UNIT - III

3 Design of Beams

3.1 Plastic Theory

Q.1 Explain plastic theory of structural steel.

Ans. : • Steel has a unique property called ductility, because of which it is able to absorb large deformation beyond the elastic limit without fracture.

- Due to this property, steel possesses reserve strength beyond its yield strength.
- The method which utilizes this reserve strength is called the plastic method of analysis.
- The plastic theory makes the design procedure more rational, since the level of safety is related to collapse load of the structure and not to the apparent failure at one point.

Q.2 Give general requirements for plastic design.

Ans. : The following assumptions are made in plastic design to simplify computations.

- The material obeys Hook's law till the stress reaches f_y.
- The yield stress and modulus of elasticity have the same value in compression and tension. (See Fig. Q.2.1).

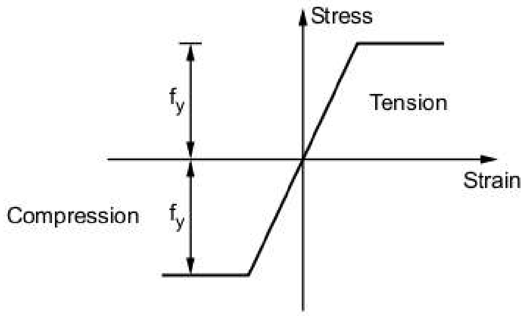

Fig. Q.2.1 Stress - Strain curve for perfectly plastic material

- The material is homogeneous and isotropic in both the elastic and plastic states.
- The material is assumed to be sufficiently ductile to permit large rotation of the section to take place.
- Plastic hinge rotations are when compared with the elastic deformations, so that all the rotations are concentrated at the plastic hinges. The segments between plastic hinges. The segments between plastic hinges are rigid.
- The influence of normal and shear forces on plastic moments is not considered.
- Plane sections remain plane even after bending and the effect of shear is neglected.
- The equilibrium of forces at the time of collapse is considered for the undeformed state of the structure.
- No instability occurs in any member of the structure upto collapse.

3.2 Plastic Hinge Concept

Q.3 Explain the plastic hinge concept with neat sketch of the section.

Ans. : • At the centre of area there will be fully plasticity over the full depth of section and section will behave like a hinge. This is called plastic hinge.

- It behaves like a frictionless mechanical hinge except that there is always a fixed moment constraint which is equal to the plastic moment capacity of the section.
- Thus the plastic hinge may be defined as a **yielded zone**, where the bending of a structural member can cause an infinite rotation to take place at a constant plastic moment M_P of the section.
- As shown in above, plastic hinges form in a member at the maximum bending moment locations.

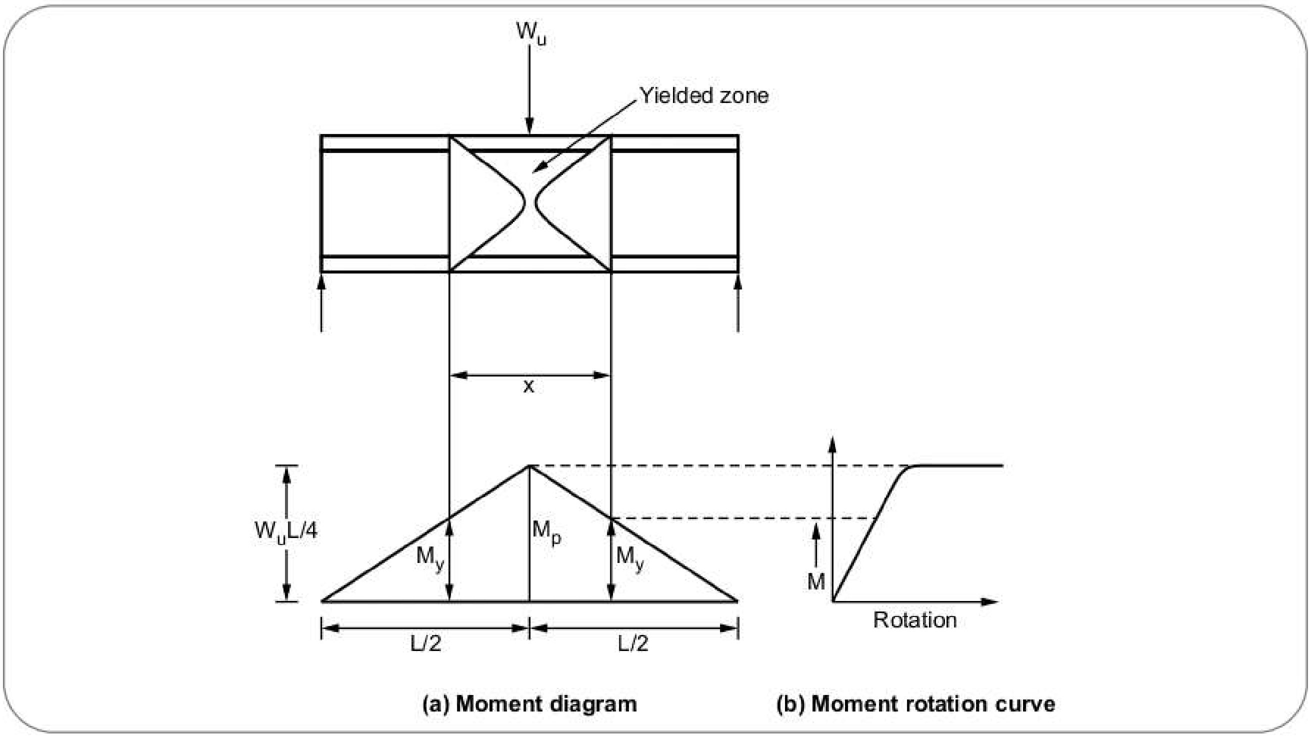

Fig. Q.3.1 Plastic hinge concept

- However, at the intersections of two members, where the bending moment is the same, a hinge forms in weaker member.
- Generally, hinges are located at restrained ends, intersections of members, and point loads.
- The hinges may not form simultaneously as the loading increases, but this is not important for calculating the final collapse load.

3.3 Theorems of Plastic Analysis

Q.4 Explain the theorems of plastic analysis ?

Ans. : Theorems in plastic analysis : Based on the number of conditions satisfied, we have three theorems in plastic analysis.

1) Static Theorem (Lower bound theorem) :

- This theorem states that the load computed from any distribution of BMDs in equilibrium with external loads (safe and statically admissible BMD) so that the maximum BM in any member shall not exceed its plastic moment, M_p ($M <$ or $= M_p$) is less than or equal to the true collapse load.
- This theorem leads to equilibrium or static method of plastic analysis.
- This theorem also called the safe theorem satisfies equilibrium and plastic moment condition.

2) Kinematic Theorem (Upper bound theorem) :

- This theorem states that the load computed from any assumed kinematically admissible mechanism is greater than or equal to the true collapse load.
- This theorem leads to kinematic or mechanism method of analysis.

3) Uniqueness Theorem :

- This theorem states that if the load evaluated by static and kinematic theorems is same, then it is the true collapse load.
- All the three conditions of plastic analysis are satisfied. According to this theorem, there is only one unique solution for a given structure, while there are innumerable possible solutions with other theorems.

3.4 Classifications of Beams as per I.S. 800 - 2007

Q.5 Explain classification of sections as per IS 800 : 2007.

Ans. : On the basis of IS 800 - 2007, classification of various cross section as follows :

1) Class 1 (Plastic) Cross Section

These sections can develop plastic hinges and have the rotation capacity required for failure of the structure by formation of plastic mehanism.

2) Class 2 (Compact) Cross Section

Such sections can develop plastic moment of resistance, but have inadeqaute plastic hinge rotation capacity for formation of plastic mechanism, due to local buckling.

3) Class 3 (Semicompact) Cross Sections

These are the sections in which the extreme fibre in compression can reach yield stress, but cannot develop the plastic moment of resistance, due to local buckling.

4) Class 4 (Slender) Cross Sections

The cross sections the element of which buckle locally even before reaching yield stress belong to this category.

Q.6 Draw neat sketches of different types of beam sections.

