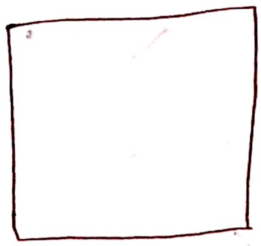
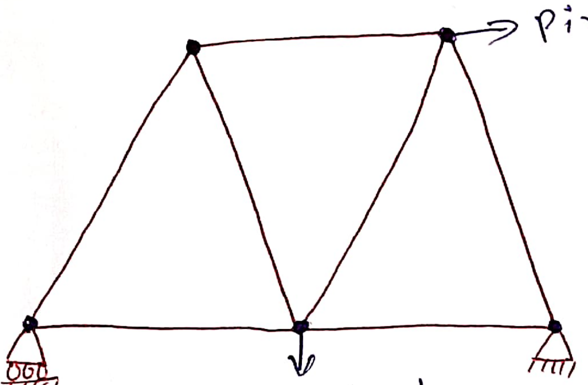
Truss :— Truss is a pin jointed structure. It is constructed by slender members and jointed at their ends by bolting, riveting, welding ~~the~~ to support loads.

slender ⇒ thin and lengthy.

slender



Non-slender



pin-joints

pin-joints

Pin joints → Truss

Rigid joints ⇒ Frames

Properties of Truss :—

(1) All members are perfectly straight.

(2) All the members of the truss carry only axial force.

i.e. SF = 0

BM = 0

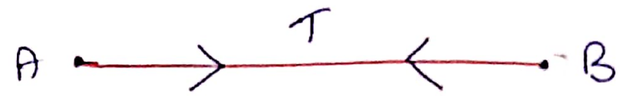
Here predominate force is axial force only.

Axial force is 2 types.

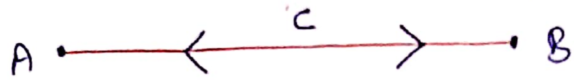
(i) Axial Tensile force

(ii) Axial compressive force

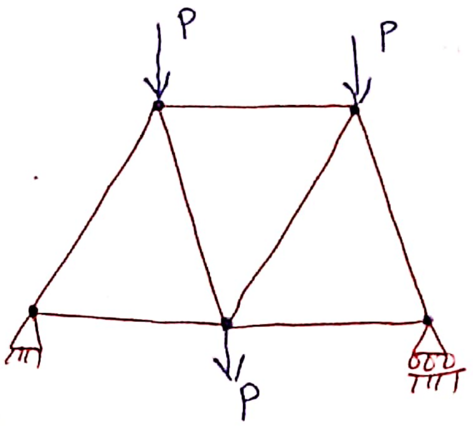
→ Axial tensile force : Away from joints.



→ Axial compressive force : Towards the joints.



(3) All the loads are acting only at the joints.



(4) All the joints carry only 2 equilibrium equations. i.e.

$$\Sigma H = 0 \quad (\text{or}) \quad \Sigma f_x = 0$$

$$\Sigma V = 0 \quad (\text{or}) \quad \Sigma f_y = 0$$

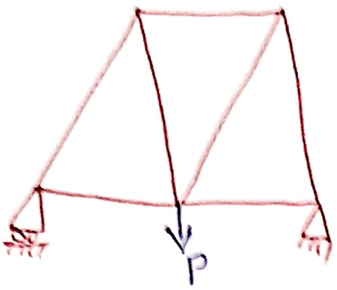
<u>classification of Truss</u> :-

(1) plane Truss (2D)

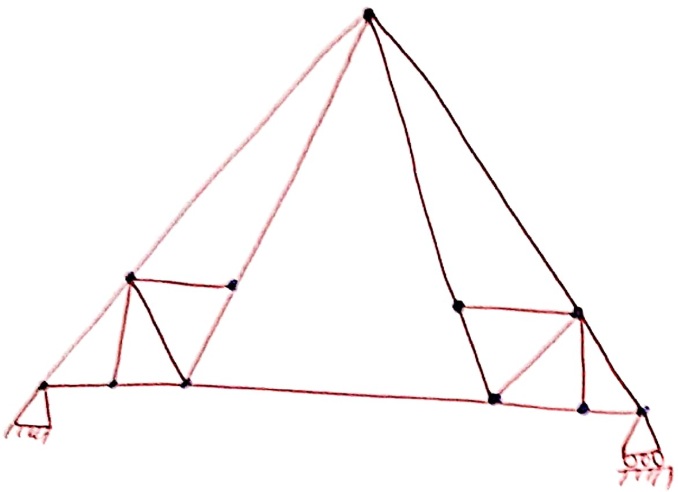
(2) space Truss (3D)

plane Truss →
- simple Truss
- compound Truss
- Complex Truss (Polygon)

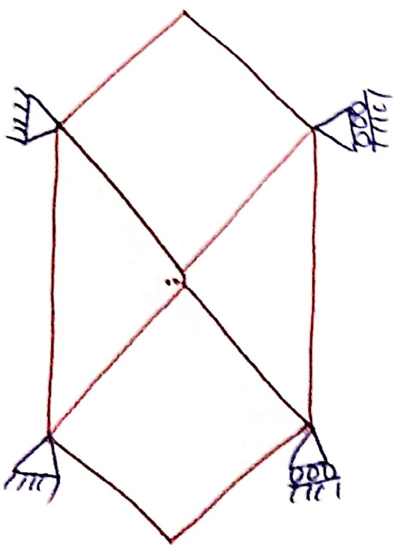
(1) Simple Truss :—



(2) Compound Truss :—



(3) Complex Truss :—



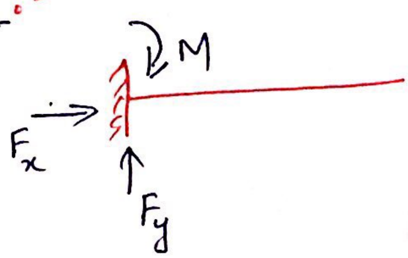
Structure :— A building that has been built (or) made from a no. of parts to arrange something in an organised way is called a structure. ①

Slab
↓
Beam
↓
column
↓
footing

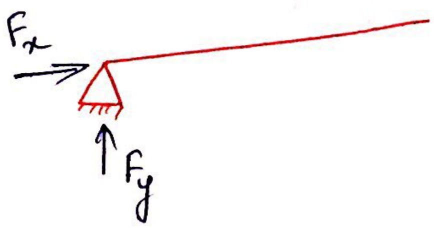
vertical loads and Horizontal loads :—

- vertical load
  - Dead load
  - Live load
- Horizontal load
  - wind load
  - Earthquake load

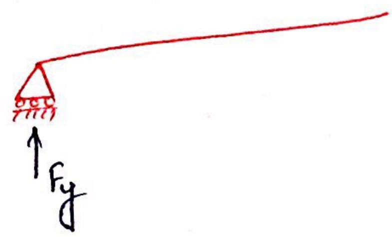
Types of supports :—

(1) Fixed — 

(3 Reactions)

(2) Hinged — 

(2 Reactions)

(3) Roller — 

(1 Reaction)

Types of forces :—

(1) Internal forces (AF, SF, BM)

(2) External forces (Loads, Reactions)

- AF
  - Compression
  - Tension

Analysis of perfect frames :—

plane frame is 2D member

space frame is 3D Member

Frame structures :— combination of beam, column, and slab to resist the lateral and gravity loads. These structures are usually used to overcome the large moments developing due to the applied loading.

Types of frames :—

~~(1) plane frame~~

(1) perfect frame

(2) Imperfect frame

(3) Redundant frame

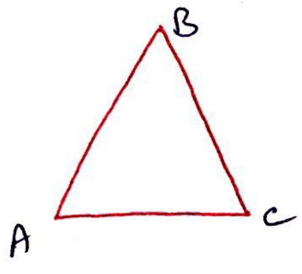
(1) Perfect frame :—

plane frame, $M = 2j - 3$

space frame, $M = 3j - 6$

Here M = Member

j = joint



$M = 3$

$j = 3$

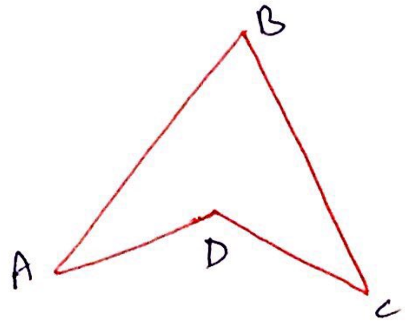
$M = 2j - 3$

$3 = 2(3) - 3$

$3 = 3$

(2) Imperfect frame :-

$$M < 2j - 3$$



M = 4

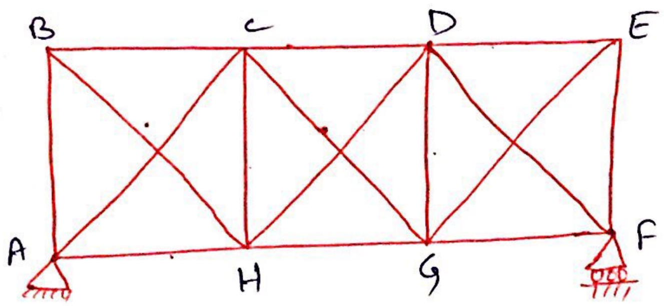
j = 4

$4 < 2(4) - 3$

$4 < 5$

(3) Redundant frame :-

$$M > 2j - 3$$



M = 16

j = 8

$16 > 2(8) - 3$

$16 > 13$

Types of structures :-

Based on equilibrium and compatibility equation

(1) statically determinate structure

(2) statically indeterminate structure.

