Code No: 132AB

R16

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B.Tech I Year II Semester Examinations, April - 2018
MATHEMATICS-II
(Common to EEE, ECE, CSE, EIE, IT, ETM)

Time: 3 hours

Max. Marks: 75

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit. Each
question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a)   Find $L\{t^2\,u(t-1)\}$. [2]

b)   Obtain the inverse Laplace transform of $F(s) = \cot^{-1} s$. [3]

c)   Find the value of $\Gamma\left(\dfrac{-1}{2}\right)$. [2]

d)   Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \sqrt{\tan\theta}\,d\theta$ using Beta and Gamma functions. [3]

e)   Evaluate $\displaystyle\iint_R \sqrt{x^2 + y^2}\,dx\,dy$ by changing to polar coordinates, where R is the region

in the xy–plane bounded by the circles $x^2 + y^2 = 1$ and $x^2 + y^2 = 4$. [2]

f)   Find the value of the triple integral $\displaystyle\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^1 r^2\,dr\,d\theta\,d\phi$. [3]

g)   Find the normal vector and unit normal vector to the surface $z^2 = x^2 - y^2$ at $\left(2,1,\sqrt{3}\right)$. [2]

h)   If $\bar{a}$ is a constant vector and $\bar{r} = x\hat{i} + y\hat{j} + z\hat{k}$, prove that $div\,(\bar{a} \times \bar{r}) = 0$. [3]

i)   Evaluate $\displaystyle\int_c (x^2 + yz)dz$, where c is given by $x = t, y = t^2, z = 3t, 1 \le t \le 2$. [2]

j)   State Gauss's divergence theorem. [3]

PART-B

(50 Marks)

2.a)   Find the Laplace transform of $f(t) = e^{-t}\left[\displaystyle\int_0^t \frac{\sin u}{u}\,du\right]$.

b)   Find the Laplace transform of the periodic function $f(t) = t,\ 0 \le t \le a,\ f(t+a) = f(t)$.

[5+5]

OR

3.a) Obtain $L^{-1}\left\{\dfrac{s}{\left(s^2+1\right)^2}\right\}$ using convolution theorem.

b) Solve the initial value problem $y''+3y'+2y=e^{-t}$, $y(0)=0$, $y'(0)=-1$ using Laplace transforms. [5+5]

4.a) Evaluate $\displaystyle\int_0^\infty x^4 e^{-2x^2}\,dx$.

b) Prove that $\beta(m,n)=\displaystyle\int_0^1 \dfrac{x^{m-1}+x^{n-1}}{(1+x)^{m+n}}\,dx$. [5+5]

OR

5.a) Prove that $\beta(m,n)=2\displaystyle\int_0^{\pi/2}\sin^{2m-1}\theta\,\cos^{2n-1}\theta\,d\theta=\beta(m+1,n)+\beta(m,n+1)$.

b) Prove that $\displaystyle\int_0^1 x^m(\ln x)^n\,dx=\dfrac{(-1)^n\,n!}{(m+1)^{n+1}}$ where $m>1$ and n is a positive integer. [5+5]

6.a) Change the order of integration in $\displaystyle\int_0^1\int_{x^2}^{2-x} xy\,dx\,dy$ and hence evaluate the same.

b) Find the area of the region bounded by the parabolas $y^2=4x$ and $x^2=4y$. [5+5]

OR

7.a) Evaluate $\displaystyle\iiint_v (1-x)\,dx\,dy\,dz$, where v is the space in the first octant below the plane $3x+2y+z=6$.

b) Find the volume of the solid enclosed between the surfaces $x^2+y^2=4$ and $x^2+z^2=4$. [5+5]

8.a) Find the values of a and b so that the surface $ax^2-byz=a+2$ is orthogonal to the surface $4x^2y+z^3=4$ at ($1,-1,2$).

b) Find the directional derivative of the scalar function $f(x,y,z)=xyz$ at ($1,4,9$) in the direction of the line from ($1,2,3$) to ($1,-1,-3$). [5+5]

OR

9.a) If $\vec{r}=x\,\hat{i}+y\,\hat{j}+z\,\hat{k}$ and $r=|\vec{r}|$, show that $\nabla\cdot\left(\dfrac{\vec{r}}{r^3}\right)=0$.

b) Prove that $\operatorname{curl}\left(\operatorname{curl}\vec{v}\right)=\nabla(\nabla\cdot\vec{v})-\nabla^2\vec{v}$. [5+5]

10.a) Show that $\int_c (2xy+3)dx + (x^2-4z)dy - 4y\,dz$, where c is any path joining $(0,0,0)$ to $(1, -1, 3)$, does not depend on the path c and evaluate the integral.

b) Apply Stoke's theorem to evaluate $\oint_c (x+y)dx + (2x-z)dy + (y+z)dz$, where c is the boundary of the triangle with vertices $(2, 0, 0)$, $(0,3,0)$ and $(0,0,6)$. [5+5]

OR

11. Verify Green's theorem for $\oint_c e^{-x}(\cos y\,dx - \sin y\,dy)$, where c is the rectangle with vertices $(0,0)$, $(\pi,0)$, $\left(\pi,\dfrac{\pi}{2}\right)$ and $\left(0,\dfrac{\pi}{2}\right)$. [10]

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