Ans. :

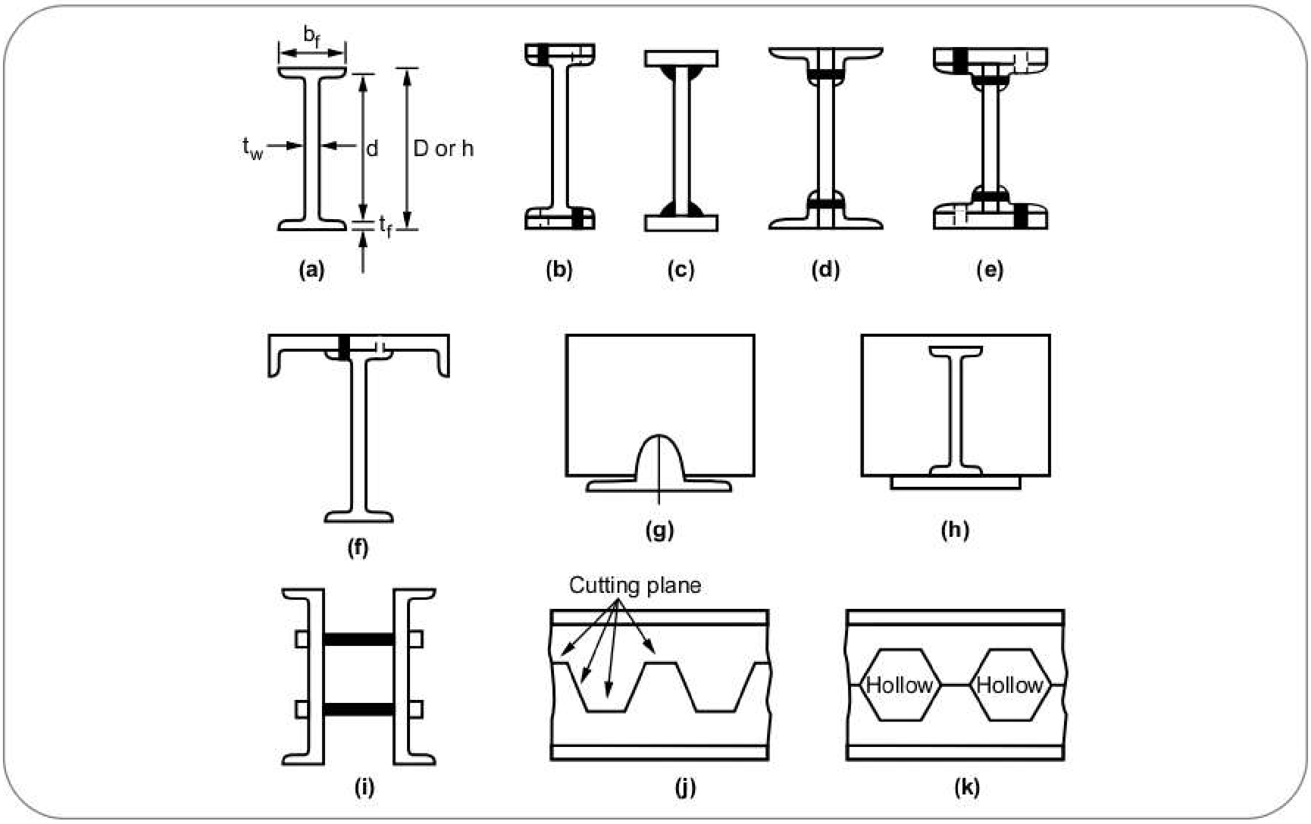

Fig. Q.6.1 Different types of open beam sections

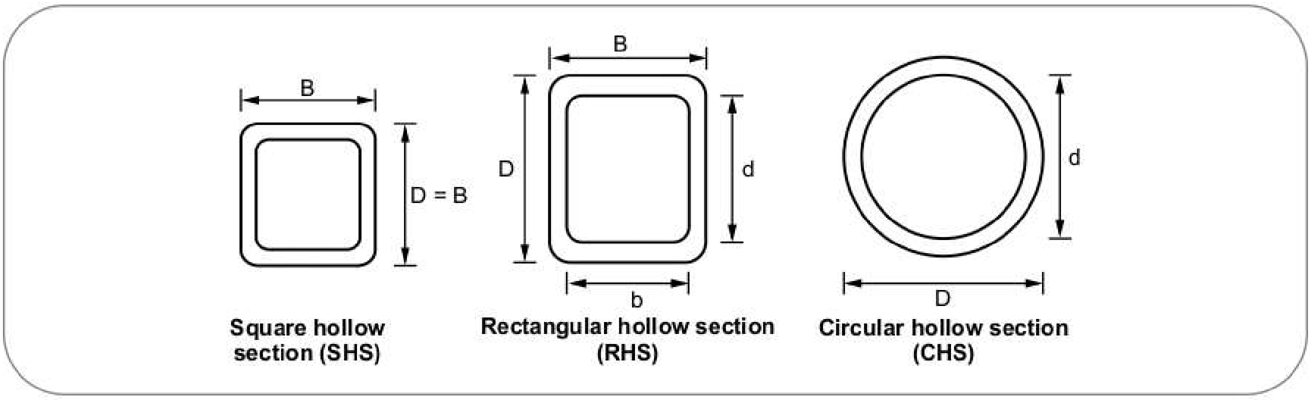

Fig. Q.6.2 Some typical types of closed beam sections.

3.5 Design of Beams

Q.7 Explain the behaviour of beam in flexure or moment.

Ans. : • Consider the cross section of a simply supported beam where the bending moment is maximum for given loading.

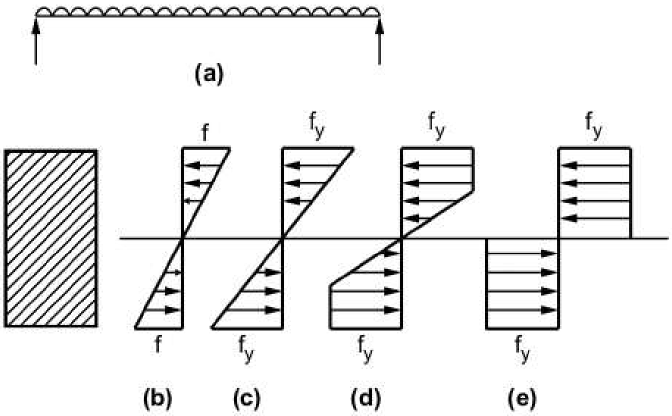

Fig. Q.7.1

1) Within the elastic limit the stress varies linearly from compression to tension as shown in Fig. Q.7.1 (b).
2) As the load is gradually increased stresses increase proportionally till extreme fibre is subjected to yield stress. Then extreme fibre yields. Fig. Q.7.1(c).
3) As the load is gradually increased by one fibre reach yield stress and stop resisting additional load. Fig. Q.7.1(d) shows partially yielded stress.
4) However, resistance to load continuous till all fibers are yielded as shown in Fig. Q.7.1(e).
5) After this condition the section will not resist further moment due to increase in load. This condition when all fibres at a section yield is called **formation of plastic hinge**.
6) After this the rotation at section will take place without resisting additional moment but the moment corresponding to yielding of all fibres is resisted. This moment capacity is called **plastic moment capacity** of section and is denoted by M_p.

3.6 Bending and Shear Strength / Buckling

Q.8 Give expression for bending strength of laterally supported beams.

Ans. : IS 800 : 2007 consider two cases with design shear strength (V_d).

a) If V 0.6 V_d

The design bending strength M_d shall be taken as,

$$\boxed{M_d = {}_b \, Z_p \, f_y \, \frac{1}{m0}}$$

$$1.2 \, Z_e \, f_y \, \frac{1}{m0} \text{ (For simply supported beam)}$$

$$1.5 \, Z_e \, f_y \, \frac{1}{m0} \text{ (For cantilever beam)}$$

where,

$${}_b = 1.0 \text{ for plastic and compact sections.}$$

$= \frac{Z_e}{Z_p}$ for semi-compact sections

Z_p = Plastic sectional modulus

Z_e = Elastic sectional modulus

b) If V > 0.6 V_d

$$\boxed{M_d = M_{dv}}$$

where, M_{dv} = Design bending strength under high shear

M_{dv} is to be calculated as given in IS 800 : 2007, Clause 9.2.2.

a) Plastic or Compact section

$$\boxed{M_{dv} = M_d - (M_d - M_{fd})} \quad 1.2\, Z_e \quad f_y \quad \frac{1}{m0}$$

where,

$$= \frac{2V}{V_d} - 1 \quad ^2$$

M_d = Plastic design moment of the whole section.

V = Factored applied shear force.

V_d = Design shear strength

M_{fd} = Plastic design strength of the area of cross section excluding the shear area

b) Semi - compact section

$$\boxed{M_{dv} = \frac{Z_e \; f_y}{m0}}$$

Q.9 Explain lateral stability of beams and lateral torsional buckling of beam.

Ans. :

- Whan a beam is loaded, one of the flanges of beam comes in compression and other in tension.
- For economy in beam design, I_z is made considerably larger than I_y.
- Such beams are quite weak in bending in the plane normal to the web and thus compression flange of the beam is liable to buckle in the direction in which it is free to move i.e. in the horizontal direction.
- This buckling tendancy increases as the ratio I_z/I_y increases. However, the bottom flange of the beam remains in tension and thus remains straight.
- But bottom flange, web and the compression flange acts a unit and thus the whole section rotates as shown in Fig. Q.9.1 below.