(1) statically determinate structure :-

unknown support reactions (or) stress resultant (or) relative forces can easily calculated with the help of equilibrium equations.

Ex:- cantilever beam, simply supported beam, over hanging beam, 3 hinged arches etc.

(2) Statically indeterminate structure :—

No. of unknowns cannot be determined by equilibrium equations alone, compatability conditions related to displacements can be used as an additional equations to solve excess unknowns.

Ex :— Fixed beams, continuous beams, fixed arches, two hinged arches, portals, multistoried frames etc.

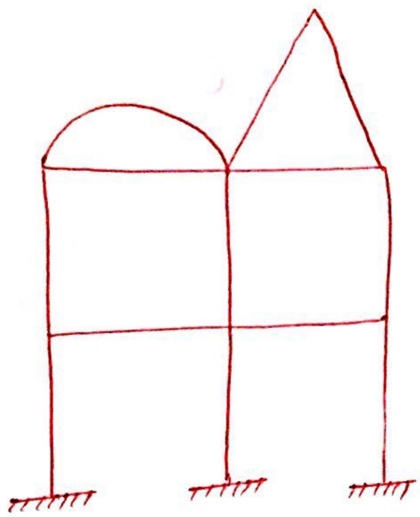
Classification of structures :—

(1) surface structure (2-D)

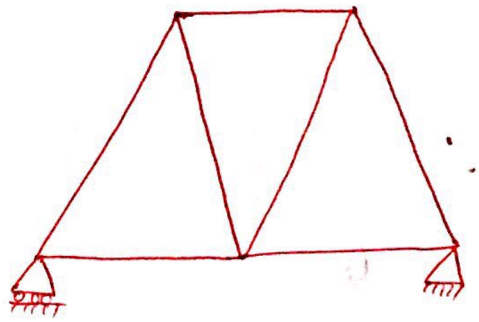
(2) Solid structure (3-D)

(3) skeletal structure (1-D)

(1) Surface structure (2-D) :—

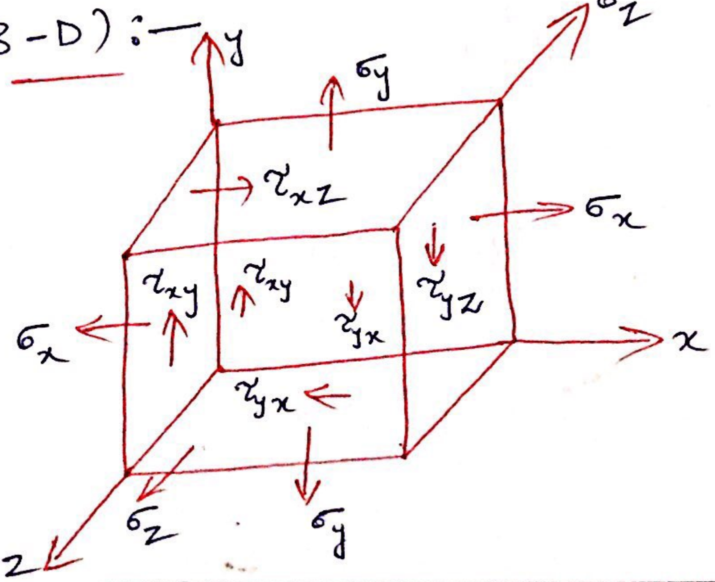


(1) Building frame



(2) Roof frame

(2) Solid structure (3-D) :—



(3) Skeletal structure (1 – D) :—

It is a combination of member and joints.

(or)

Group of joints.

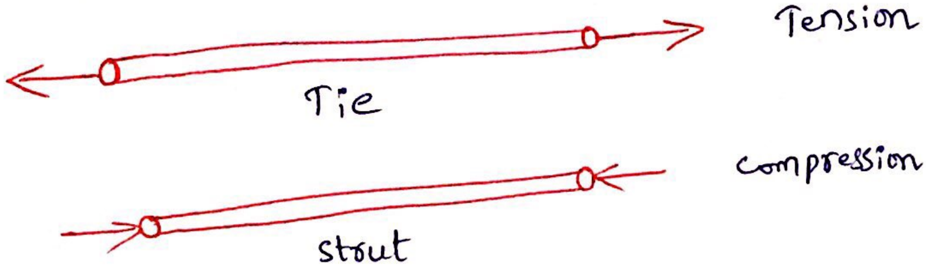
∴ There are 2 types.

(i) Pin-jointed structure

(ii) Rigid jointed structure

(i) Pin-jointed structure :—

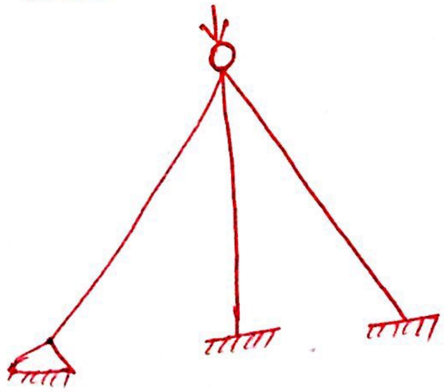
(a) plane pin jointed :— A structure connected by a no. of pins (or) no. of hinges is called pin-jointed. Two forces are set up in pin-jointed structure, they are tension and compression.



Tension member is called as tie.

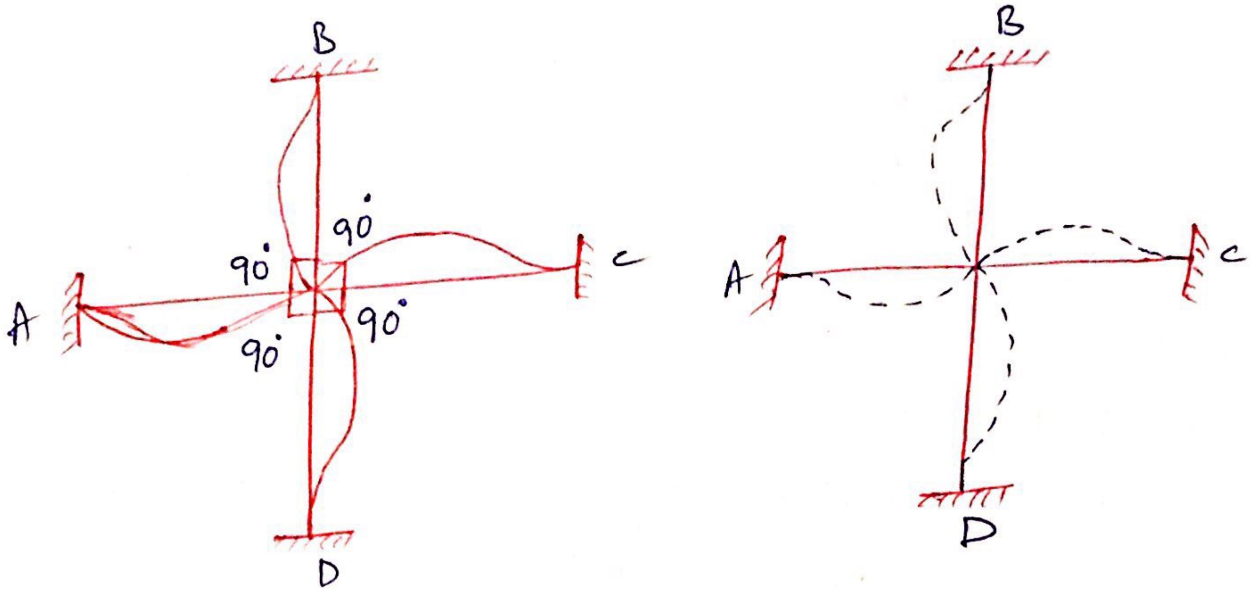
compression member is called as strut.

