

Note: This question paper contains two parts A and B. Part A is compulsory which carries 25 marks. Answer all questions in Part A. Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A

(25 Marks)

- Solve $y = a \sqrt{1 + p^2}$. [2]
- Solve $\frac{1}{D^2} x^4$. [2]
- Evaluate $\int_{x=1}^3 \int_{y=0}^1 xy^2 dy dx$. [2]
- If $\vec{r} = xi + yj + zk$, then evaluate $\nabla^2(r^2)$. [2]
- Find the value of $\int_V (i + j + k) dV$. [2]
- Find the integrating factor of $\frac{dy}{dx} + 2xy = e^{-x^2}$. [3]
- Solve $(D^3 - 4D^2)y = 5$. [3]
- Find the limits after changing the order of integration for $\int_0^{a/b} \int_0^{b/\sqrt{b^2-y^2}} f(xy) dy dx$. [3]
- Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$. [3]
- If $\vec{F}(t) = xi + 2yj + zk$ then evaluate $\int_1^2 \text{curl } \vec{F}(t) dt$. [3]

PART-B

(50 Marks)

- Solve $(1 + x^2) \frac{dy}{dx} + 2xy = 4x^2$, $y(0) = 0$. [5]
- If 30% of a radioactive substance disappears in 10 days, how long will it take for 90% of it to disappear? [5+5]

OR

- Solve $(y + y^2)dx + xy dy = 0$. [5]
- Solve $(x + 2y^3) \frac{dy}{dx} = y$. [5]

- Solve $(D^2 + 4)y = \tan 2x$ by variation of parameters. [5]
- Solve $(D^3 + 4D)y = 5 + \sin 2x$. [5]

OR

- Solve $(D^2 + 4D + 3)y = e^{ex}$. [5]
- Solve $(D^2 + 1)y = x^2 \sin 2x$. [5]

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6.a) Evaluate $\int_0^\pi \int_0^{a(1+\cos\theta)} r^2 \cos\theta dr d\theta$.

b) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} dz dy dx$.

OR

[5+5]

7.a) Change into polar co-ordinates and evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dy dx$.

b) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$.

[5+5]

8.a) Find the angle between the normal to the surface $xy = z^2$ at the points $(4, 1, 2)$ and $(3, 3, -3)$.

b) Prove that $\nabla \cdot (\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$.

[5+5]

OR

9.a) Find the angle of intersection of the spheres $x^2 + y^2 + z^2 = 39$ and $x^2 + y^2 + z^2 + 4x - 6y - 8z + 52 = 0$ at the point $(4, -3, 2)$.

b) A vector field is given by $\vec{A} = (x^2 + xy^2)\vec{i} + (y^2 + x^2y)\vec{j}$. Show that the field is irrotational and find the scalar potential.

[5+5]

10. Find the work done in moving a particle in the force field $\vec{F} = 3x^2\vec{i} + (2xz - y)\vec{j} + z\vec{k}$ along the straight line from $(0, 0, 0)$ to $(2, 1, 3)$.

[10]

OR

11.a) Evaluate $\iint_S \vec{F} \cdot \hat{n} ds$ if $\vec{F} = 2xy\vec{i} + yz^2\vec{j} + xz\vec{k}$ over the parallelepiped $x = 0, y = 0, z = 0, x = 2, y = 1, z = 3$.

b) If $\vec{F} = (3x^2 - 2z)\vec{i} - 4xy\vec{j} - 5x\vec{k}$, Evaluate $\int_V \text{curl } \vec{F} dv$, where V is volume bounded by planes $x = 0, y = 0, z = 0$ and $3x + 2y - 3z = 6$.

[5+5]

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