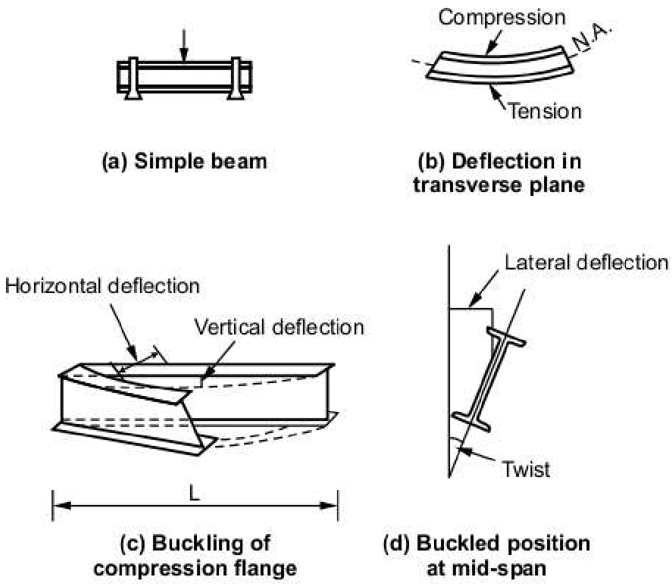

Fig. Q.9.1 Lateral torsional buckling of I-beam section

Elastic Critical Moment

It is the bending moment at which the beam fails by lateral buckling when subjected to uniform moment.

- The lateral buckling of compression flange can be prevented then flexural strength of the beam can be used to its full value.

3.7 Built - up Beams

Q.10 Explain built - up beam ?

Ans. : • A built - up beam is also known as **compound beam**.

- The built - up beams are used when the span, load and corresponding bending moment are of such magnitudes that rolled steel beam section becomes inadequate to provide required section modulus.
- Built - up beams are also used when rolled steel beams are inadequate for limited depth.
- In building construction, the depth of beam is limited by a space provided by the architect.

- Drawing beam of small depth of beam is limited by a space provided by the architect.
- The strength of rolled steel beams is increased by adding plates to its flange which is one of the method forming built-in section.

3.8 Laterally Supported Beams

Q.11 What do you understand laterally restrained beams ? Explain with diagram ?

☞[JNTU : May-16, 17, Marks 3]

Ans. : 1) Laterally Supported Beams :

- A beam can fail by reaching the plastic moment M_P. If it can remain stable upto the fully plastic condition.
- This situation is possible where compression flange buckling of the beam is restrained and constituent elements (flange or web) do not buckle locally and are referred to as **laterally restrained beams**.
- With increasing traverse loads, these laterally restrained beams will attain full moment capacity. (See Fig. Q.11.1)

Q.12 Give design procedure and checks for rolled beam sections.

Ans. : Design Procedure :

1) A trial section is selected assuming it is going to be plastic section (class 1 section).
2) Then it is checked for the class it belongs.
3) Check for bending strength.
4) Check for shear strength.
5) Check for the deflection.

If any check fails the section is revised.

Checks required for beam design,

1) Check for Shear Capacity

- **Design shear = V**
- Design shear strength of section,

$$V_d = \frac{f_y \; h \; t_w}{\sqrt{3} \quad m0}$$

- For safe design, $V_d > V$
- In case of low shear, $V < 0.6\,V_d$

2) Check for design bending strength

- Design bending strength,

$$M_d = {}_b \, Z_p \frac{f_y}{m0} \qquad 1.2 \frac{Z_e \; f_y}{m0}$$

- If $\frac{d}{t_w} < 67$, then ${}_b = 1$

3) Check for deflection

- Permissible deflection for simply supply supported beam,

$$= \frac{l}{300}$$

- Maximum deflection,

$$_{cal} = \frac{5}{384} \frac{wl^4}{EI}$$

for safe condition, $> {}_{cal}$

4) Check for web bearing

- Bearing strength,

$$F_w = A_e \frac{f_{yw}}{m0} = (b + n_1) t_w \frac{f_{yw}}{m0}$$

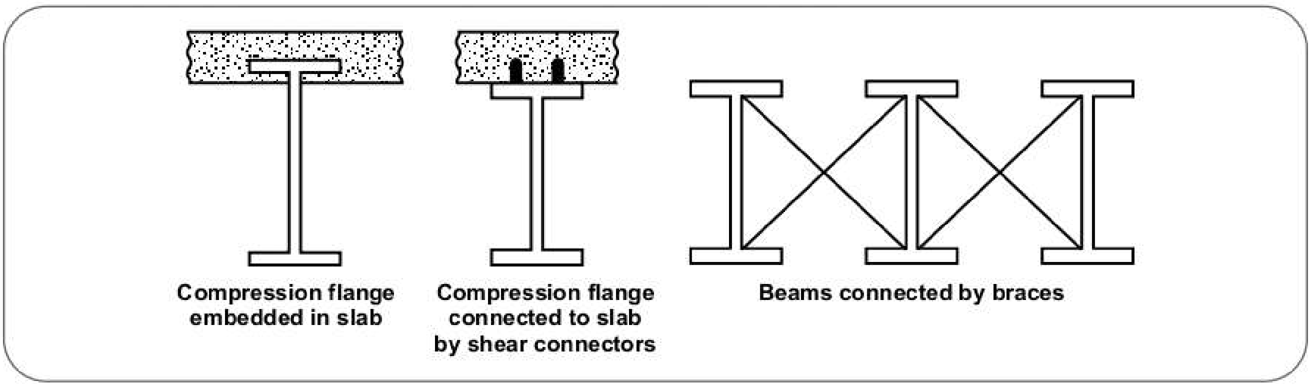

Fig. Q.11.1 Laterally supported beams

• Design shear strength = V_d

For safe condition, $F_w > V_d$

5) Check for web buckling (at support)

• Capacity of web section,

$$F_{wb} = A_b \; f_{cd}$$

$$A_b = B_1 \; t_w = (b+n) t_w$$

For safe condition, $F_{wb} > V_d$

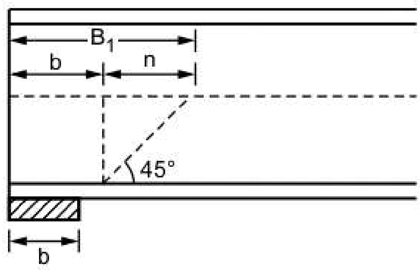

Fig. Q.12.1

3.9 Design of Eccentric Connections

Q.13 What is *eccentric* connections ? Explain with diagram.

Ans. : • If the force applied does not pass through the CG of the joint then such joint carries moment in addition to an axial direct force. Such type of connections are called as **eccentric connections**.

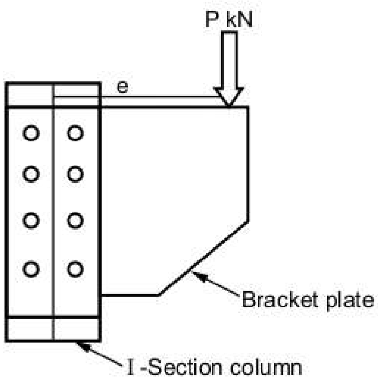

Fig. Q.13.1

Types of eccentric connections :

1) Bracket connections :

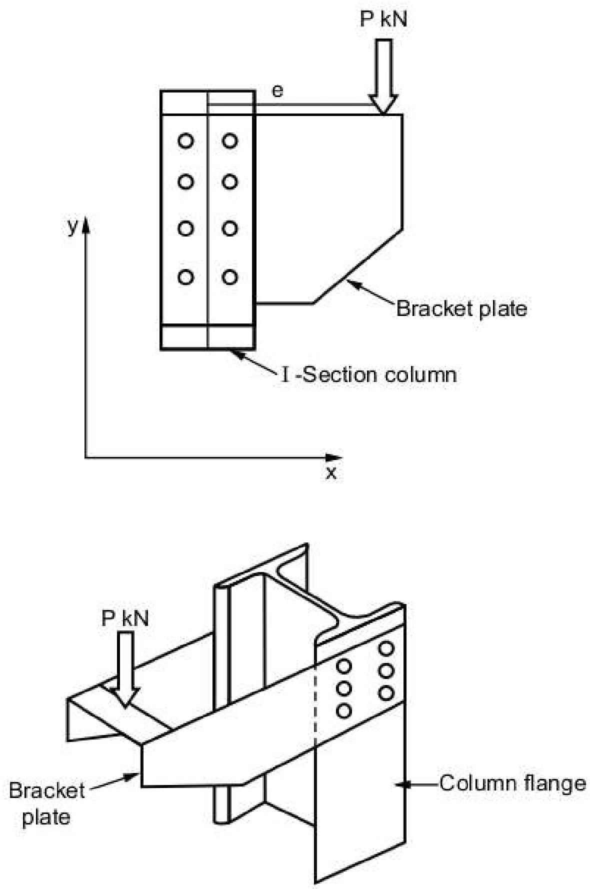

Fig. Q.13.2

2) Framed connections :