(b) space pin jointed :—

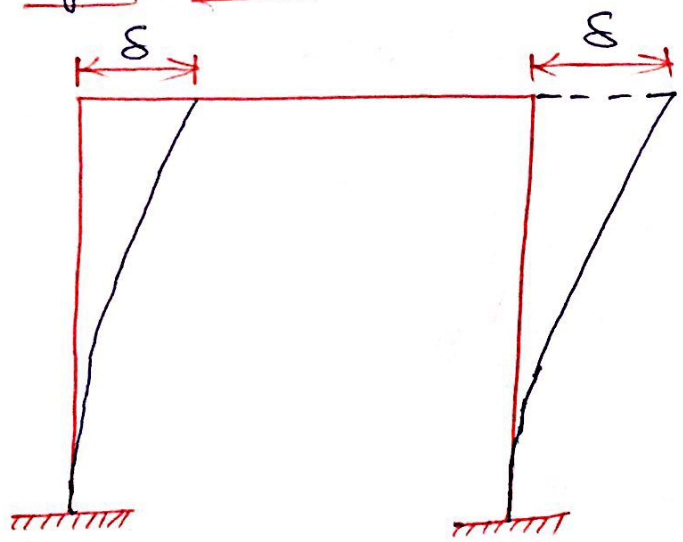


(ii) Rigid - jointed structure :—

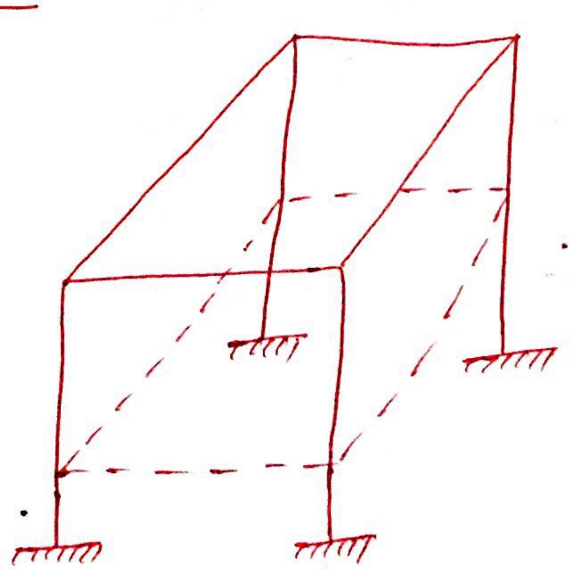
All the members are connected at the point.



(a) plane rigid jointed :—



(b) space Rigid jointed :—



Method of Joints :—

1. Find support reactions

↓

Equilibrium equations

$$\Sigma F_x = 0$$

$$\Sigma F_y = 0$$

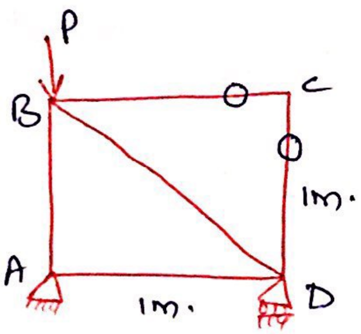
$$\Sigma M = 0$$

2. Joint equilibrium equations

↓

Find member forces (Tensile)

Ex:- Find forces in the member

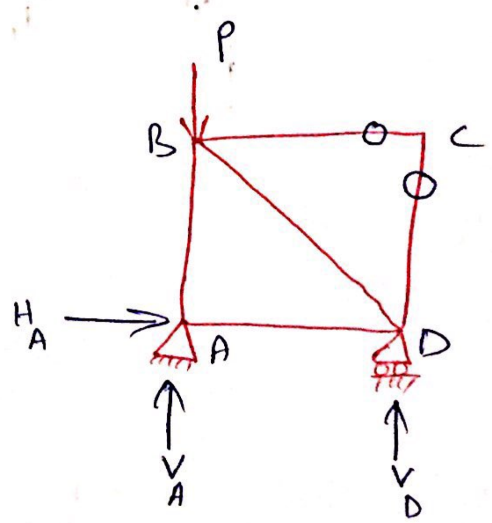


$$\sin\theta = \frac{Opp}{Hyp}$$

$$\cos\theta = \frac{Adj}{Hyp}$$

$F_{CB} = F_{CD} = 0$ (Non colinear member with no external load acting at the joint)

Support Reactions :—



$\Sigma F_x = 0 \Rightarrow H_A = 0$

$\Sigma F_y = 0 \Rightarrow V_A + V_D = P \rightarrow (1)$

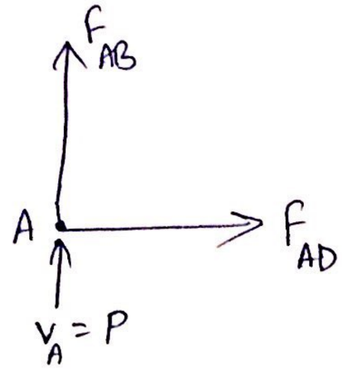
$\Sigma M_A = 0 \Rightarrow -V_D \times 1 = 0 \Rightarrow V_D = 0$

'V_D' sub. in (1)

$V_A + 0 = P$

$V_A = P$

Joint A :-



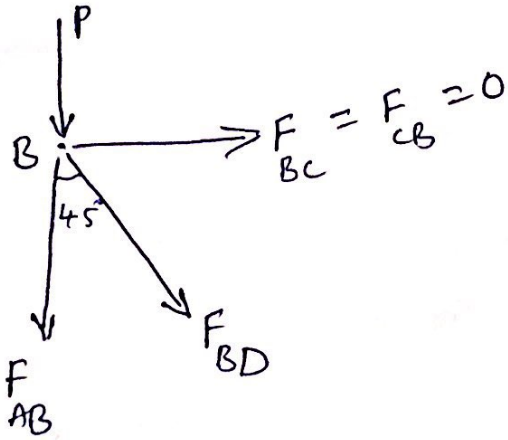
$$\Sigma F_x = 0 \Rightarrow F_{AD} = 0$$

$$\Sigma F_y = 0 \Rightarrow P + F_{AB} = 0$$

$$F_{AB} = -P$$

$$F_{AB} = P \text{ (Compressive)}$$

Joint B :-



$$\Sigma F_x = 0 \Rightarrow F_{BD} \cdot \sin 45^\circ + F_{BC} = 0$$

$$F_{BD} \cdot \frac{1}{\sqrt{2}} + 0 = 0$$

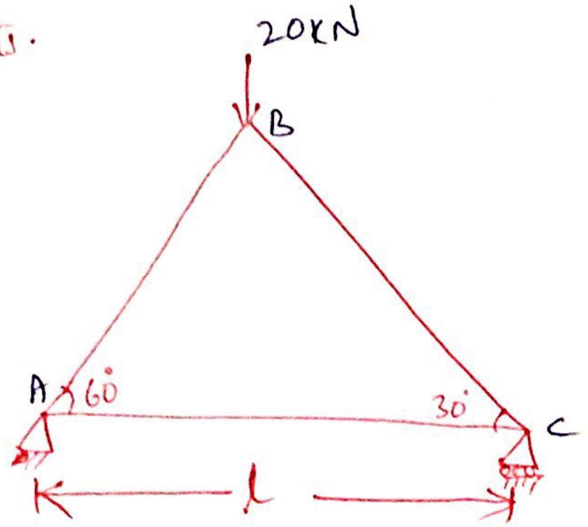
$$F_{BD} = 0$$

$$\Sigma F_y = 0 \Rightarrow -P - F_{AB} - F_{BD} \cdot \cos 45^\circ = 0$$

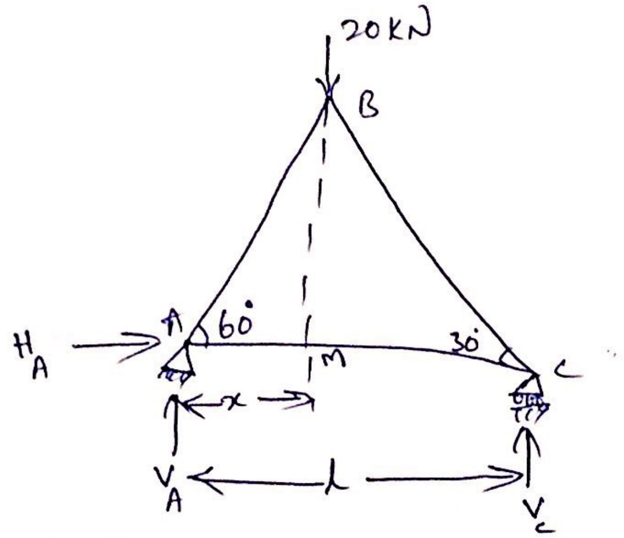
$$-P - (-P) - F_{BD} \cdot \frac{1}{\sqrt{2}} = 0$$

$$F_{BD} = 0$$

① Find the forces in all the members by using of method of joints. ②



Ⓐ



Support Reactions :—

$\Sigma F_x = 0 \Rightarrow H_A = 0$

$\Sigma F_y = 0 \Rightarrow V_A + V_C - 20 = 0$

$V_A + V_C = 20$

↺ +ve

↻ −ve

$\Sigma M_A = 0 \Rightarrow (V_C \times l) - (20 \times x) = 0 \longrightarrow ①$

By right angle triangle AB~~C~~ (By property)

