

Time: 3 hours

Max. Marks: 75

**Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

## PART-A

(25 Marks)

- 1.a) Solve the following differential equation  $n(2y - x^3)dx + x dy = 0$ . [2]  
 b) Find the Particular Integral of the equation  $(D^2 - 2D + 1)y = x e^x \sin x$ . [3]  
 c) Examine whether the vectors are linearly dependent or not  $(3, 1, 1), (2, 0, -1), (4, 2, 1)$ . [2]  
 d) If  $\alpha, \beta$ , and  $\gamma$  are the roots of the equation  $x^3 + px + q = 0$  then the value of the determinant  $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$  is [3]  
 e) Compute the Eigen values and Eigen vectors of  $\begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$ . [2]  
 f) Find the Eigen values of the following system  $\begin{cases} 8x - 4y = \lambda x \\ 2x + 2y = \lambda y \end{cases}$  [3]  
 g) Find the value of  $\frac{\partial x}{\partial y} \cdot \frac{\partial y}{\partial z} \cdot \frac{\partial z}{\partial x}$  if  $f(x, y, z) = 0$ . [2]  
 h) Find  $\frac{dy}{dx}$  if  $x^y = y^x$ . [3]  
 i) Form the partial differential equation by eliminating the arbitrary function  $z = f(x^2 + y^2)$ . [2]  
 j) Solve the following partial differential equation  $yq - xp = z$ . [3]

## PART-B

(50 Marks)

- 2.a) Find the value of the constant  $d$  such that the parabolas  $y = c_1 x^2 + d$  are the orthogonal trajectories of the family of ellipses  $x^2 + 2y^2 - y = c_2$ .  
 b) In a culture of yeast, the active ferment doubles itself in 3 hours. Determine the number of times it multiplies itself in 15 hours. [5+5]  
 OR  
 3.a) Solve  $(D^2 + 5D + 6)y = e^x \cos 2x$ .  
 b) Solve by the method of variation of parameters  $y'' + y = \sec x$ . [5+5]

$$2x + 3y + 4z = 11$$

- 4.a) Discuss the consistency of the system of equations  $x + 5y + 7z = 15$   
 $3x + 11y + 13z = 25$

- b) Find an LU decomposition of the Matrix A and solve the linear system  $AX=B$

$$\begin{bmatrix} -3 & 12 & -6 \\ 1 & -2 & 2 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -33 \\ 7 \\ -1 \end{bmatrix} \quad [5+5]$$

OR

- 5.a) Solve the system of equations by the Gauss Seidel method

$$10x + y + z = 12$$

$$2x + 10y + z = 13$$

$$2x + 2y + 10z = 14$$

- b) Convert the matrix into echelon form  $\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix}$  [5+5]

- 6.a) Find  $A^{39}$  if  $A = \begin{bmatrix} 4 & 1 \\ 2 & 3 \end{bmatrix}$ .

- b) Compute the Modal matrix for  $\begin{bmatrix} 5 & 4 \\ 12 & 7 \end{bmatrix}$ . [5+5]

OR

7. Reduce the quadratic form  $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_3x_2 + 4x_3x_1$  to the sum of squares and find the corresponding linear transformation. Find the index and signature. [10]

- 8.a) Determine the functional dependence and find the relation between  $u = \frac{x-y}{x+y}$ ,  $v = \frac{xy}{(x-y)^2}$ .

- b) If  $u = x^2 + y^2 + z^2$ ,  $v = xyz$  find  $J \begin{pmatrix} x, y \\ u, v \end{pmatrix}$ . [5+5]

OR

- 9.a) Expand  $x^2y + 3y - 2$  in powers of  $x - 1$  using Taylor's theorem.

- b) Find the maximum and minimum distances of the point  $(3, 4, 12)$  from the sphere  $x^2 + y^2 + z^2 = 1$ . [5+5]

10. Solve the partial differential equations:

a)  $px(z - 2y^2) = (z - qy)(z - y^2 - 2x^3)$

b)  $xp - yq + x^2 - y^2 = 0$ . [5+5]

OR

11. Solve the partial differential equations:

a)  $p(1 + q) = qz$

b)  $z^2(p^2x^2 + q^2) = 1$ . [5+5]

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