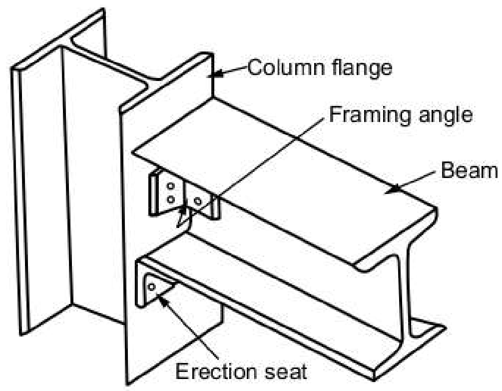

Fig. Q.13.3

3) Seat connection :

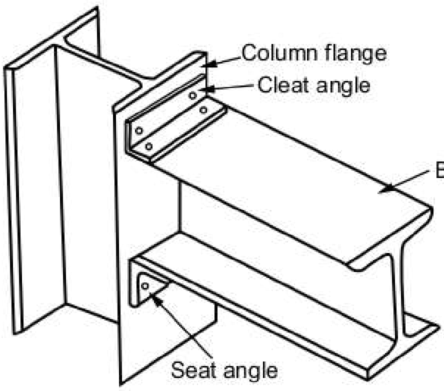

Fig. Q.13.4

Q.14 Design a simply supported plated rolled steel beam section to carry a uniformly distributed load 20 kN/m inclusive of self weight. Effective span of the beam is 3 m. The compression flange of the beam is laterally supported.

☞ **[JNTU : April-18, Marks 10]**

Ans. : Given data :

Span = 3 m = 3000 mm

Total UDL = 20 kN/m

Factored UDL = 1.5 × 20 = 30 kN/m

Assume Fe410 grade of steel,

f_y = 250 MPa, f_u = 410 MPa

1) Maximum bending moment and shear force :

i) $$M = \frac{wL^2}{8} = \frac{30\ \ 3^2}{8} = \mathbf{33.75\ kN - m}$$

$$= \mathbf{33.75\ \ 10^6\ N.mm}$$

ii) $$V = \frac{wL}{2} = \frac{30\ \ 3}{2} = \mathbf{45\ kN}$$

2) Trial section :

$$Z_{p(req)} = \frac{M\ \ _{m0}}{f_y} = \frac{33.75\ \ 10^6\ \ 1.1}{250}$$

$$= \mathbf{148.5\ \ 10^3\ mm^3}$$

Let us provide ISLB 200 @ 194 N/m.

The properties of a section are,

h = 200 mm

b_f = 100 mm

t_f = 7.3 mm

$d = h - 2(t_f + r_1)$

$= 200 - 2\,(7.3 + 9.5) = 166.40$ mm

t_w = 5.4 mm

$I_{zz} = 1696.6\ \ 10^4\ mm^4$

$Z_e = 169.7\ \ 10^3\ mm^4$

$Z_p = 184.3\ \ 10^3\ mm^3$

3) Section classification :

$$= \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

$$b = \frac{b_f}{2} = \frac{100}{2} = 50\ mm$$

$$\frac{b}{t_f} = \frac{50}{7.3} = 6.85 < 9.4$$

$$\frac{d}{t_w} = \frac{166.40}{5.4} = 30.81 < 84$$

Hence, section is a **plastic section.**

4) Check for shear capacity :

- Design shear force = Maximum shear force

V = 45 kN

- Design shear strength of section,

$$V_d = \frac{f_y}{\sqrt{\ }\ \ _{m0}}\ \ h\ \ t_w$$

$$= \frac{250}{\sqrt{\ }\ \ 1.1}\ \ 200\ \ 5.4\ \ 10^{-3}$$

$$= \mathbf{141.713\ kN > 45\ kN}$$

Hence, OK.

5) Check for high / low shear :

$0.6\,V_d = 0.6\ \ 141.713 = \mathbf{85.02\ kN > V}$

Hence OK.

6) Check for design bending strength :

Since section is plastic, $_b = 1$

Design bending strength,

$$M_d = \ _b\ Z_p\ \frac{f_y}{_{m0}}$$

$$= 1 \quad 184.3 \quad 10^3 \quad \frac{250}{1.1} \quad 10^{-6}$$

$$= \mathbf{41.895\ kN.m}$$

$$1.2\, Z_e \quad \frac{f_y}{_{m0}}$$

$$= 1.2 \quad 169.7 \quad 10^3 \quad \frac{250}{1.1} \quad 10^{-6}$$

$$= \mathbf{46.281\ kN.m}$$

Hence, safe for maximum bending moment.

7) Check for deflection :

- Permissible deflection,

$$= \frac{l}{300} = \frac{3000}{300} = 10 \text{ mm}$$

- Maximum actual deflection,

$$_{cal} = \frac{5}{384} \frac{wl^3}{EI}$$

$$= \frac{5}{384} \quad \frac{30 \quad 10^3 \quad 3000^3}{2 \quad 10^5 \quad 1696.6 \quad 10^4}$$

$$= \mathbf{3.11\ mm < 10\ mm}$$

Hence, safe against deflection.

Q.15 Design a stiffened seated connection for an ISMB 350 @ 514 N/m with the column section ISHB 300 @ 576.8 N/m. The beam transmits an end reaction of 320 kN due to factored loads. The steel is of grade of Fe410.

☞ **[JNTU : May-17, Marks 10]**

Ans. : Design with bolted connections :

For Fe 410 grade of steel; f_u = 410 MPa, f_{yw} = 250 MPa

For bolts of grade of 4.6 : f_{ub} = 400 MPa

Partial safety factor for material of bolt : $_{mb}$ = 1.25

Partial safety factor for the material : $_{m0}$ = 1.10

$$= \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1.0$$

Step 1 : The relevant properties of the sections to be connected from steel table are :

Property	ISMB 350	ISHB 300
Width of flange, b_f	140 mm	250 mm
Thickness of flange, t_f	14.2 mm	10.6 mm
Thickness of web, t_w	8.1 mm	7.6 mm
Gauge, g	80 mm	
Radius at the root, R_1	14 mm	

The length of seat angle, B = Width of beam flange

= **140 mm**, (b_f = 140 mm).

Bearing length of seat leg,

$$b = \frac{R}{t_w} \quad \frac{_{m0}}{f_{yw}}$$

$$= \frac{320 \quad 10^3}{8.1} \quad \frac{1.10}{250} = \mathbf{173.82\ mm}$$

Provide a clearance c of 5 mm between the beam and the column flange.

Required length of outstanding leg = 173.82 mm $\simeq$ 200 mm

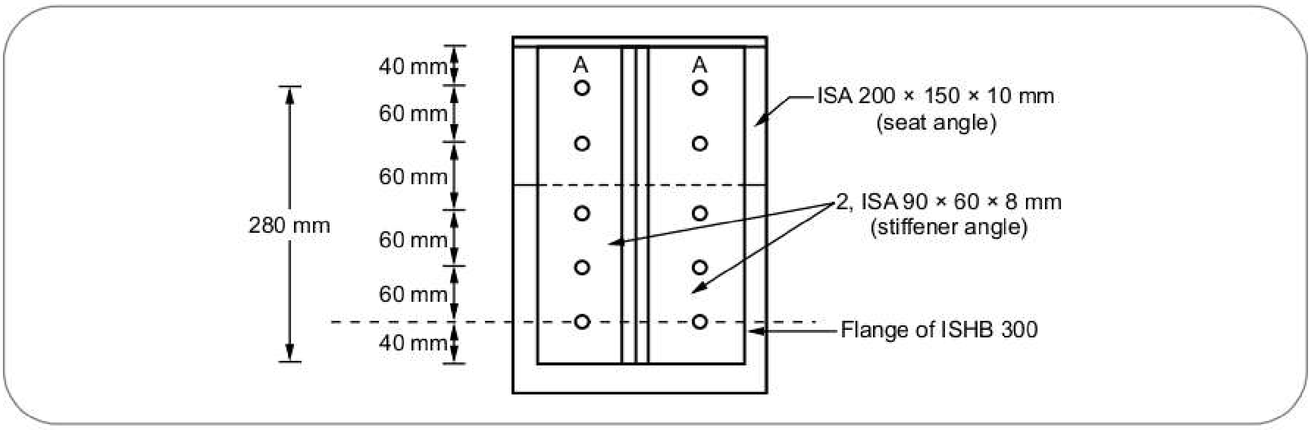

Fig. Q.15.1

Step 2 : Let us provide seat angle **200 × 150 × 10 mm** with **seating leg of 200 mm** connected to the flange of beam with 2, 24 diameter bolts of grade 4.6.

