

# Unit **1.1**

## Strain Energy

### Introduction

**Strain energy** is the energy stored in the body associated with the deformation of the member.

When an elastic body is loaded within elastic limits, it deforms and some work is done which is stored within the body. This energy stored is strain energy. It is recoverable without loss as soon as load is removed.

But if elastic limit is exceeded, permanent deformation occurs and particles of material of body slide one over the other. The work done in overcoming the cohesion of the particles is again stored as strain energy, but in this case it is not completely recoverable.

### 1.1. Strain Energy due to Gradual or Static Load

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### Resilience

It is the ability of a material to absorb energy when it is deformed **elastically** and release that energy upon unloading.

- It is defined as strain energy in simple tension and compression.
- It is also referred as **strain energy density** denoted by ' $\mu$ '.

NOTE

Strain energy is always a positive scalar quantity whether in compression or tension.

### Proof Resilience/Modulus of Resilience

Value of resilience at the elastic limit or at proof stress.

NOTE

Proof resilience is area under the stress-strain curve upto elastic limit.

### Modulus of Toughness

Strain energy per unit volume required to cause the material to rupture.

NOTE

Modulus of toughness is given by the area under the stress-strain curve upto rupture point.

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### 1.2. Strain Energy due to Volumetric Stress

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### 1.3 Strain Energy in a Member subjected to Torsion

#### 1.4. Strain Energy in a Member due to Shear Stress

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#### 1.5. Strain Energy in a Member subjected to Bending

### 1.6. Stress due to Various Types of Axial Loads

The magnitude of the stress produced in a member due to a load depends upon the manner in which the load is applied.

#### Gradually applied load

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**Suddenly applied load*****For Micro Notes by the Student***

NOTE

The maximum stress due to suddenly applied load is twice the stress intensity produced by the load of same magnitude applied gradually.

**Previous JNTU Conventional Questions***For Micro Notes by the Student*

01. A load  $P$  falls from a height of 25 mm on a collar at the lower end of a vertical steel bar 1.5 m long and  $\phi 28$  mm in diameter. If the maximum instantaneous elongation is 3 mm, determine the corresponding stress and the magnitude of the load  $P$ ? **(Dec - 2014)**
02. A titanium bar with square cross section with 75 mm side and length 3.0 m is subjected to tensile load of 900 kN along the length direction. Determine the change in the dimensions and increase in volume of the bar. Young's modulus and Poisson's ratio for titanium are 100 GPa and 0.33 respectively. Determine the strain energy developed in the body. What is the increase in the strain energy if the same load is suddenly applied?
03. A steel rod of length 1.25 m and 22 mm diameter hangs vertically with a collar firmly attached at the lower end of the rod. Find the maximum stress induced in the rod when a block of weight 25 kg falls on the collar from a clear height of 300 mm. Also find the energy absorbed and the modulus of resilience. Use modulus of elasticity =  $2 \times 10^5$  N/mm<sup>2</sup> **(May - 2018)**

**Previous Gate Questions****One Mark Questions**

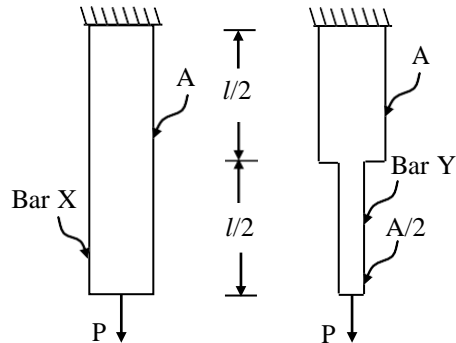
01. For linear elastic systems, the type of displacement function for the strain energy is **(GATE - 03)**
- (a) Linear                      (b) Quadratic  
(c) Cubic                      (d) Quartic

**KEY**

01. (b)

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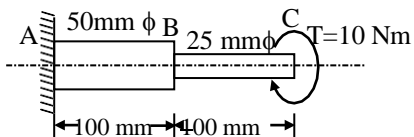
01. What is the ratio of the strain energy in bar X to that in bar Y when the material of the two bars is the same? The cross-sectional areas are as indicated over the indicated lengths. (GATE-2015)



- (a)  $1/3$       (b)  $2/3$       (c)  $4/3$       (d)  $1/6$

02. For a ductile material, toughness is a measure of (GATE-2013)

A stepped steel shaft is subjected to a clockwise torque of 10 Nm at its free end. Shear modulus of steel is 80 GPa. The strain energy stored in the shaft is (GATE-2017)



- (a) 1.73 Nmm      (b) 2.52 Nmm  
(c) 3.46 Nmm      (d) 4.12 Nmm

**KEY**

01. (b)      02. (b)      03. (a)

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**Previous TSPSC Questions**

01. A cantilever beam of length  $L$  is subjected to a point load  $P$  at free end. Then strain energy in usual notations is expressed as
- (a)  $PL^3/(3EI)$  (b)  $PL/(6EI)$   
 (c)  $PL^2/(36EI)$  (d)  $P^2L^3/(6EI)$
02. The strain energy stored in a solid circular shaft, under pure torque, per unit its volume (take max shear stress =  $q$  and modulus of rigidity =  $C$ ) is expressed as
03. The ratio of energy stored in a rectangular cantilever beam loaded at the free end without producing permanent set to the energy stored in the same bar in simple tension is
- (a)  $1/3$  (b)  $1/6$  (c)  $1/9$  (d)  $1/2$
04. The strain energy stored due to bending for a cantilever beam of span , modulus of elasticity  $E$ , moment of inertia  $I$  and a point load  $P$  at the free end is
- (a)  $\frac{P^2L^3}{6EI}$  (b)  $\frac{P^2L^3}{40EI}$  (c)  $\frac{P^2L^3}{96EI}$  (d)  $\frac{P^2L^3}{240EI}$
05. Strain energy is the
- (a) energy stored in a body when strained within elastic limits  
 (b) energy stored in a body when strained up to the breaking of a specimen  
 (c) maximum strain energy which can be stored in a body  
 (d) proof resilience per unit volume of a material
06. A prismatic bar 1 m long and 4 sq. cm in cross sectional area is compressed by a force of 80 kN. If  $E = 200 \text{ kN/sq.mm}$ , the total strain energy stored in the bar is equal to
- (a) 80 kN-mm (b) 40 kN-mm  
 (c) 0.05 kN-mm (d) 400 kN-mm

06. As compared to uniaxial tension or compression, the strain energy stored in bending is only

- (a)  $1/4$                       (b)  $1/3$                       (c)  $1/8$                       (d)  $1/2$

07. The stress induced in a body, when suddenly loaded, is how many times the stress induced when the same load is applied gradually

- (a) equal to                                      (b) Four times  
 (c) one-half times                              (d) two times

08. A specimen  $160 \text{ mm}^2$  in cross section stretches by  $0.06 \text{ mm}$  over a  $50 \text{ mm}$  gauge length under an axial load of  $35 \text{ kN}$ . The strain energy is

- (a)  $1436 \text{ N-mm}$                               (b)  $780 \text{ N-mm}$   
 (c)  $1050 \text{ N-mm}$                               (d)  $1298 \text{ N-mm}$

10. Elastic strain energy due to direct force is given by

- (A)  $P^2L/2AE$                                       (B)  $P^2L/2G$   
 (C)  $P^2L/2EI$                                       (D)  $P^2L/G$

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### KEY

01. (d)    02.  $q^2/4c$     03. (c)    04. (a)    05. (a)  
 06. (b)    07. (b)    08. (d)    09. (c)