

Time: 3 Hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit.

Each question carries 10 marks and may have a, b, c as sub questions.

## Part- A

(25 Marks)

- 1.a) Find  $n$ , if  $\vec{f} = r^n \vec{r}$  is solenoidal. [2M]
- b) Find the angle between the surfaces  $x^2 + y^2 + z^2 = 9$  and  $z = x^2 + y^2 - 3$  at the point  $(2, 1, 2)$ . [3M]
- c) If  $f(x) = x^4$  in  $(-1, 1)$  then find the Fourier coefficient  $b_n$ . [2M]
- d) Write any three properties of Fourier transforms. [3M]
- e) Evaluate  $\Delta^{10}[(1-ax)(1-bx^2)(1-cx^3)(1-dx^4)]$ . [2M]
- f) Write the normal equations to fit a curve  $y = ae^{bx}$  for the given data by the method of least squares. [3M]
- g) Define transcendental equation and give an example. [2M]
- h) Write short notes on iteration method to find a root for  $f(x) = 0$ . [3M]
- i) Define initial value problem and give an example. [2M]
- j) Write the finite difference formula for  $y'(x)$  and  $y''(x)$ . [3M]

## Part- B

(50 Marks)

- 2.a) With usual notations of vector calculus, prove that  $\nabla^2 f(r) = f''(r) + \frac{2}{r} f'(r)$ .
- b) Apply Green's theorem to evaluate  $\oint_C (xy + y^2) dx + x^2 dy$ , where  $C$  is the boundary of the area enclosed by the  $x$ -axis and the upper half of the circle  $x^2 + y^2 = a^2$ .

OR

- 3.a) Find the work done by  $\vec{F} = (2x - y - z)i + (x + y - z)j + (3x - 2y - 5z)k$  along a curve  $C$  in the  $xy$ -plane given by  $x^2 + y^2 = 9$ ,  $z = 0$ .
- b) Use Gauss divergence theorem, to evaluate  $\iiint_S (x dy dz + y dz dx + z dx dy)$ , where  $S$  is the portion of the plane  $x + 2y + 3z = 6$ , which lies in the first octant.

- 4.a) Expand  $f(x) = x \sin x$  as a Fourier series in the interval  $0 < x < 2\pi$ .
- b) Express  $f(x) = \begin{cases} 1, & \text{for } 0 \leq x \leq \pi \\ 0, & \text{for } x > \pi \end{cases}$ , as a Fourier sine integral and hence evaluate  $\int_0^{\infty} \frac{1 - \cos(\pi\lambda)}{\lambda} \sin(x\lambda) d\lambda$ .

OR

- 5.a) Find the half-range cosine series for  $f(x) = x^2$  in the range  $0 \leq x \leq \pi$ .
- b) Find the Fourier sine transform of  $\frac{e^{-ax}}{x}$ .

- 6.a) If  $P$  is the pull required to lift a load  $W$  by means of a pulley block, find a linear law of the form  $P = mW + c$  connecting  $P$  and  $W$ , using the following data:

$P=12$	15	21	25
$W=50$	70	100	120

- where  $P$  and  $W$  are taken in  $kg.wt$ . Compute  $P$  when  $W = 150 kg.wt$ .  
 b) Find the missing values in the following data

$x:$	45	50	55	60	65
$y:$	3.0	-	2.0	-	-2.4

**OR**

- 7.a) The pressure  $p$  of wind corresponding to velocity  $v$  is given by the following data. Estimate  $p$  when  $v = 25$ .

$v:$  10 20 30 40

$p:$  1.1 2 4.4 7.9

- b) Find the polynomial  $f(x)$  by using Lagrange's formula and hence find  $f(3)$  for  
 $x:$  0 1 2 5  
 $f(x):$  2 3 12 147.

- 8.a) Using Newton's iterative method, find the real root of  $x \log_{10} x = 1.2$  correct to four decimal places.

- b) Solve:  $20x + y - 2z = 17$ ,  $3x + 20y - z = -18$ ,  $2x - 3y + 20z = 25$  by Gauss-Seidel iterative method.

**OR**

- 9.a) Solve by L-U decomposition method:  $x + y + z = 9$ ,  $2x - 3y + 4z = 13$ ,  
 $3x + 4y + 5z = 40$ .

- b) Use the method of false position to find the fourth root of 32 correct to three decimal places.

- 10.a) Using Runge-Kutta method of order 4, find  $y(0.2)$  for the equation  
 $\frac{dy}{dx} = \frac{y-x}{y+x}$ ,  $y(0) = 1$ . Take  $h = 0.2$ .

- b) A solid of revolution is formed by rotating about the  $x$ -axis, the area between the  $x$ -axis, the lines  $x = 0$  and  $x = 1$  and a curve through the points with the following co-ordinates

$x:$  0.00 0.25 0.50 0.75 1.00

$y:$  1.0000 0.9896 0.9589 0.9089 0.8415.

Estimate the volume of the solid formed using Simpson's rule.

**OR**

11. Use power method to find the numerically largest Eigen value and the

corresponding Eigen vector of  $A = \begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$ . Find also the least Eigen value

and hence the third Eigen value also.

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