Radius at root of angle, R_a = 13.5 mm (From IS Handbook No. 1)

Step 3 : Stiffener angles :

Bearing area required by stiffener angles,

$$\text{Required A} = R\frac{_{m0}}{f_y}$$

$$= 320 \quad 10^3 \quad \frac{1.1}{250} = \mathbf{1408\ mm^2}$$

Let us provide **two angles ISA 90 × 60 × 8 mm.**

Area provided by the stiffening legs of the angles = 2 × (90 × 8) = **1440 mm^2**

Length of outstanding leg = 90 – 8 = **82 mm**

Thickness of the angle, t_a = 8 mm should be more than t_w i.e. (8.1 mm).

Since the thickness is almost same and stiffener angle section may be used.

Length of outstand of stiffener | 14 t_a i.e. 14 × 8 × 1 = **112 mm** (= 1) which is as it should be.

Distance of end reaction from column flange,

$$e_x = \frac{200}{2} = \mathbf{100\ mm}$$

Stiffener angles provide some rigidity to seat angle and the reaction is assumed to act at the middle of the seat leg. Thus, the eccentricity is increased.

Step 4 : Design of connections :

Let us provide 24 mm diameter bolts of grade 4.6, at a pitch of 60 mm. The bolts connecting the legs of stiffener angles with column flange will be in single shear and bearing.

For 24 mm diameter bolt, A_{nb} = 353 mm^2

Minimum pitch, p = 2.5 × 24 = 60 mm

Edge distance, e = 39 mm 40 mm

Diameter of bolt hole, d_0 = 24 + 2 = 26 mm

a) Strength of the bolt in single shear,

$$V_{dsb} = A_{nb}\frac{f_{ub}}{\sqrt{3} \quad _{mb}}$$

$$= 353 \quad \frac{400}{\sqrt{3} \quad 1.25} \quad 10^{-3} = \mathbf{65.22\ kN}$$

b) Strength of the bolt in bearing,

$$V_{dpb} = 2.5\, k_b\, dt\frac{f_u}{_{mb}}$$

k_b is at least of $\frac{e}{3\,d_0} = \frac{40}{3 \quad 26} = 0.513;$

$$\frac{p}{3\,d_0} - 0.25 = \frac{60}{3 \quad 26} - 0.25 = 0.519;$$

$$\frac{f_{ub}}{f_u} = \frac{400}{410} = 0.975; \text{ and } 1.0$$

Hence, k_b = 0.513

$$V_{dpb} = 2.5 \quad 0.513 \quad 24 \quad 8 \quad \frac{410}{1.25} \quad 10^{-3}$$

$$= \mathbf{80.76\ kN}$$

Hence, **strength of the bolt = 65.22 kN**

There will be two vertical rows of bolts connecting legs of the two stiffener angles with the column flange.

Number of bolts in one row,

$$n = \sqrt{\frac{6\,M}{p n \ V_{sd}}} = \sqrt{\frac{6 \quad 320 \quad 10^3 \quad 100}{60 \quad 2 \quad 65.22 \quad 10^3}}$$

$$= 4.95 \simeq 5$$

The depth of stiffener angle = 4 × 60 + 2 × 40 = 320 mm

$$h = 320 - 40 = 280 \text{ mm}$$

$$h/7 = 280/7 = 40 \text{ mm}$$

Refer Fig. Ex. 14.6.

The critical bolt will be A

$$y_i = 2 \quad [0 \quad 60 \quad 120 \quad 180 \quad 240] = \mathbf{1200\ mm}$$

$$y_i^2 = 2 \quad [0 \quad 60^2 \quad 120^2 \quad 180^2 \quad 240^2]$$

$$= \mathbf{216{,}000\ mm^2}$$

Step 5 :

a) Moment shared by the critical bolt,

$$M = \frac{M}{1 + \frac{2\,h}{2\,l}\frac{y_i}{y_i^2}}$$

$$= \frac{320 \quad 100}{1+\frac{2 \quad 180}{2\,l} \quad \frac{1200}{216000}}$$

$$= \mathbf{27.87 \quad 10^3 \ kNmm}$$

b) Tensile force in the critical bolt,

$$T_b = \frac{M \ y_n}{y_i^2} = \frac{27.87 \quad 10^3 \quad 240}{216000} = \mathbf{30.96\ kN}$$

c) Shear force in the critical bolt,

$$V_{sb} = \frac{P}{\text{Number of bolts}} = \frac{320}{2 \quad 5} = \mathbf{32\ kN}$$

Step 6 : Check :

$$\frac{V_{sb}}{V_{dsb}}^{2} \quad \frac{T_b}{V_{db}}^{2} \qquad 1$$

Strength of bolt in tension, $T_{db} = \frac{T_{nb}}{_{mb}}$

$$T_{nb} = 0.9\, f_{ub}\, A_{nb}$$

$$= 0.9 \quad 400 \quad 353 \quad 10^{-3} = \mathbf{127.08\ kN}$$

$$| \ f_{yb} \frac{_{mb}}{_{m0}} A_{sb}$$

$$= 240 \quad \frac{1.25}{1.10} \quad 452 \quad 10^{-3} = 123.27 \text{ kN}$$

Hence, T_{nb} = **123.27 kN**

and $T_{db} = \frac{123.27}{1.25} = \mathbf{98.61\ kN}$

$$\frac{32}{65.22}^{2} \quad \frac{30.96}{98.61}^{2} = 0.339$$

$$< 1.0$$

which is it should be.

Q.16 Solve Q.15 with welded connection.

Ans. :

For Fe 410 grade of steel : f_u = 410 MPa

Partial safety factor for shop welding : $_{mw}$ = 1.25

Step 1 : The calculation for the required bearing length and its value will remain same as that in design with bolted connection and will be 178.82 mm.

Provide a seat plate of 180 mm length.

The width of the seat plate > width of beam flange = (b_f = 140 mm)

Hence, provide width of seat plate = 160 mm. Refer to Fig. Q.16.1 (a).

The thickness of seat plate | thickness of the beam flange (14.2)

$$t_f = 14.2 \text{ mm}$$

Provide a seat plate **180 × 160 × 16 mm in size**.

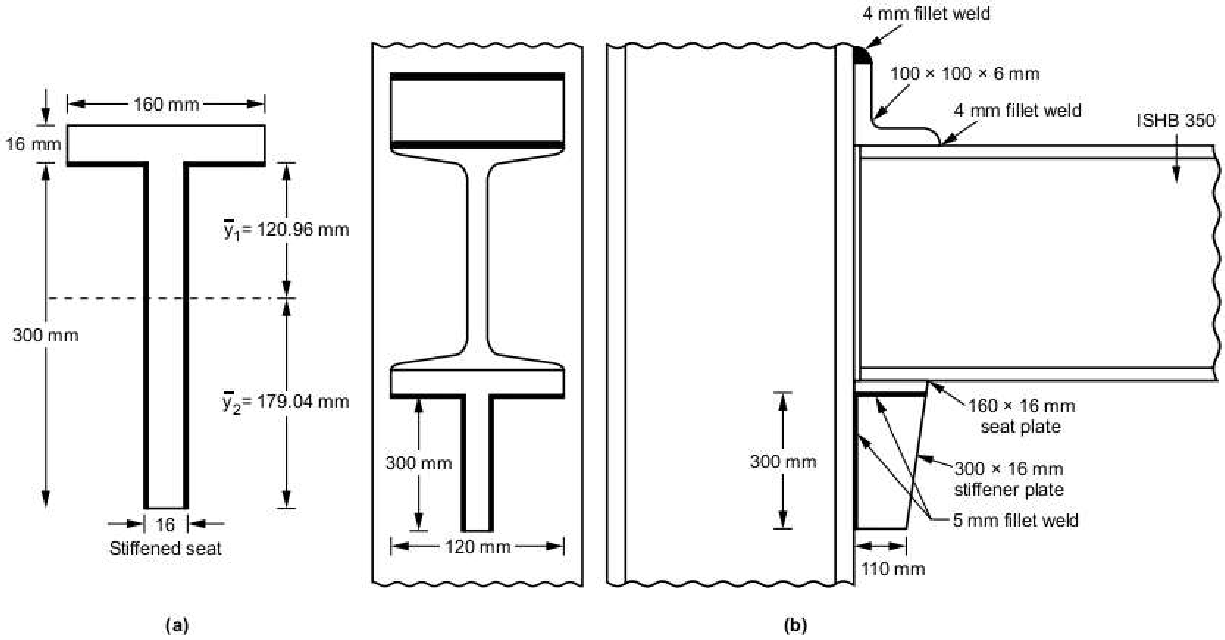

Fig. Q.16.1

Step 2 : Stiffening plate :

Thickness of stiffening plate | thickness of beam web (8.1 mm)

$$t_w = 8.1 \text{ mm}$$

Provide 16 mm thick stiffening plate.

