

Code No: 132AB

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech I Year II Semester Examinations, May/June - 2017

MATHEMATICS-II

(Common to EEE, ECE, CSE, EIE, IT)

Time: 3 hours

Max. Marks: 75

Note: This question paper contains two parts A and B.

Part A is compulsory which carries 25 marks. Answer all questions in Part A.

Part B consists of 5 Units. Answer any one full question from each unit. Each question carries 10 marks and may have a, b, c as sub questions.

PART-A**(25 Marks)**

- 1.a) Find the Laplace transform of $f(t) = \begin{cases} K, & 0 < t < 2 \\ 0, & 2 < t < 4 \end{cases}$, $f(t+4) = f(t), \forall t > 0$. [2]
- b) Find the Laplace transform of $f(t) = \frac{1-e^{-t}}{t}$. [3]
- c) Evaluate $\beta\left(\frac{9}{2}, \frac{7}{2}\right)$. [2]
- d) Evaluate $\int_0^{\infty} e^{-x^2} dx$. [3]
- e) Evaluate $\int_0^{\frac{\pi}{2}} \sin^2 \theta \cos^2 \theta d\theta$ using beta and gamma functions. [2]
- f) Show that the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ is $\frac{16}{3}a^2$. [3]
- g) Find a vector normal to the surface $xyz^2 = 20$ at the point $(1, 1, 2)$. [2]
- h) If $u\bar{F} = \nabla u$, where u, v are scalar fields and \bar{F} is a vector field, show that $\bar{F} \cdot \text{curl } \bar{F} = 0$. [3]
- i) State Green's theorem. [2]
- j) Find the work done by a force $y\bar{i} + x\bar{j}$ which displays a particle from origin to a point $(\bar{i} + \bar{j})$ along the line $y = x$. [3]

PART-B**(50 Marks)**

- 2.a) Express the function $f(t)$ in terms of unit step function, where
- $$f(t) = \begin{cases} t-1, & 1 < t < 2 \\ 3-t, & 2 < t < 3 \end{cases}$$
- Hence find its Laplace transform. [5+5]
- b) Find the Laplace transform of $\int_0^{\infty} te^{-3t} \sin t dt$. [5+5]

OR

- 3.a) State the convolution theorem on Laplace transforms. Using it find the inverse Laplace transform of $\frac{1}{s(s^2+a^2)}$. [5+5]
- b) Solve $y'' + 2y' + 5y = e^{-t} \sin t$, $y(0) = 0$ and $y'(0) = 1$ using Laplace transforms. [5+5]

- 4.a) Evaluate $\int_0^1 x^{\frac{3}{2}} (1-x)^{\frac{3}{2}} dx$ using Beta, Gamma functions. [5+5]

- b) Evaluate $\int_0^1 \frac{dx}{(1-x^2)^{\frac{1}{n}}}$. [5+5]

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OR

5.a) Show that $\int_0^\infty \frac{t^{m-1}}{(a+bt)^{m+n}} dt = \frac{\beta(m,n)}{a^n b^m}$, where m, n, a, b are positive integers.

b) Evaluate $\int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx$. [5+5]

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6.a) Evaluate $\int_0^1 \int_0^1 \frac{x^3 dx dy}{\sqrt{x^2+y^2}}$ by changing into polar coordinates.

b) By double integration, calculate the area bounded by the curve $a^2x^2 = y^3(2a - y)$. [5+5]

OR

7.a) Find the area enclosed in the first quadrant by the curve $(\frac{x}{a})^\alpha + (\frac{y}{b})^\beta = 1$, $\alpha > 0, \beta > 0$, using beta gamma functions.

b) Find the center of gravity of the area of the cardioid $r = a(1 + \cos \theta)$. [5+5]

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8.a) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$.

b) If $f = (x^2 + y^2 + z^2)^{-n}$, find $div grad f$ and determine n if $div grad f = 0$. [5+5]

OR

9.a) Show that the vector $\vec{F} = (x+3y)\vec{i} + (y-3z)\vec{j} + (x-2z)\vec{k}$ is solenoidal and also find $\vec{F} \cdot curl \vec{F}$.

b) In what direction from $(3, 1, -2)$ is the directional derivative of $\phi = x^3y^2 + yz$ maximum? Find also the magnitude of this maximum. [5+5]

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10. State Stokes theorem. Verify it for the vector field $\vec{F} = (2x-y)\vec{i} - yz^2\vec{j} - y^2z\vec{k}$ over the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$, bounded by its projection on the xy -plane. [10]

OR

11.a) Using Green's theorem, find the area of the region in the first quadrant bounded by the curves $y = x, y = \frac{1}{x}, y = \frac{x}{4}$.

b) Evaluate $\iiint_V div \vec{F} dV$, where $\vec{F} = y\vec{i} + x\vec{j} + z^2\vec{k}$ over the surface of the cylinder $x^2 + y^2 = a^2, z = 0, z = h$. [5+5]

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