Code No: 113AH

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JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD

B.Tech II Year I Semester Examinations, November - 2015

MATHEMATICS-III

(Common to EEE, ECE, EIE, AGE, ETM)

Time: 3 Hours **Max. Marks: 75**

Note: This question paper contains two parts A and B.
Part A is compulsory which carries 25 marks. Answer all questions in Part A.
Part B consists of 5 Units. Answer any one full question from each unit.
Each question carries 10 marks and may have a, b, c as sub questions.

PART- A **(25 Marks)**

1.a) Determine the nature of the point x= 0 for the equation $x^2(x^2+1)\,y'' + (x^2-1)\,y' + 2y = 0$. [2M]

b) Find the series solution of the equation $y'' - y = 0$. [3M]

c) Express $2 - 3x + 4x^2$ in terms of Legendre polynomial. [2M]

d) Express $J_3(x)$ in terms of J_0 and J_1. [3M]

e) Prove that $f(z) = \bar{z}$ is not analytic at any point. [2M]

f) Show that the function f(z) = sinx cosh y + i cosx sinh y is continuous as well as analytic every where. [3M]

g) State the Cauchy's Residue theorem. [2M]

h) Expand log z by Taylor's series about z = 1. [3M]

i) Define conformal transformation. [2M]

j) Find the points at which w = coshz is not conformal. [3M]

PART-B **(50 Marks)**

2.a) Obtain the series solution of the equation $y'' + x\,y' + y = 0$

b) Find the series solution of $4x\,y'' + 2\,y' + y = 0$. [4+6]

OR

3.a) Solve in series the equation $y'' + xy = 0$

b) Solve in series the equation $(1 - x^2) - 2xy + n(n+1)y = 0$ about x = 0. [4+6]

4.a) Prove that $nP_n = (2n-1) \times P_{n-1} - (n-1)\,P_{n-2}$, $n \geq 2$

b) State and prove generating function of Bessel's function. [5+5]

OR

5.a) Prove that $\int_{-1}^{1} P_n(x)P_m(x)dx = 0$, if $m \neq n$, $2/(2n+1)$ if $m = n$.

b) Prove that $J_0^2 + 2(J_1^2 + J_2^2 + J_3^2 + \ldots\ldots) = 1$. [5+5]

6.a) Find the analytic function whose real part is $u = e^{2x}(x\cos 2y - y\sin 2y)$.

b) Evaluate $\int_C \mathrm{Re}\, z\, dz$ where C is the shortest paoth from 1+i to 3+2i. [5+5]

OR

7.a) State and prove Cauchy's integral theorem.

b) Evaluate using Cauchy's integral formula $\int_C \frac{e^{2z}}{(z-1)(z-2)}dz$, where C is the circle $|z| = 3$. [5+5]

8.a) Express $f(z)=\frac{z}{(z-1)(z-3)}$ in series of positive and negative powers of (z-1).

b) Evaluate $\int_0^{2\pi}\frac{1}{(5-3\cos\theta)}d\theta$ using residue theorem. [5+5]

OR

9.a) Give two Laurent's series expansions in powers of z for $f(z)=\frac{1}{(1-z)z^2}$ and specify the region in which these expansions are valid.

b) Evaluate $\int_C\frac{z^2+2z-2}{z(z-4)(z-1)}dz$ where C is $|z|=1.5$. [5+5]

10.a) Under the transformation w = 1/z find the image of the circle $|z-2i|=2$.

b) Find the bilinear transformation which maps the points (-1,0,1) into the points (0, i, 3i) [5+5]

OR

11.a) Find the image of the region in the z-plane between the line y = 0 and y = $\pi/2$ Under the transformation $w=e^z$.

b) Show the bilinear transformation preserves the cross ratio. [5+5]

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