Distance of end reaction from the outer edge of seat plate $= 180 - \frac{178.82}{2} = $ **90.59 mm**

Bending moment

$$= 320 \times 90.59 = \mathbf{28{,}988.8 \text{ Nmm}}$$

Depth of stiffener plate can be determined from buckling criteria.

$$\frac{d}{t} < 18.9$$

$$d < 18.9 \times 16 = \mathbf{302.4 \text{ mm}}$$

Hence, **adopt depth of stiffener plate 300 mm**.

Provide a stiffener plate **300 × 16 mm**. Refer to Fig. Q.16.1(a). Provide fillet weld to connect stiffener as shown in Fig. Q.16.1.

$$\bar{y}_1 = \frac{2 \; (300 \;\; t_1) \;\; 150+0}{2 \;\; 300 \;\; t_1 + (160-16) \;\; t_1}$$

$$= 120.96 \text{ mm}$$

$$\bar{y}_2 = 300 - 120.96 = 179.04 \text{ mm}$$

Assuming unit weld thickness,

I_{xx} of weld group

$$= (160-16) \;\; t_1 \;\; 120.96^2 + 2 \;\; \frac{300^3}{12} \;\; t_1$$

$$+2 \;\; 300 \;\; t_1 \;\; \left(179.04 - \frac{300}{2}\right)^2$$

$$= 711.29 \;\; 10^4 \; t_1 \text{ mm}^4$$

Vertical shear/mm,

$$q_1 = \frac{320 \;\; 10^3}{2 \;\; 300 \;\; t_1 + (160-16) t_1}$$

$$= \frac{430.10}{t_1} \text{ N/mm}$$

Horizontal shear/mm,

$$q_2 = \frac{28988.8 \;\; 10^3 \;\; 120.96}{711.29 \;\; 10^4 \;\; t_1} = \frac{492.98}{t_1} \text{ N/mm}$$

Resultant stress, $q_r = \sqrt{q_1^2 \;\; q_2^2}$

$$= \sqrt{\left(\frac{430.10}{t_1}\right)^2 \;\; \left(\frac{492.98}{t_1}\right)^2}$$

$$= \frac{654.23}{t_1} \text{ N/mm}$$

$$\frac{654.23}{t_1} = \frac{f_u}{\sqrt{3} \;\; _{mw}} = \frac{410}{\sqrt{3} \;\; 1.25}$$

$$\mathbf{t_1 = 3.455 \text{ mm}}$$

Size of weld S $= \frac{3.455}{0.7} = 4.935 \text{ mm} \quad 5 \text{ mm}$

Provide 5 mm fillet weld.

Provide a nominal size flange cleat angle **ISA 100 × 100 × 6 mm** and connect it **by 5 mm fillet weld** as shown in Fig. Q.16.1(b).

Q.17 Design a seat connection for the factored beam end reaction of 110 kN. The beam section is ISMB 250 @ 365.9 N/m connected to the flange of column section ISHB200 @ 365.9 N/m using bolted connection. Steel is of 410 and bolts are of grade 4.6. **[JNTU : May-17, Marks 10]**

Ans. : Given Data :

For Fe 410 grade of steel : f_u = 410 MPa, f_{yw} = 250 MPa

For bolts of grade 4.6 : f_{ub} = 400 MPa

Partial safety factor for material of bolt : $_{mb}$ = 1.25

Partial safety factor for material : $_{m0}$ = 1.10

Step 1 : The relevant properties of the sections to be connected, from are :

Property	ISMB 250	ISHB 300
Width of flange, b_f	125 mm	200 mm
Thickness of flange, t_f	12.5 mm	9.00 mm
Thickness of web, t_w	6.9 mm	6.1 mm
Gauge, g	65 mm	55 mm
Radius at the root, R_1	13 mm	

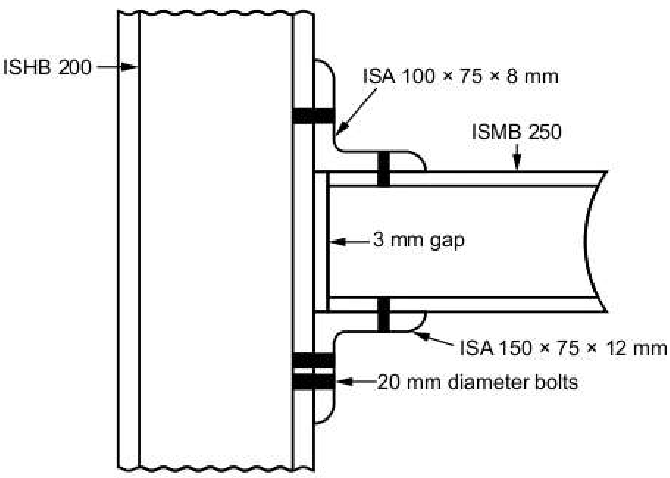

Fig. Q.17.1

The length of seat angle, B = width of beam flange = 125 mm, (b_f = 125 mm)

Bearing length of the seat leg,

$$b = \frac{R}{t_w} \frac{{}_{m0}}{f_{yw}} = \frac{110 \quad 10^3}{6.9} \frac{1.10}{250}$$

$$= 70.14 \text{ mm}$$

- Let us provide an angle section with leg length of 75 mm.
- Provide a clearance of 3 mm between the beam and the column flange.
- Required length of outstanding leg = 70.14 + 3 = 73.14 mm < 75 mm which is all right.

Length of bearing on seat, $b_1 = b - (t_1 \quad R_1)$

$$b_1 = 70.14 - (12.5 \quad 13) = 44.64 \text{ mm}$$

The reaction of 110 kN may be assumed to be distributed uniformly over the bearing length of 44.64 mm.

- Let us try **a seat angle 150 × 75 × 12 mm**.

Radius at the root of the seat angle, R_a = 10 mm

The distance of end of bearing on seat to root of angle,

$$b_2 = b_1 + c - (t_a \quad R_a) = 44.64 + 3 - (12 + 10)$$

$$= \mathbf{25.64 \text{ mm}}$$

Step 2 :

a) Moment at the critical section,

$$M = R \frac{b_2}{b_1} \frac{b_2}{2}$$

$$= 110 \frac{25.64}{44.64} \frac{25.64}{2} = \mathbf{809.98 \text{ kNmm}}$$

b) Moment capacity of the angle leg,

$$M_d = 1.2 Z_e \frac{f_y}{{}_{m0}}$$

$$= 1.2 \frac{125 \quad 12^2}{6} \frac{250}{1.10} 10^{-3}$$

$$= \mathbf{818.18 \text{ kNmm} > 809.98 \text{ kNmm}}$$

which is as it should be.

Hence, **provide a seat angle 150 × 75 × 12 mm**.

c) Shear capacity of the outstanding leg (seating leg),

$$V_{dp} = Bt_a \frac{f_y}{\sqrt{3} \quad {}_{m0}}$$

$$= 125 \quad 12 \frac{250}{\sqrt{3} \quad 1.10} = \mathbf{196.82 \text{ kN}}$$

$$> \mathbf{110 \text{ kN}}$$

which is all right.

Step 3 : Connection of seat angle leg with the column flange :

Let us provide 20 mm diameter 4.6 grade bolts. The bolts will be in single shear and bearing, $A_{nb} = 245 \text{ mm}^2$ (From Table 5.1)

a) Strength of the bolt in single shear,

$$V_{dsb} = A_{nb} \frac{f_{ub}}{\sqrt{3} \quad {}_{mb}}$$

$$= 245 \frac{400}{\sqrt{3} \quad 1.25} 10^{-3} = 45.26 \text{ kN}$$

Number of bolts required, $n = \frac{110}{45.26} = 2.43 \simeq 4.$

- Provide 20 mm diameter bolts of grade 4.6, 4 in numbers, in two rows at a pitch of 60 mm.
- Provide 20 mm diameter bolts of grade 4.6, 2 in numbers to connect seat leg with the beam flange.

- Provide a nominal cleat angle ISA $100 \times 75 \times 8$ mm over the top of the beam flange.
- Connect the legs of this angle with the flanges of beam and column with two 20 mm diameter bolts each.

Q.18 A simply supported steel joist of 4 m effective span is laterally supported throughout. It carries a total uniformly distributed load of 40 kN inclusive of self weight. Design an appropriate section using steel of grade Fe410. Sketch the details of the section. ☞ **[JNTU : May-16, Marks 10]**

Ans. : Given data :

Effective span = 4 m = 4000 mm

Total UDL = 40 kN/m

Factored UDL = 40 × 1.5 = 60 kN/m

- For steel of grade Fe410,

$$f_y = 250 \text{ MPa}, \quad f_u = 410 \text{ MPa}$$

1) Maximum Bending moment and Shear force :

i) $$M = \frac{wL^2}{8} = \frac{60 \quad 4^2}{8} = \mathbf{120 \text{ kN - m}}$$

ii) $$V = \frac{wL}{2} = \frac{60 \quad 4}{2} = \mathbf{120 \text{ kN}}$$

2) Trial section :

$$Z_{p(req)} = \frac{M \quad _{m0}}{f_y} = \frac{120 \quad 10^6 \quad 1.1}{250}$$

$$= \mathbf{528 \quad 10^3 \text{ mm}^3}$$

Let us try, ISLB200 @ 370 N/m.

