Code No: 154AQ

R18

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech II Year II Semester Examinations, March - 2022
DISCRETE MATHEMATICS
(Common to CSE, IT, ITE)

Time: 3 Hours **Max. Marks: 75**

Answer any five questions
All questions carry equal marks

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1.a) Construct a truth table for each of these compound propositions.
i) $p \to \neg p$ ii) $p \leftrightarrow \neg p$ iii) $p \wedge q \to p \vee q$ iv) $(q \to \neg p) \leftrightarrow (p \leftrightarrow q)$

b) Show that if p, q, and r are compound propositions such that p and q are logically equivalent and q and r are logically equivalent, then p and r are logically equivalent.[8+7]

2.a) Suppose that the domain of the propositional function $P(x)$ consists of the integers -2, -1, 0, 1, and 2. Write out each of these propositions using disjunctions, conjunctions, and negations.
i) $\exists x \neg P(x)$ ii) $\forall x \neg P(x)$ iii) $\neg \exists x P(x)$ iv) $\neg \forall x\, P(x)$

b) Translate each of these statements into logical expressions using predicates, quantifiers and logical connectives.
i) Something is not in the correct place.
ii) All tools are in the correct place and are in excellent condition.
iii) Everything is in the correct place and in excellent condition.
iv) Nothing is in the correct place and is in excellent condition.
v) One of your tools is not in the correct place, but it is in excellent condition. [7+8]

3.a) Determine whether each of these statements is true or false:
i) $x \in \{x\}$ ii) $\{x\} \subseteq \{x\}$ iii) $\{x\} \in \{x\}$ iv) $\{x\} \in \{\{x\}\}$ v) $\emptyset \subseteq \{x\}$

b) Let A and B be sets. Show that: i) $(A \cap B) \subseteq A$ ii) $A \cap (B - A) = \emptyset$

c) Suppose that the universal set is $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ each of these sets can be represented with bit strings where the i^{th} bit in the string is 1 if i is in the set and 0 otherwise. Find the set specified by each of these bit strings.
i) 11 1100 1111 ii) 01 0111 1000 iii) 10 0000 0001 [5+5+5]

4.a) Determine whether each of these functions is a bijection from **R** to **R**
i) $f(x) = -3x + 4$ ii) $f(x) = -3x^2 + 7$
iii) $f(x) = (x + 1)/(x + 2)$ iv) $f(x) = x^5 + 1$

b) Consider these functions from the set of students in a discrete mathematics class. Under what conditions is the function one-to-one if it assigns to a student his or her
i) mobile phone number ii) student identification number.
iii) final grade in the class iv) home town.

c) List the first 10 terms of each of these sequences.
i) the sequence obtained by starting with 10 and obtaining each term by subtracting 3 from the previous term
ii) the sequence whose n^{th} term is the sum of the first n positive integers
iii) the sequence whose n^{th} term is $3^n - 2^n$
iv) the sequence whose n^{th} term is $\lfloor \sqrt{x} \rfloor$
v) the sequence whose first two terms are 1 and 5 and each succeeding term is the sum of the two previous terms. [5+5+5]

5.a) Describe an algorithm that takes as input a list of n integers and finds the number of negative integers in the list.

b) In this problem, to establish a big-O relationship, find witnesses C and k such that $|f(x)| \leq C|g(x)|$ whenever $x > k$. Determine whether each of these functions is $O(x)$.
i) $f(x) = 5 \log x$ ii) $f(x) = \lceil x/2 \rceil$

c) Suppose that an element is known to be among the first four elements in a list of 32 elements. Would a linear search or a binary search locate this element more rapidly?
[5+5+5]

6.a) Let $P(n)$ be the statement that $1^2 + 2^2 + \cdots + n^2 = n(n + 1)(2n + 1)/6$ for the positive integer n.
i) What is the statement $P(1)$?
ii) Show that $P(1)$ is true, completing the basis step of the proof.
iii) What is the inductive hypothesis?
iv) What do you need to prove in the inductive step?
v) Complete the inductive step, identifying where you use the inductive hypothesis

b) Give a recursive definition of the:
i) the set of even integers
ii) the set of positive integers congruent to 2 modulo 3.
iii) the set of positive integers not divisible by 5

c) Give a recursive algorithm for finding the sum of the first n positive integers. [5+5+5]

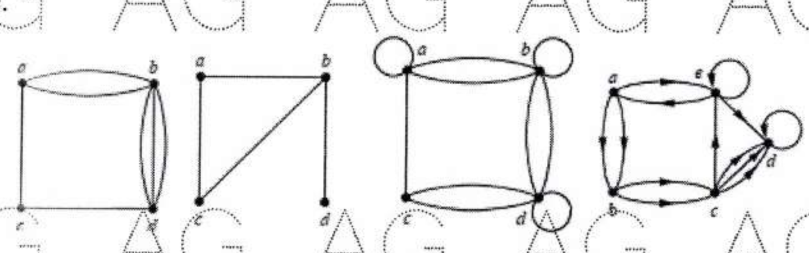
7.a) Which is more likely: rolling a total of 8 when two dice are rolled or rolling a total of 8 when three dice are rolled?

b) Find the probability that a family with five children does not have a boy, if the sexes of children are independent and if a boy and a girl are equally likely.

c) Suppose that 8% of all bicycle racers use steroids, that a bicyclist who uses steroids tests positive for steroids 96% of the time, and that a bicyclist who does not use steroids tests positive for steroids 9% of the time. What is the probability that a randomly selected bicyclist who tests positive for steroids actually uses steroids? [5+5+5]

8.a) How can a graph that models e-mail messages sent in a network be used to find electronic mail mailing lists used to send the same message to many different e-mail addresses?

b) For each undirected graph that is not simple, find a set of edges to remove to make it simple. [8+7]



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