R15

Code No: 123BT

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, November/December - 2016

PROBABILITY THEORY AND STOCHASTIC PROCESSES

(Common to ECE, ETM)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit.
Each question carries 10 marks and may have a, b, c as sub questions.

PART- A

(25 Marks)

1.a) A discrete random variable can be defined on a continuous sample space. State whether it is true or false. Give an example to support your claim. [2]

b) Write the conditions to be satisfied by a function to be a random variable. [3]

c) Write the properties of probability density function. [2]

d) Determine whether the following function is a valid probability distribution function or not? Write the properties used. $G_X(x) = \frac{x}{a}[u(x-a) - u(x-2a)]$. [3]

e) Write two properties of joint distribution function of random variables. [2]

f) State Central limit theorem. [3]

g) Give an example of a deterministic random process. [2]

h) Auto correlation function of a stationary random process is $R_{XX}(\tau) = 25 + \frac{4}{1+6\tau^2}$. Find its variance. [3]

i) Check whether the function below is a valid power density spectrum or not.

$\frac{\omega}{j\omega^6 + \omega^2 + 3}$. [2]

j) Autocorrelation function of a random process is given by $R_{XX}(\tau) = 3\delta(\tau)$. Find and sketch its power density spectrum. [3]

PART-B

(50 Marks)

2.a) State and prove Bayes Theorem.

b) Define the terms outcome, event, sample space, mutually exclusive events. Consider the experiment of rolling of two fair dice simultaneously and represent its sample space. Also give examples of terms mentioned above related to this experiment. [5+5]

OR

3.a) Discuss the relative frequency approach and axiomatic approach of probability.

b) In a box there are 100 resistors whose resistances and tolerances are as shown in the table below. Let A be the event of drawing a 47Ω resistor, B be the event of drawing a resistor with 5% tolerance, and C be the event of drawing a 100Ω resistor. Find ~~P(A/B),~~ P(A/C) and P(B/C). [5+5]

	Tolerance		
Resistance (Ω)	5%	10%	Total
22	10	14	24
47	28	16	44
100	24	8	32
Total	62	38	100

4.a) Find the mean of Binomial random variable.

b) In a sports event javelin throw distances are well approximated by a Gaussian distribution for which mean is 30m and standard deviation is 5m. In a qualifying round, contestants must throw farther than 27m to qualify. In the main event the record throw is 44m.

i) What is the probability of being disqualified in the first round?

ii) In the main event what is the probability the record will be broken? [5+5]

OR

5.a) Obtain the characteristic function of Poisson random variable.

b) X and Y are two statistically independent random variables related to W as W= X + Y. Obtain the probability density function of Y in terms of probability density functions of X and Y. [5+5]

6.a) Obtain the expression for conditional density $f_X(X/B)$ where event B is defined as $\{ y_a \le Y \le y_b \}$.

b) Write short notes on jointly Gaussian random variables. [5+5]

OR

7.a) Two random variables X and Y have joint characteristic function $\varphi_{XY}(\omega_1,\omega_2) = \exp(-2\omega_1^2 - 8\omega_2^2)$. Show that X and Y are uncorrelated zero mean random variables.

b) Two statistically independent random variables X and Y have mean values $E[X] = 2$ and $E[Y] = 4$. They have second moments $E[X^2] = 8$ and $E[Y^2] = 25$. Find Variance of $W = 3X-Y$. [5+5]

8.a) A random process is defined as $X(t) = A\cos(\omega_o t + \Theta)$, where Θ is a uniformly distributed random variable in the interval $(0, \pi/2)$. Check for its wide sense stationarity? A and ω_o are constants.

b) Classify random processes and explain. [6+4]

OR

9.a) Define autocorrelation function of a random process. Write its properties and prove any two of them.

b) Explain the concept of time average and ergodicity. Write the conditions for a random process to be ergodic in mean and autocorrelation. [5+5]

10.a) Derive the expression for power density spectrum of a random process.

b) Write the properties of power spectral density. [6+4]

OR

11.a) Prove $S_{YY}(\omega) = \left|H(\omega)^2\right| S_{XX}(\omega)$. Where X(t) is input random process of an LTI system and Y(t) its output. $|H(\omega)|$ is the transfer function of the LTI system.

b) Define cross power density spectrum and write its properties. [5+5]

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