The properties of a section,

$$h = 300 \text{ mm}$$

$$b_f = 150 \text{ mm}$$

$$t_f = 9.4 \text{ mm}$$

$$d = h - 2(t_f + r_1) = 300 - 2(9.4 + 15)$$

$$= 251.2 \text{ mm}$$

$$t_w = 6.7$$

$$I_{zz} = 7332.9 \quad 10^4 \text{ mm}^4$$

$$Z_e = 488 \quad 10^3 \text{ mm}^3$$

$$Z_p = 554.3 \quad 10^3 \text{ mm}^3$$

3) Section classification :

$$= \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

$$b = \frac{b_f}{2} = \frac{150}{2} = 75 \text{ mm}$$

$$\frac{b}{t_f} = \frac{75}{9.4} = 7.98 < 9.4$$

$$\frac{d}{t_w} = \frac{251.2}{6.7} = 37.49 < 84$$

Hence, section is a **plastic section**.

4) Check for shear capacity :

Design shear force = Maximum shear force

$$V = 120 \text{ kN}$$

- Design shear strength of section,

$$V_d = \frac{f_y}{\sqrt{\quad} \quad _{m0}} \quad h \quad t_w$$

$$= \frac{250}{\sqrt{\quad} \quad 1.1} \quad 300 \quad 6.7 \quad 10^{-3}$$

$$= \mathbf{263.744 \text{ kN} > 120 \text{ kN}}$$

Hence, safe in shear.

Check for high / low shear :

$$0.6\, V_d = 0.6 \quad 263.744 = \mathbf{158.25 \text{ kN} > 120 \text{ kN}}$$

Hence, safe in low shear.

5) Check for design bending strength :

Since section is plastic, $_b = 1$

Design bending strength,

$$M_d = \ _b \ Z_p \frac{f_y}{_{m0}}$$

$$= 1 \quad 554.3 \quad 10^3 \quad \frac{250}{1.1} \quad 10^{-6} = \mathbf{125.98 \text{ kN.m}}$$

$$1.2\, Z_e \frac{f_y}{_{m0}} = 1.2 \quad 488 \quad 10^3 \quad \frac{250}{1.1} \quad 10^{-6}$$

$$= \mathbf{133.09 \text{ kN.m}}$$

Hence, section is safe for maximum bending moment.

6) Check for deflection :

- Permissible deflection,

$$= \frac{l}{300} = \frac{4000}{300} = \mathbf{13.33 \text{ mm}}$$

• Maximum actual deflection,

$$_{cal} = \frac{5}{384} \frac{wl^3}{EI}$$

$$= \frac{5}{384} \frac{120 \quad 10^3 \quad 4000^3}{2 \quad 10^5 \quad 7332.9 \quad 10^4}$$

$$= \mathbf{6.82\ mm < 13.33\ mm}$$

Hence, section is safe against deflection.

Q.19 Determine the safe load P that can be carried by the joint shown in Fig. Q.19.1. The bolts used are 20 mm diameter of grade 4.6. The thickness of the flange of I - section is 9.1 mm and that of bracket plate 10 mm.

☞[JNTU : May-16, Marks 10]

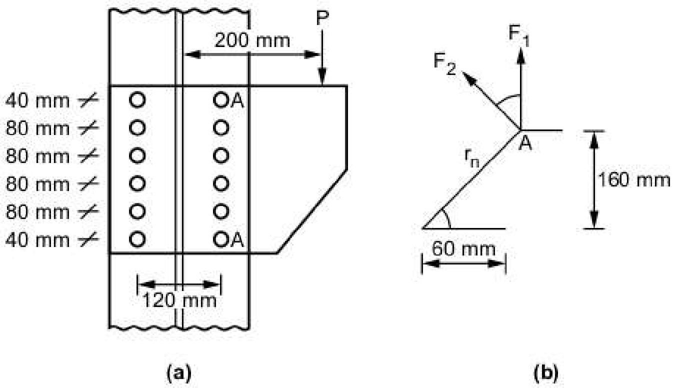

Fig. Q.19.1

Ans. :

For Fe 410 grade of steel : $f_u = 410$ MPa

For bolts of grade 4.6 : $f_{ub} = 400$ MPa

Partial safety factor for the material of bolt :
$_{mb} = 1.25$

A_{nb} = Stress area of 20 mm diameter bolt = 245 mm^2 (From Table 5.1)

1) **Given** : Diameter of bolt, d = 20 mm; pitch, p = 80 mm; edge distance, e = 40 mm, For d = 20 mm,
$d_0 = 20 + 2 = 22$ mm.

a) Strength of the bolt in single shear,

$$\mathbf{V_{dsb}} = A_{nb} \frac{f_{ub}}{\sqrt{3} \quad _{mb}}$$

$$= 245 \quad \frac{400}{\sqrt{3} \quad 1.25} \quad 10^{-3} = \mathbf{45.26\ kN}$$

b) Strength of the bolt in bearing,

$$V_{dpb} = 2.5\, k_b\, dt \frac{f_u}{_{mb}}$$

From Table 5.3, for 20 mm diameter bolt the diameter of bolt hole, $d_0 = 22$ mm

k_b is at least of $\frac{e}{3\, d_0} = \frac{40}{3 \quad 22} = 0.606;$

$$\frac{p}{3\, d_0} - 0.25 = \frac{80}{3 \quad 22} - 0.25 = 0.96;$$

$$\frac{f_{ub}}{f_u} = \frac{400}{410} = 0.975; \text{ and } 1.0$$

Hence, $k_b = 0.606$

$$\mathbf{V_{dpb}} = 2.5 \quad 0.606 \quad 20 \quad 9.1 \quad \frac{410}{1.25} \quad 10^{-3}$$

$$= \mathbf{90.44\ kN}$$

Hence, strength of the bolt, $V_{sd} = 45.26$ kN

Let, P_1 be the factored load,

Service load, $P = \frac{P_1}{\text{Load factor}} = \frac{P_1}{1.50}$

The bolt which is stressed maximum is A.

Total number of bolts in the joint, n = 10

2) The direct force, $F_1 = \frac{P_1}{n} = \frac{P_1}{10}$

The force in bolt due to torque,

$$\boxed{F_2 = \frac{P e_0\, r_n}{r^2}}$$

$$r_n = \sqrt{(80 \quad 80)^2 \quad \frac{120}{2}^2} = 170.88 \text{ mm}$$

$$r^2 = 4 \quad [(160^2 \quad 60^2) \quad (80^2 \quad 60^2)] \quad 2 \quad 60^2$$

$$= 164{,}000 \text{ mm}^2$$

$$\mathbf{F_2} = \frac{P_1 \quad 200 \quad 170.88}{164000} = \mathbf{0.20839\ P_1}$$

$$\cos = \frac{60}{\sqrt{60^2 \quad 160^2}} = 0.3511$$

The resultant force on the bolt should be less han or equal to the strength of bolt.

$$45.26 \quad \sqrt{\frac{P_1}{10} \quad (0.20839\,P_1)^2 \atop +2 \quad P_1 \quad 0.20839\,P_1 \quad 0.3511}$$

$$P_1 = 173.49 \text{ kN}$$

The service load,

$$P = \frac{P_1}{\text{Load factor}} = \frac{173.49}{1.5} = \mathbf{115.65 \text{ kN}}$$

Q.20 Design a laterally supported beam of effective span 6 m for the following data. Grade of steel = Fe410.
Maximum bending moment M = 150 kNm
Maximum shear force V = 210 kN
Check for deflection is not required.

☞[JNTU : May-17, Marks 10]

Ans. : Given data :

1) Effective span = 6 m

2) Maximum BM = 150 kN/m

Factored BM = 1.5 × 150 = 225 kN/m

3) Maximum shear force = 210 kN

Factored SF = 1.5 × 210 = 315 kN/m

4) For Fe410, f_y = 250 MPa

• To design a laterally supported beam,

I) $$Z_{p(req)} = \frac{M \quad _{m0}}{f_y} = \frac{225 \quad 10^6 \quad 1.1}{250}$$

$$= 990 \quad 10^3 \text{ mm}^3$$

Let us try, ISLB400 @ 558 N/m.

