and the equivalent transfer admittance during fault is 0.8 pu. The 100 MVA generator has an inertia constant of 8 MJ/MVA and if was delivering 1.0 pu power prior of the fault with rotor power angle of 30°. The system frequency is 50 Hz. The initial accelerating power (in pu) will be

(c) 0.56

(D) 0.6

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#### Load Flow Analysis

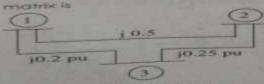
 The bus admittance matrix of a power system is given as

[-js0 j10 j5 j10 -j30 j10 j5 j10 -j25]

The impedance of line between bus 2 and 3 will be equal to

(a) 10.1 (b) 10.2 (c) 10.2

2. A single line diagram of a power system is shown in the given figure. The per unit reactance are specified. The diagonal elements of the Y<sub>min</sub>



(a) 10.7.10.75.10.45 pu (b) 10.7.10.75.10.45 pu (c) 17.0.16.0.19.0 pu (d) 17.0.16.0.19.0 pu o3. The equivalent Thevenin's bus admitted matrix of a two – bus system with identical

generators on both buses is  $\begin{vmatrix} -j30 & +j10 \\ +j10 & -j30 \end{vmatrix}$ 

the generator reactance and interconnection reactance will be respectively

(a) | 0.05 and |0.1

(b) - j0.05 and j 0.1

(e) - jo.05 and - jo.1

(d) j0:1 and j0:05

04. The bus admittance matrix of a three-ti-

If each transmission line between the two but is represented by an equivalent s - netwo the magnitude of the shunt susceptance of the connecting bus 1 and 2 is

(a) 4 (b) 2 (d) 0

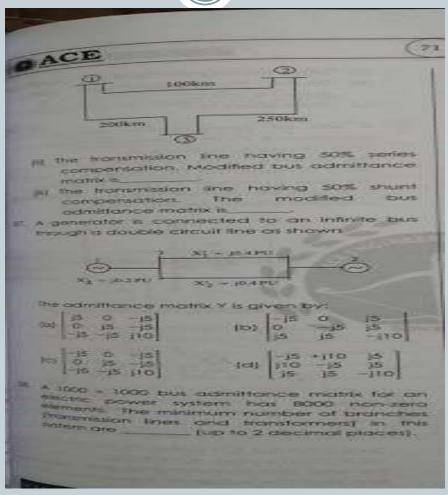
05. The Ym, of a 3-Bus, 3- transmission line is

$$Y_{ba} = \begin{bmatrix} -14.4 & 10 & 5 \\ 10 & -11.5 & 2.5 \\ 5 & 2.5 & -6.3 \end{bmatrix}$$

Each transmission line is represented a network. The Shunt susceptance of transmission line between buses I and I

of transmission line is 10.001/km for and it susceptance is 1 0.0016/km for bus dominamater is

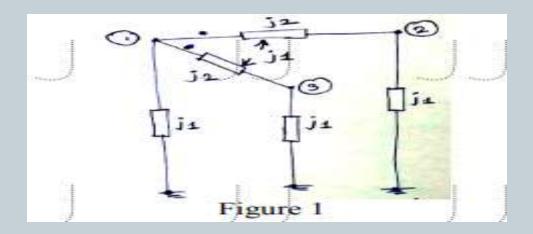




## Previous JNTUH questions



- 1. Define the terms TREE, Co-TREE and LINK of a graph.
- 2. What is an incidence Matrix? Explain with a suitable example
- 3. Form the YBUS for the system shown in below figure 1, using singular transformation method.





- Give the steps for modification of existing ZBUS, when a branch Zb is added from existing bus(k) to the reference bus.
- Show that YBUS = A (transport) Ypre A.
- Form the ZBUS for the system shown in below figure