$\cos 60° = \frac{AB}{AC}$

$\cos 60° = \frac{AB}{l}$

$AB = l \cdot \cos 60°$

$AB = \frac{l}{2}$

From ΔABM

$$AM = AB \cdot \cos 60^\circ$$

$$x = \frac{l}{2} \cdot \frac{1}{2}$$

$$x = \frac{l}{4}$$

Sub. 'x' in (1)

$$(V_c \times l) - (20 \times \frac{l}{4}) = 0$$

$$V_c \times l = 20 \times \frac{l}{4}$$

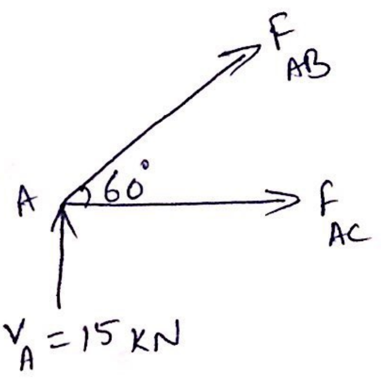
$$V_c = 5\,KN.$$

$$V_A + V_c = 20$$

$$V_A + 5 = 20$$

$$V_A = 15\,KN.$$

Joint A :—



$$\Sigma F_x = 0 \Rightarrow F_{AC} + F_{AB} \cdot \cos 60^\circ = 0$$

$$F_{AC} + F_{AB} \cdot \frac{1}{2} = 0 \rightarrow (2)$$

$$\sin 60 = \frac{\sqrt{3}}{2}$$

$$\Sigma F_y = 0 \Rightarrow 15 + F_{AB} \cdot \sin 60^\circ = 0$$

$$F_{AB} = \frac{-15}{\sin 60^\circ} = \frac{-15}{\sqrt{3}/2}$$

$$F_{AB} = \frac{-15 \times 2}{\sqrt{3}}$$

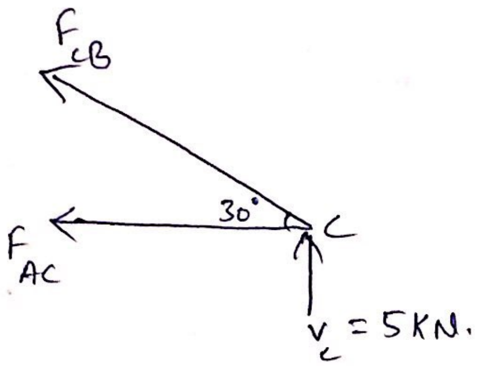
F_{AB} sub. in (2)

$$F_{AC} + F_{AB} \cdot \frac{1}{2} = 0$$

$$F_{AC} + \left(\frac{-15 \times \cancel{2}}{\sqrt{3}}\right) \cdot \frac{1}{\cancel{2}} = 0$$

$$F_{AC} = \frac{15}{\sqrt{3}} \text{ KN.}$$

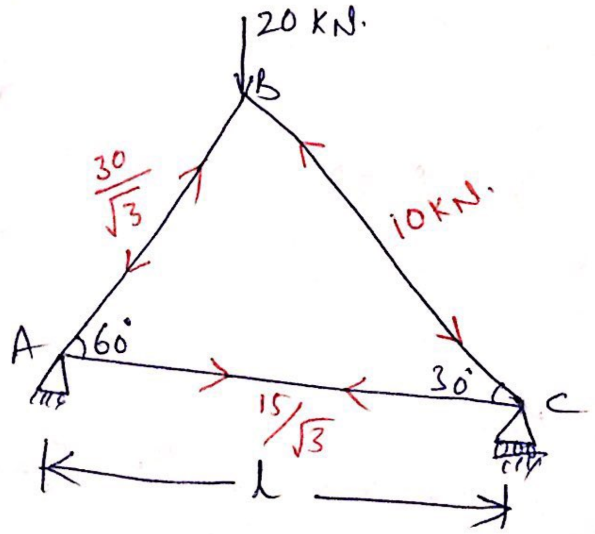
Joint C :-



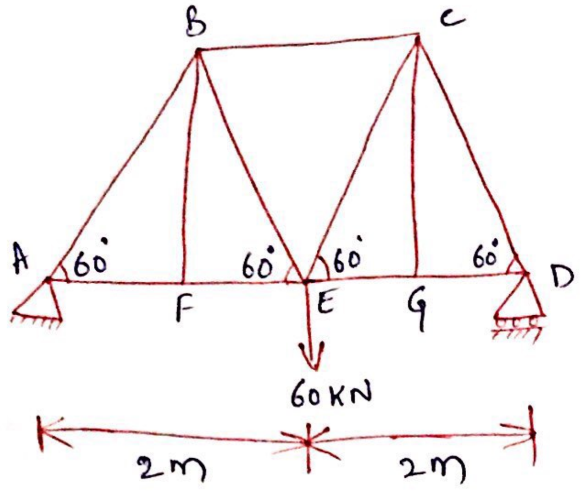
$$\Sigma F_y = 0 \Rightarrow 5 + F_{CB} \cdot \sin 30^\circ = 0$$

$$5 + F_{CB} \cdot \frac{1}{2} = 0$$

$$F_{CB} = -10 \text{ KN.}$$



(2) Find the forces in all the members.



(A)

$\Sigma f_x = 0$; $H_A = 0$

$\Sigma M_A = 0$; $(60 \times 2) - V_D \times 4 = 0$

$V_D = 30$ KN.

c.w. = +ve

A.c.w. = −ve

↑ = +ve

↓ = −ve

→ = +ve

← = −ve

$\Sigma f_y = 0$; $V_A + V_D - 60 = 0$

$V_A + 30 - 60 = 0$

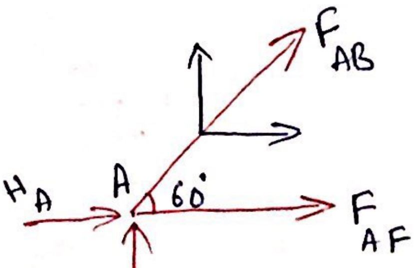
$V_A = 30$ KN.

symmetrical Truss.

Joint A :—

$\Sigma f_y = 0$; $V_A + F_{AB} \cdot \sin 60^\circ = 0$

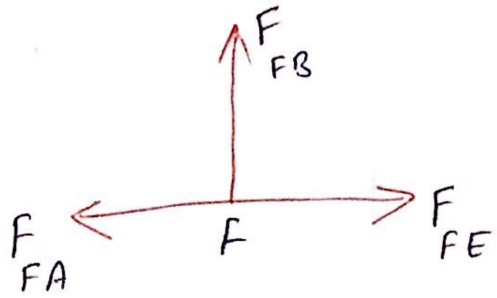
$F_{AB} = -34.641$ KN. (C)



$\Sigma f_x = 0$; $H_A + F_{AF} + F_{AB} \cdot \cos 60^\circ = 0$

$F_{AF} = 17.32$ KN. (T)

Joint F :—



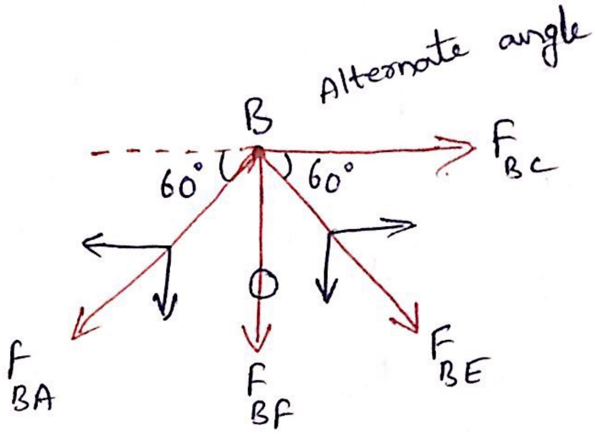
$\Sigma f_x = 0;\ F_{FE} - F_{FA} = 0$

$F_{FE} - 17.32 = 0$

$F_{FE} = 17.32\,KN\ (T)$

$\Sigma f_y = 0;\ F_{FB} = 0$

Joint B :—



$\Sigma f_y = 0;$

$-F_{BE} \cdot \sin 60^\circ - F_{BA} \cdot \sin 60^\circ - F_{BF} = 0$

$-F_{BE} \cdot \sin 60^\circ - (-34.641 \cdot \sin 60^\circ) - 0 = 0$

$F_{BE} = 34.641\,KN.$

$\Sigma f_x = 0;$

$F_{BC} + F_{BE} \cdot \cos 60^\circ - F_{BA} \cdot \cos 60^\circ = 0$

$F_{BC} + 34.641 \cdot \cos 60^\circ - (-34.641) = 0$

$F_{BC} = -34.641\,KN.$

Due to symmetry of structure :