II) Properties of section :

Depth of section h = 400 mm

Width of flange, b_f = 165 mm

Thickness of flange, t_f = 12.5 mm

Thickness of web, t_w = 8 mm

$$d = h - 2\,(t_f + r_1) = 400 - 2\,(12.5 + 16)$$

$$= 343 \text{ mm}$$

$$I_{zz} = 19305.3 \quad 10^4 \text{ mm}^4$$

$$Z_e = 965.3 \quad 10^3 \text{ mm}^3$$

$$Z_p = 1099 \quad 10^3 \text{ mm}^3$$

III) Section classification :

$$= \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

$$b = \frac{b_f}{2} = \frac{165}{2} = 82.5 \text{ mm}$$

$$\frac{b}{t_f} = \frac{82.5}{12.5} = \mathbf{6.6 < 9.4}$$

$$\frac{d}{t_w} = \frac{343}{8} = \mathbf{42.87 < 84}$$

Hence, section is a **plastic section.**

• Factored self weight of beam = 1.5 × 0.558

$$= \mathbf{0.837 \text{ kN/m}}$$

$$\text{Total maximum BM} = \frac{w\,l^2}{8} \quad 225$$

$$= \frac{0.837 \quad 6^2}{8} \quad 225 = \mathbf{228.77 \text{ kN.m}}$$

IV) Plastic section modulus required :

$$Z_p = \frac{M \quad _{m0}}{f_y} = \frac{228.77 \quad 10^6 \quad 1.1}{250}$$

$$= \mathbf{1006.59 \quad 10^3 \text{ mm}^3}$$

$$< 1099.5 \quad 10^3 \text{ mm}^3$$

Hence OK.

• **Design shear force :**

$$\text{Shear force (v)} = \frac{w\,l}{2} \quad 315$$

$$= \frac{0.837 \quad 6}{2} \quad 315$$

$$= \mathbf{317.51 \text{ kN}}$$

• **Design shear strength of section,**

$$V_d = \frac{f_y}{\sqrt{\;} \quad _{m0}} \quad h \quad t_w = \frac{250}{\sqrt{\;} \quad 1.1} \quad 400 \quad 8$$

$$= \mathbf{419.89 \text{ kN} > 317.51 \text{ kN}}$$

Hence, OK.

V) Check for design capacity of section :

$$\frac{d}{t_w} = \frac{343}{8} = \mathbf{42.87 < 67}$$

$$M_d = {}_b Z_p \frac{f_y}{{}_{m0}}$$

$$M_d = \frac{1 \quad 1099 \quad 10^3 \quad 250}{1.1}$$

$$= \mathbf{249.772\ kN.m}$$

$$M_d \quad \frac{1.2 \quad Z_e \quad f_y}{{}_{m0}} \quad \frac{1.2 \quad 965.3 \quad 10^3 \quad 250}{1.1}$$

$$= \mathbf{263.26\ kN.m}$$

$$M_d = \mathbf{249.77\ kN.m} > 263.26\ kN$$

Hence OK.

Q.21 A simply supported steel of 5 m effective span is laterally supported throughout. It carries a total uniformly distributed load of 50 kN (including self - weight). Design an appropriate section using steel grade Fe410.

☞[JNTU : May-17, Marks 10]

Ans. : Given Data :

I) Span = 5 m = 5000 mm

II) Total uniformly distributed load = 50 kN

Total factored load = 75 kN

III) For Fe 415 grade of steel,

$$f_y = 250\ MPa$$

IV) Partial safety factors,

$${}_{m0} = 1.1, \quad {}_f = 1.50$$

1) Maximum bending moment and shear force :

i) $$M = \frac{wl^2}{8} = \frac{75 \quad 5^2}{8} = 234.375\ kN - m$$

ii) $$V = \frac{wl}{2} = \frac{75 \quad 5}{2} = 187.5\ kN$$

Maximum section modulus required,

$$Z_{p(req)} = \frac{M \quad {}_{m0}}{f_y} = \frac{234.375 \quad 1.1 \quad 10^6}{250}$$

$$Z_{p(req)} = \mathbf{1031.25 \quad 10^3\ mm^3}$$

Let us try, ISLB400 @ 558 N/m.

2) Properties of Section :

Depth of section h = 400 mm

Width of flange, b_f = 165 mm

Thickness of flange, t_f = 12.5 mm

Thickness of web, t_w = 8 mm

$$d = h - 2(t_f + r_1)$$

$$= 400 - 2(12.5 + 16)$$

$$= 343\ mm$$

$$I_{zz} = 19305.3 \quad 10^4\ mm^4$$

$$Z_e = 965.3 \quad 10^3\ mm^3$$

$$Z_p = 1099.5 \quad 10^3\ mm^3$$

3) Section classification :

$$= \sqrt{\frac{250}{f_y}} = \sqrt{\frac{250}{250}} = 1$$

$$b = \frac{b_f}{2} = \frac{165}{2} = 82.5\ mm$$

$$\frac{b}{t_f} = \frac{82.5}{12.5} = \mathbf{6.6 < 9.4}$$

$$\frac{d}{t_w} = \frac{343}{8} = \mathbf{42.87 < 84}$$

Hence, section is a **plastic section**.

4) Check for shear force :

Design shear force :

$$\text{Shear force (v)} = \frac{w\,l}{2}$$

$$= \frac{50.837 \quad 5}{2} = \mathbf{127.09\ kN}$$

- **Design shear strength of section,**

$$V_d = \frac{f_y}{\sqrt{\ } \quad {}_{m0}} \quad h \quad t_w$$

$$= \frac{250}{\sqrt{\ } \quad 1.1} \quad 400 \quad 8$$

$$= \mathbf{419.89\ kN > 127.09\ kN} \qquad \textbf{...OK}$$

5) Check for design capacity of section :

$$\frac{d}{t_w} = \frac{343}{8} = 42.87 < 67$$

$$M_d = {}_b Z_p \frac{f_y}{{}_{m0}}$$

($_b$ = 1, since section is plastic)

$$M_d = \frac{1 \quad 1099.5 \quad 10^3 \quad 250}{1.1}$$

$$= \mathbf{249.88 \ kN.m}$$

$$M_d \quad \frac{1.2 \quad Z_e \quad f_y}{{}_{m0}} \quad \frac{1.2 \quad 965.3 \quad 10^3 \quad 250}{1.1}$$

$$= \mathbf{263.26 \ kN.m}$$

$$M_d = \mathbf{249.88 \ kN.m} > 263.26 \text{ kN}$$

6) Check for deflection :

$$_{cal} = \frac{5}{384} \frac{wl^3}{EI}$$

$$= \frac{5 \quad 50 \quad 5000^4}{384 \quad 2 \quad 10^5 \quad 19305.3 \quad 10^4}$$

$$_{cal} = \mathbf{10.54 \ mm}$$

- Allowable maximum deflection,

$$_{allow} = \frac{l}{300} = \frac{5000}{300} = \mathbf{16.67 \ mm}$$

$$\boxed{_{cal} < \ _{allow}}$$

Hence, **the design is safe.**

Review Questions

1. *Give general requirements for plastic design.*
2. *Explain the theorems of plastic analysis.*
3. *Explain the behaviour of Beam in flexure or moment.*
4. *What do you understand laterally restrained beams ? Explain with diagram ?*

[JNTU : May-17, Marks 2]

Fill in the Blanks for Mid Term Exam

Q.1 Purlins as per S800 - 2007 are designed as ____.

Q.2 The imperfection factor ( ) for welded steel section is ____.

Q.3 Beam column is a member which subjected to both action (i)___ as well as (ii)____.

Q.4 Lateral buckling of beam involves three kinds of deformations namely ____, _____ and ____.

Q.5 The problem of web crippling in beams is significant when _____.

Multiple Choice Questions for Mid Term Exam

Q.1 The maximum permissible deflection of steel beams in buildings other than industrial building is limited to ______.

a) Span/250 b) Span/300
c) Span/180 d) Span/500

Q.2 A section is required to be designed as high shear case in case factored shear force is ____.

a) less than 0.6 V_d b) more than 0.6 V_d
c) less than 0.4 V_d d) less than 0.8 V_d

Q.3 For an I-Beam, web buckling is less critical than flange buckling because _____.

a) web can develop part buckling strength.
b) edge conditions of flange are more favorable for buckling
c) web is thicker than flange
d) flange is thicker than web

Q.4 As per Indian standard codal requirements beam should be _____.

a) rolled to have maximum sectional modulus.
b) plastic or at least about one axis.
c) at least symmetrical about one axis.
d) all of the above.

Q.5 Design bending strength of laterally supported beam is given by ____.

a) $M_d = {}_b \ Z_p \ f_{bd}$ b) $M_d = \frac{{}_b \ Z_p \ f_y}{{}_{m0}}$
c) $M_d = \frac{{}_b \ Z_e \ f_{bd}}{{}_{m1}}$ d) $M_d = \frac{{}_b \ Z_e \ f_y}{{}_{m0}}$