

Code No: 123AA

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, November/December - 2016

MATHEMATICS-II

(Common to CE, MMT, AE, PTE, CEE)

Time: 3 Hours

Max. Marks: 75

**Note:** This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

**PART- A**

(25 Marks)

- 1.a) What is the greatest rate of increase of  $\phi = xyz^2z^2$  at the point  $(-1,1,2)$ ? [2]
- b) If  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$  then show that  $\nabla^2(r^n) = n(n+1)r^{n-2}$  where  $r = |\vec{r}|$ . [3]
- c) Write the Euler's formula in the interval  $(c, c+2\pi)$ , for finding Fourier series. [2]
- d) Find the value of  $a_0$  for the function  $f(x) = e^{-x}$  in the interval  $0 < x < 2\pi$ . [3]
- e) Evaluate  $\frac{d}{dx} e^x$ . [2]
- f) Express the function  $f(x) = 2x^4 - 6x^3 + 5x^2 - 20x + 10$  in factorial notation. [3]
- g) Show that the rate of convergence of Bisection method is linear. [2]
- h) Establish Newton Raphson's method for determining the approximate value of the root of the equation  $f(x) = 0$ . [3]
- i) Write Simpson's  $\frac{1}{3}$  rule. [2]
- j) Evaluate  $K_3$  for the equation  $\frac{dy}{dx} = y - x$ ,  $y(0) = 1.5$  by using Runge-Kutta 4<sup>th</sup> order method. [3]

**PART-B**

(50 Marks)

- 2.a) Find the directional derivative of  $f = xy + yz + zx$  in the direction of vector  $i + 2j + 2k$  at the point  $(1,2,0)$ .
  - b) Find the scalar potential of  $\vec{F} = (z + \sin y)\vec{i} + (-z + x \cos y)\vec{j} + (x - y)\vec{k}$ . [5+5]
- OR**
- 3.a) Prove that  $(y^2 - z^2 + 3yz - 2x)\vec{i} + (3xz + 2xy)\vec{j} + (3xy - 2xz + 2z)\vec{k}$  is both solenoidal and irrotational.
  - b) Find the flux of the vector field  $\vec{A} = (X - 2Z)\vec{i} + (x + 3y + z)\vec{j} + (5x + y)\vec{k}$  through the upper side of the triangular ABC with vertices at the points  $A(1,0,0)$ ,  $B(0,1,0)$ ,  $C(0,0,1)$  [5+5]

4.a) Obtain a Fourier expansion for  $\sqrt{1 - \cos x}$  in  $-\pi < x < \pi$ .

b) Find the Fourier transform of  $f(x) = \begin{cases} 1 & \text{if } |x| < a \\ 0 & \text{if } |x| \geq a \end{cases}$  where  $a$  is a positive real

number. Hence deduce that: i)  $\int_0^\pi \frac{\sin t}{t} dt = \frac{\pi}{2}$  and ii)  $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$ . [5+5]

5.a) Express  $\cos x$  in Fourier series in  $0 < x < 2\pi$ .

b) Find the Fourier transform of  $f(x)$  given by  $f(x) = \begin{cases} x^2 & \text{if } |x| \leq a \\ 0 & \text{if } |x| > a \end{cases}$ . [5+5]

6.a) Find the cubic polynomial interpolation which takes on the values:  $f_0=5, f_1=1, f_2=9, f_3=25, f_4=55$ .

b) The mode of a certain frequency curve  $y = f(x)$  is very near  $x = 9$  and the value of the frequency density  $f(x)$  for  $x=8.9, 9.0$  and  $9.3$  are respectively equal to  $0.30, 0.35$  and  $0.25$ . Calculate the approximate value of the mode. [5+5]

OR

7.a) From the following table, find the number of students who obtained less than 45 marks:

Marks	30-40	40-50	50-60	60-70	70-80
No of Students	31	42	51	35	31

b) Fit a second degree parabola to the following data, taking  $x$  as the independent variable. [5+5]

$x$ :	2	3	4	5	6	7	8	9	
$y$ :	2	6	7	8	10	11	11	10	9

8.a) Evaluate  $\sqrt{29}$  by Newton-Raphson formula. Correct to four places of decimals.

b) Apply Gauss-Seidal iteration method to solve equations.

$10x_1 + x_2 + x_3 = 12, 2x_1 + 10x_2 + x_3 = 13$  and  $2x_1 + 2x_2 + 10x_3 = 14$ . [5+5]

OR

9.a) By iteration method, find the root of  $\tan x = x$  up to four decimal places.

b) Apply Jacobi iteration method to solve equations.

$27x + 6y - z = 85, 6x + 15y + 2z = 72$  and  $x + y + 54z = 110$ . [5+5]

10.a) Calculate the approximate value of  $\int_0^{\frac{1}{2}\pi} \sin x \cdot x dx$ .

i) By Trapezoidal rule

ii) By Simpson's rule, Using 11 ordinates.

b) Given the differential equation  $\frac{dy}{dx} = \frac{x^2}{y^2+1}$  with the initial condition  $y=0$  when  $x=0$ , use Picard's method to obtain  $y$  for  $x=0.25, 0.5$  and  $1.0$  correct to three decimal places. [5+5]

OR

11.a) Use Simpson's three-eights rule to obtain the value of  $\int_0^{0.3} (1 - 8x^3)^{\frac{1}{2}} dx$ .

b) Solve the boundary-value problem  $y'' = y(x), y(0) = y(1) = 0$  by the shooting method. [5+5]