$F_{CD} = F_{AB}$

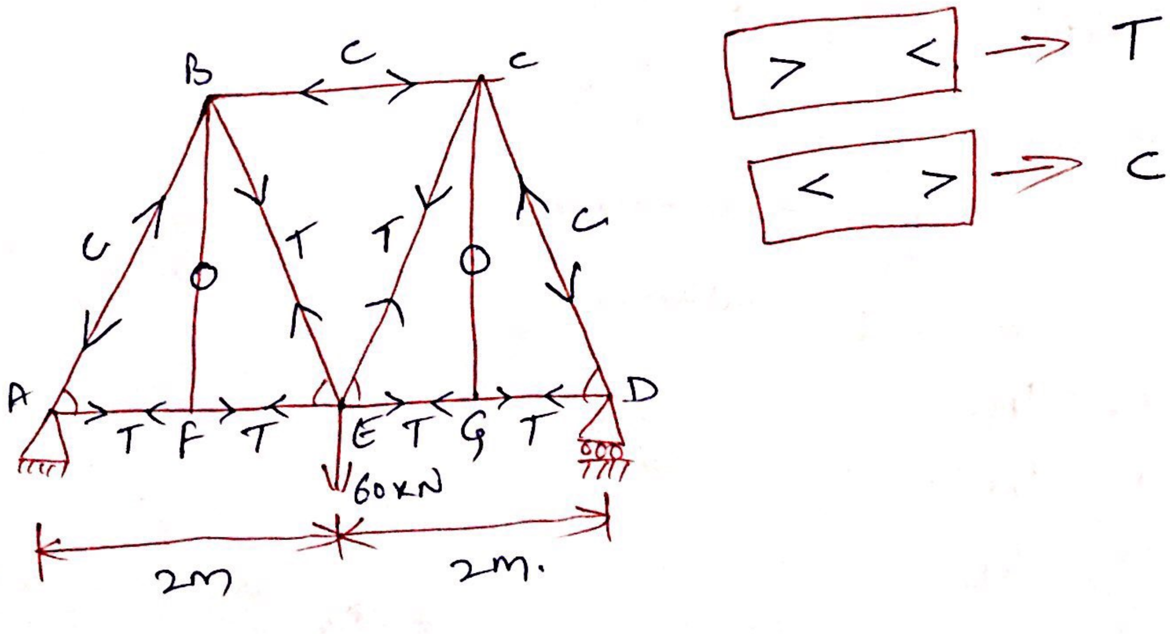
$F_{GD} = F_{AF}$

$F_{GE} = F_{EF}$

$F_{CG} = F_{FB}$

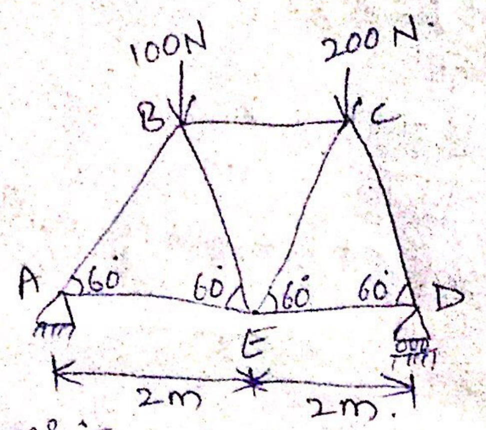
$F_{CE} = F_{BE}$

S.No.	Force in member	Magnitude	Nature
1	F_{AB}, F_{CD}	34.641 kN	C
2	F_{AF}, F_{GD}	17.32 kN	T
3	F_{FB}, F_{CG}	0	—
4	F_{BE}, F_{CE}	34.641 kN	T
5	F_{EF}, F_{GE}	17.32 kN	T
6	F_{BC}	34.641 kN	C



Problem :-

1



A Reactions :-

$$R_A + R_D = 100 + 200$$

$$R_A + R_D = 300N \rightarrow (1)$$

$$\Sigma M_A = 0 \Rightarrow (100 \times 1) + (200 \times 3) - (R_D \times 4) = 0$$

$$R_D = 175N.$$

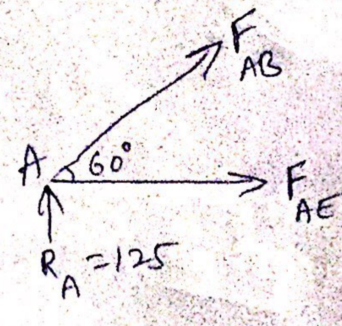
sub. 'R_D' in eqn (1)

$$R_A + 175 = 300 \Rightarrow R_A = 300 - 175$$

$$R_A = 125N.$$

Assume all unknowns of tensile nature.

Joint A :-



$\Sigma H = 0$

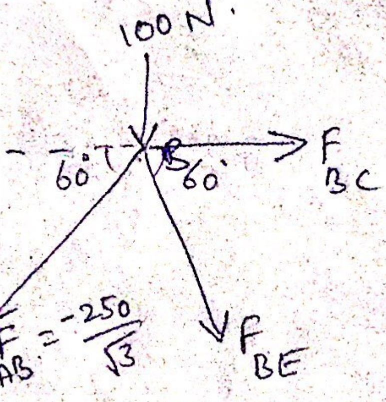
$F_{AE} + F_{AB} \cdot \cos 60° = 0$

$\Sigma V = 0$

$F_{AB} \cdot \sin 60° + 125 = 0.$

$$F_{AB} = \frac{-250}{\sqrt{3}}$$

Joint B :-



$\Sigma H = 0$

$F_{BC} + F_{BE} \cdot \cos 60° - F_{AB} \cdot \cos 60° = 0$

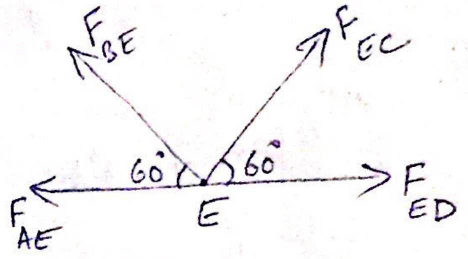
$\Sigma V = 0$

$-100 - F_{AB} \cdot \sin 60° - F_{BE} \cdot \sin 60° = 0$

$$F_{BE} = \frac{50}{\sqrt{3}}$$

$$F_{BC} = \frac{-150}{\sqrt{3}}$$

Joint E :—



$\Sigma V = 0$

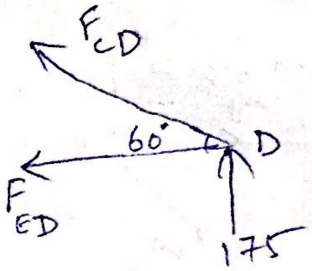
$$F_{BE} \cdot \sin 60° = F_{EC} \cdot \sin 60°$$

$$F_{EC} = \frac{50}{\sqrt{3}}$$

$\Sigma H = 0$

$$F_{AE} = F_{ED} = \frac{125}{\sqrt{3}}$$

Joint D :—



$\Sigma V = 0$

$$F_{CD} \cdot \sin 60° + 175 = 0$$

$$F_{CD} = \frac{-350}{\sqrt{3}}$$

$$F_{AB} = \frac{-250}{\sqrt{3}}$$

$$F_{BC} = \frac{-150}{\sqrt{3}}$$

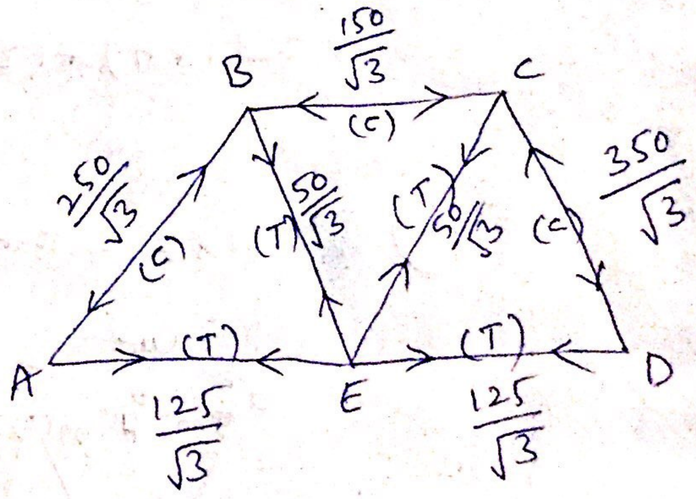
$$F_{CD} = \frac{-350}{\sqrt{3}}$$

$$F_{ED} = \frac{125}{\sqrt{3}}$$

$$F_{AE} = \frac{125}{\sqrt{3}}$$

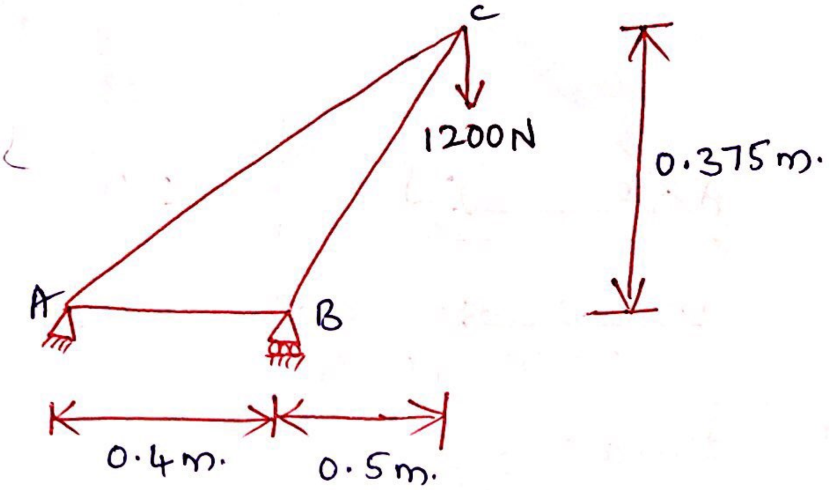
$$F_{BE} = \frac{50}{\sqrt{3}}$$

$$F_{EC} = \frac{50}{\sqrt{3}}$$



① A truss is loaded and supported as shown in the fig. Find by the method of joints the axial forces in each member of the truss.

①



Ⓐ

$\Sigma f_x = 0;$ $\boxed{H_A = 0}$

$\Sigma f_y = 0;$ $R_A + R_B - 1200 = 0$

$R_A + R_B = 1200N \rightarrow ①$

$\Sigma M_A = 0;$ $(-R_B \times 0.4) + (1200 \times 0.9) = 0$

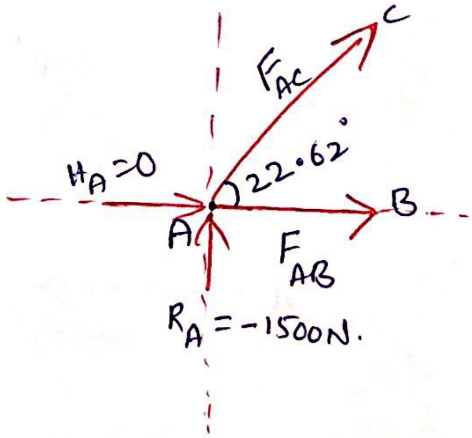
$\boxed{R_B = 2700N.}$

'R_B' sub. in ①

$R_A + 2700N = 1200N$

$\boxed{R_A = -1500N.}$

Joint A :-

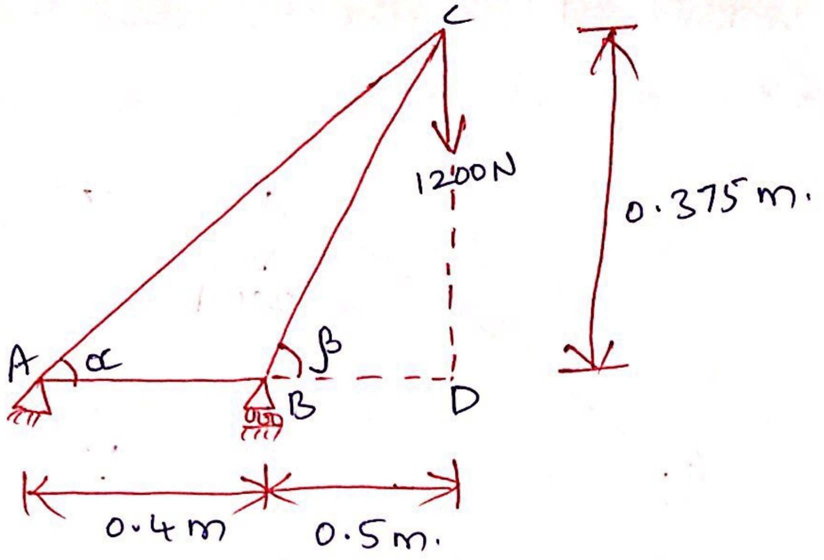


$\Sigma f_y = 0;$ $R_A + F_{AC} \cdot \sin. 22.62° = 0$

$F_{AC} = 3900 N. (T)$

$\Sigma f_x = 0;$ $H_A + F_{AB} + F_{AC} \cdot \cos 22.62° = 0$

$F_{AB} = -3600N \ (C)$



From ΔACD; $\tan\alpha = \frac{CD}{AD}$

$= \frac{0.375}{0.9}$

$\alpha = 22.62^\circ$

From ΔCBD; $\tan\beta = \frac{CD}{BD}$

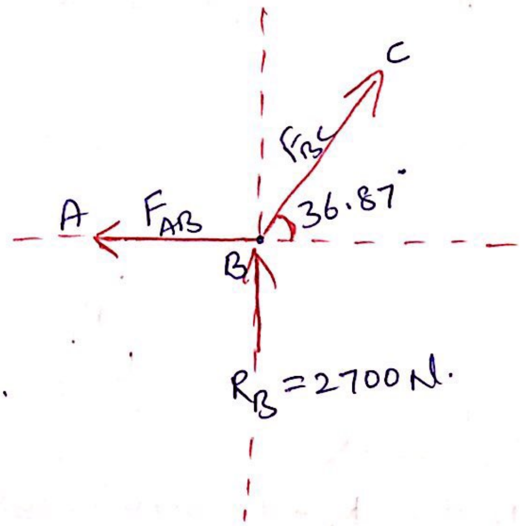
$= \frac{0.375}{0.5}$

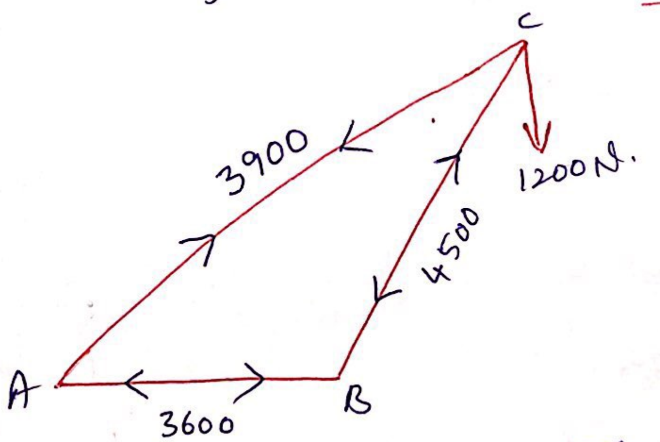
$\beta = 36.87^\circ$

Joint 'B' :—

$\Sigma f_y = 0$; $R_B + F_{BC} \cdot \sin 36.87^\circ = 0$

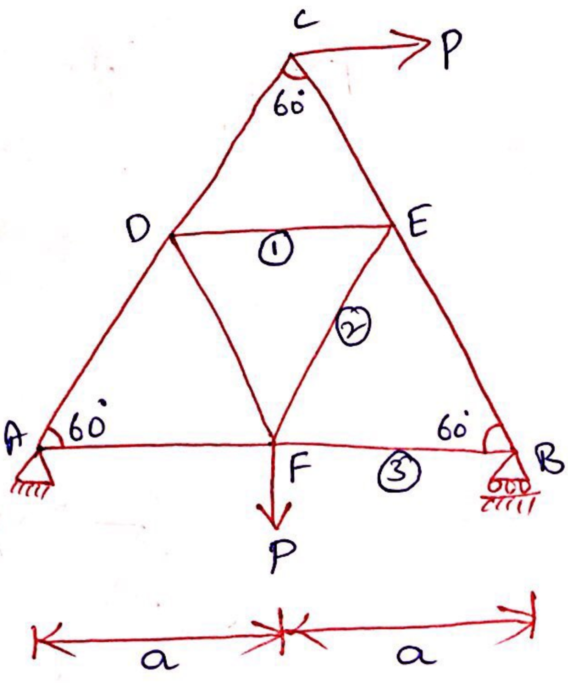
$F_{BC} = -4500 N. (C)$





② Find the axial forces in the members 1, 2, 3 by the method of joints.

②



Ⓐ $\Sigma f_x = 0;\ H_A + P = 0 \Rightarrow \boxed{H_A = -P}$

$\Sigma f_y = 0;\ R_A + R_B - P = 0 \Rightarrow R_A + R_B = P \rightarrow ①$

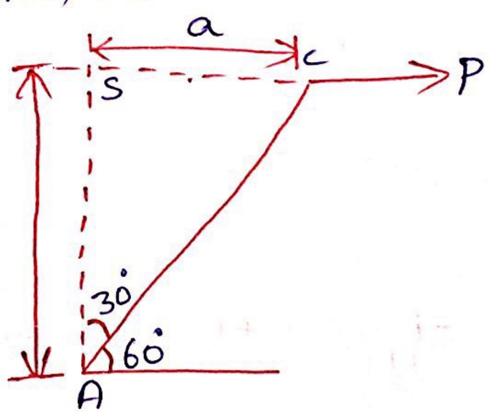
$\Sigma M_A = 0;\ (-R_B \times 2a) + (P \times a) + (P \times AS) = 0$

$(-R_B \times 2a) + (P \times a) + (P \times a\sqrt{3}) = 0$

$\boxed{R_B = 1.366P}$

'R_B' sub. in ①

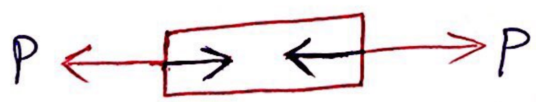
$\boxed{R_A = -0.366P}$



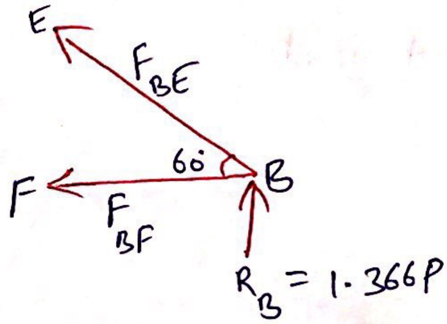
From ΔASC,

$\tan 30° = \frac{a}{AS}$

$AS = a\sqrt{3}$



Joint B :—



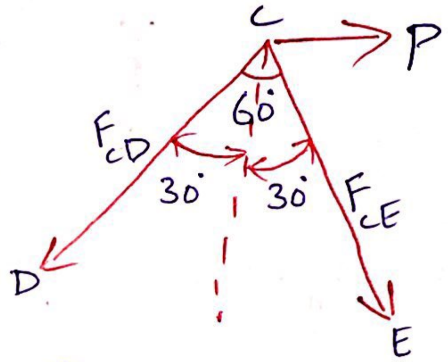
$\Sigma f_y = 0;\ R_B + F_{EB} \cdot \sin 60^\circ = 0$

$F_{EB} = -1.577P$

$\Sigma f_x = 0;\ -F_{BF} - F_{BE} \cdot \cos 60^\circ = 0$

$F_{BF} = 0.788P$

Joint c :—



$\Sigma f_x = 0;\ P + F_{CE} \cdot \sin 30^\circ - F_{CD} \cdot \sin 30^\circ = 0$

$F_{CD} \cdot \sin 30^\circ - F_{CE} \cdot \sin 30^\circ = P \rightarrow (2)$

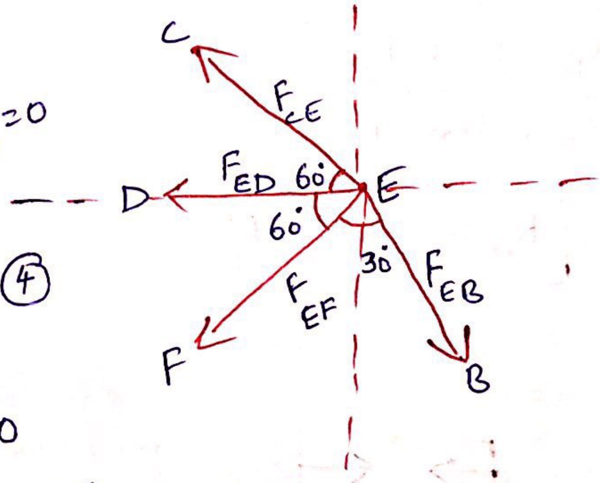
$\Sigma f_y = 0;\ -F_{CE} \cdot \cos 30^\circ - F_{CD} \cdot \cos 30^\circ = 0$

$F_{CD} \cdot \cos 30^\circ + F_{CE} \cdot \cos 30^\circ = 0 \rightarrow (3)$

solving (2), (3)

$F_{CD} = P,\ F_{CE} = -P$

Joint E :—



$\Sigma f_x = 0;$

$-F_{CE} \cdot \cos 60^\circ - F_{EF} \cdot \cos 60^\circ + F_{EB} \cdot \sin 30^\circ = -F_{ED} = 0$

$F_{ED} + F_{EF} \cdot \cos 60^\circ = -0.2885P \rightarrow (4)$

$\Sigma f_y = 0;$

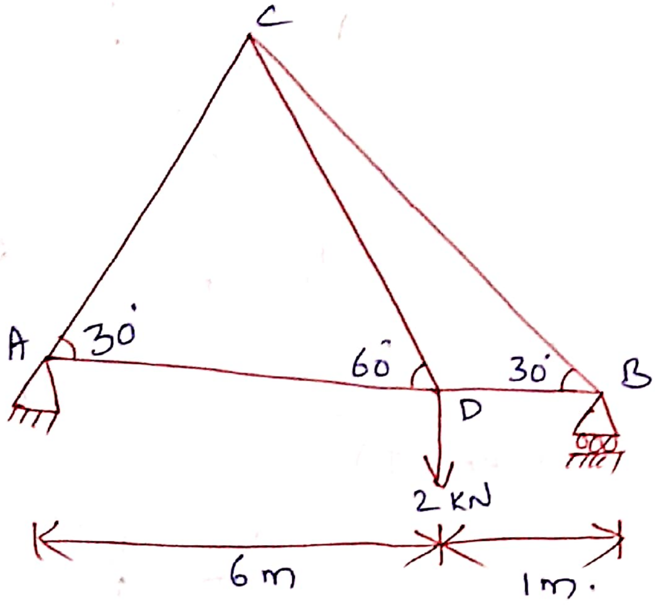
$F_{CE} \cdot \sin 60^\circ - F_{EF} \cdot \sin 60^\circ - F_{EB} \cdot \cos 30^\circ = 0$

$-P \cdot \sin 60^\circ - F_{EF} \cdot \sin 60^\circ - (-1.577P) \cdot \cos 30^\circ = 0$

$F_{EF} = -0.577P$; 'F_{EF}' sub. in (4)

$F_{ED} = -0.577P$

(1) Find forces in all the members of truss by the method of joints. (1)



(A)

$\Sigma H = 0 \Rightarrow H_A = 0$

$\Sigma V = 0 \Rightarrow R_A + R_B - 2 = 0$

$R_A + R_B = 2\,KN \rightarrow (1)$

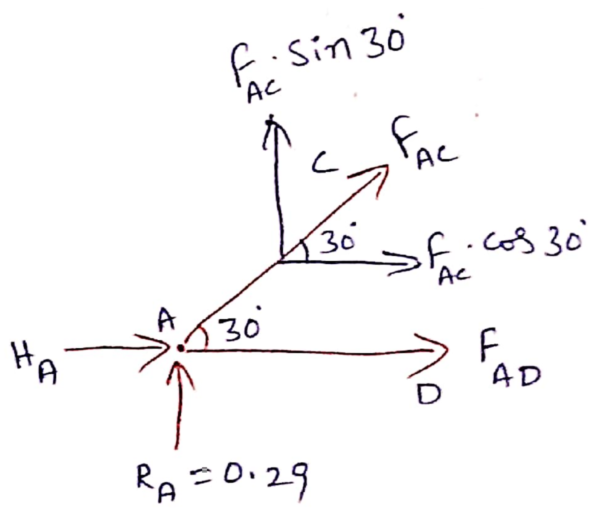
$\Sigma M_A = 0 \Rightarrow (2 \times 6) - (R_B \times 7) = 0$

$R_B = 1.71\,KN.$

'R_B' sub. in (1)

$R_A + 1.71 = 2$

$R_A = 0.29\,KN.$



Joint A :-

$\Sigma V = 0$; $R_A + F_{AC} \cdot \sin 30° = 0$

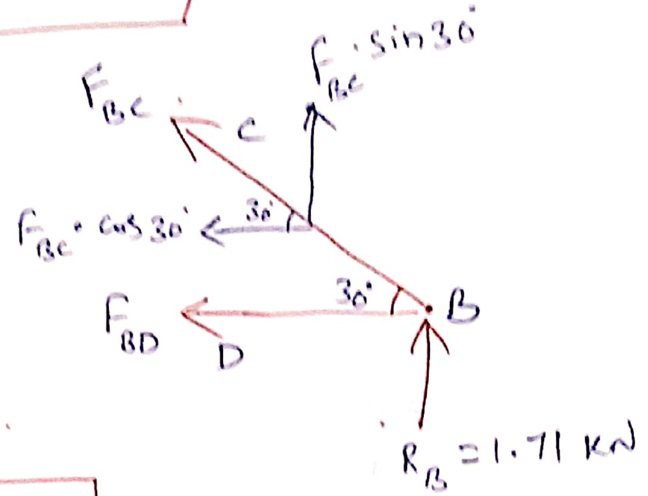
$0.29 + F_{AC} \cdot \sin 30° = 0$

$\boxed{F_{AC} = -0.58\,KN.\,(C)}$

$\Sigma H = 0;\quad F_{AD} + F_{AC} \cdot \cos 30^\circ = 0$

$$\boxed{F_{AD} = 0.50\ \text{kN}\ (T)}$$

Joint B :–



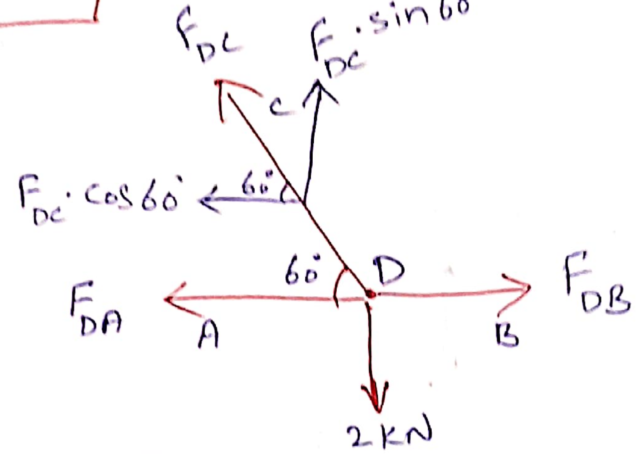
$\Sigma V = 0;\quad R_B + F_{BC} \cdot \sin 30^\circ = 0$

$$\boxed{F_{BC} = -3.42\ \text{kN}\ (C)}$$

$\Sigma H = 0;\quad -F_{BD} - F_{BC} \cdot \cos 30^\circ = 0$

$$\boxed{F_{BD} = 2.96\ \text{kN}\ (T)}$$

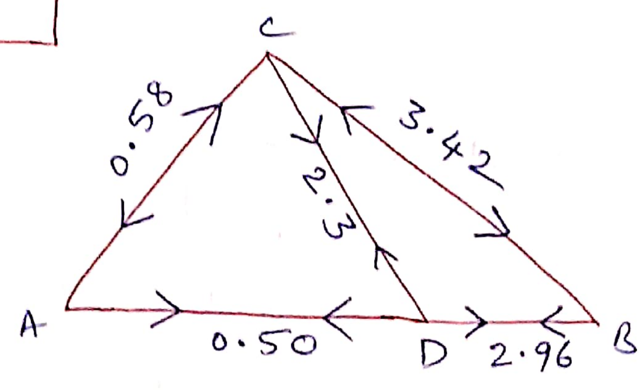
Joint D :–



$\Sigma V = 0;\quad F_{DC} \cdot \sin 60^\circ - 2 = 0$

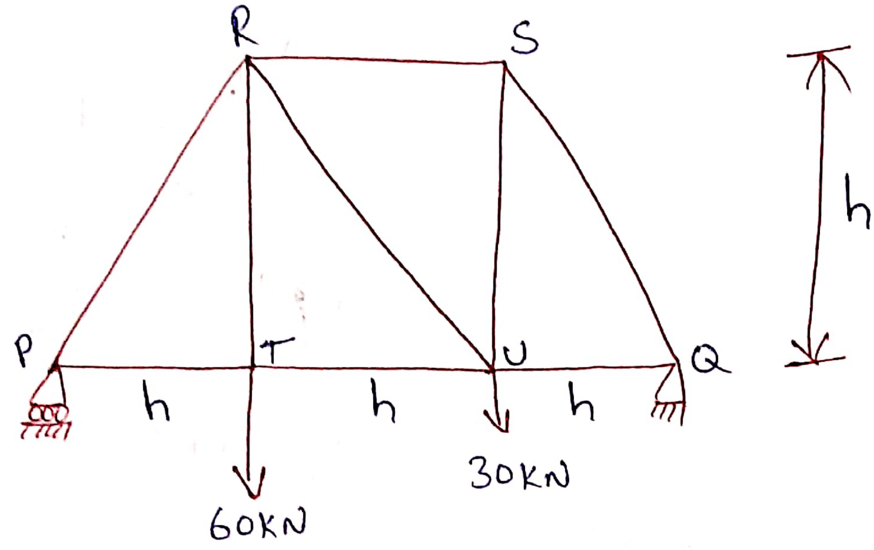
$$\boxed{F_{DC} = 2.3\ \text{kN}\ (T)}$$

Members	Magnitude (kN)	Nature of force
AC	−0.58	C
AD	0.50	T
BC	−3.42	C
BD	2.96	T
DC	2.3	T



problem on method of Joints :—

1. Find the forces in the members by using of method of joints ?



(A)

Equilibrium equations :

$\Sigma f_x = 0 \Rightarrow \Sigma H = 0$

$-H_Q = 0 \Rightarrow H_Q = 0$

$\Sigma f_y = 0 \Rightarrow \Sigma V = 0$

$R_P + R_Q = 60 + 30$

$R_P + R_Q = 90 \text{ KN} \rightarrow (1)$

$\Sigma M_P = 0 \Rightarrow (60 \times h) + (30 \times 2h) - (R_Q \times 3h) = 0$

$R_Q = 40 \text{ KN}.$

R_Q Sub. in (1)

$R_P + 40 = 90$

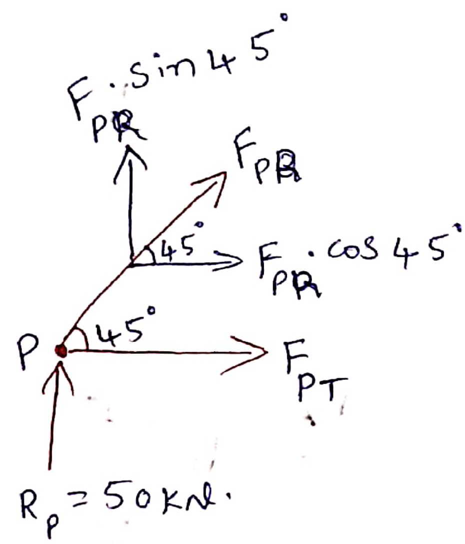
$R_P = 50 \text{ KN}.$

$$\tan\theta = \frac{Opp}{Adj}$$

$$\tan\theta = \frac{h}{h}$$

$$\theta = \tan^{-1}(1)$$

$$\theta = 45^\circ$$



Joint P :—

$\Sigma f_y = 0 \Rightarrow \Sigma V = 0$

$$R_p + F_{PR} \cdot \sin 45^\circ = 0$$

$$50 + F_{PR} \cdot \frac{1}{\sqrt{2}} = 0$$

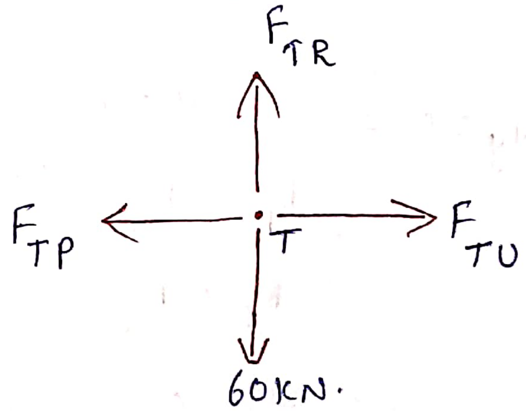
$$\boxed{F_{PR} = -50\sqrt{2}\ KN.\ (C)}$$

$\Sigma f_x = 0 \Rightarrow \Sigma H = 0$

$$F_{PT} + F_{PR} \cdot \cos 45^\circ = 0$$

$$F_{PT} + \left(-50\sqrt{2} \cdot \frac{1}{\sqrt{2}}\right) = 0$$

$$\boxed{F_{PT} = 50\,KN.\ (T)}$$



Joint T :—

$\Sigma f_y = 0 \Rightarrow \Sigma V = 0$

$$F_{TR} - 60 = 0$$

$$\boxed{F_{TR} = 60\,KN\ (T)}$$

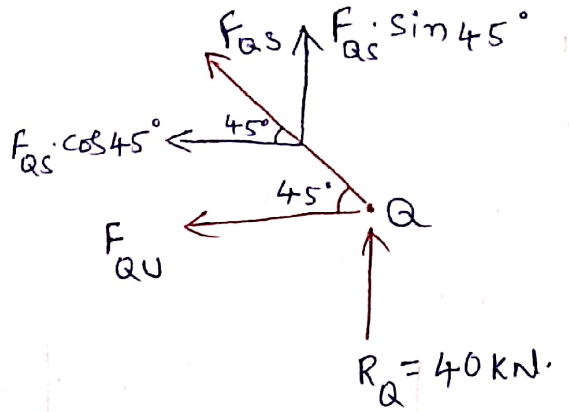
$\Sigma f_x = 0 \Rightarrow \Sigma H = 0$

$$F_{TU} - F_{TP} = 0$$

$$F_{TU} - 50 = 0$$

$$\boxed{F_{TU} = 50\,KN\ (T)}$$

Joint Q :—



$\Sigma f_y = 0 \Rightarrow \Sigma V = 0$

$$40 + F_{QS} \cdot \sin 45° = 0$$

$$40 + F_{QS} \cdot \frac{1}{\sqrt{2}} = 0$$

$$F_{QS} \cdot \frac{1}{\sqrt{2}} = -40$$

$$\boxed{F_{QS} = -40\sqrt{2}\ KN.\ (C)}$$

$\Sigma f_x = 0 \Rightarrow \Sigma H = 0$

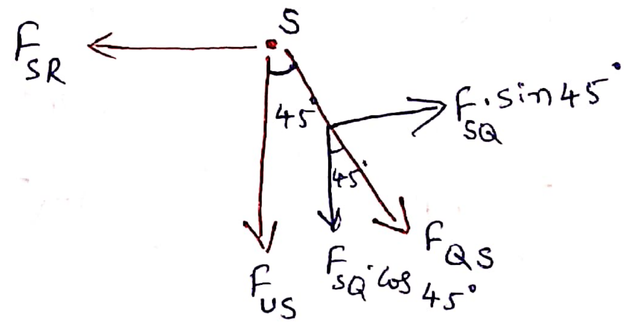
$$-F_{QU} - F_{QS} \cdot \cos 45° = 0$$

$$-F_{QU} - \left(-40\cancel{\sqrt{2}} \cdot \frac{1}{\cancel{\sqrt{2}}}\right) = 0$$

$$-F_{QU} + 40 = 0$$

$$\boxed{F_{QU} = 40\,KN\ (T)}$$

Joint S :—



$\Sigma f_y = 0 \Rightarrow \Sigma V = 0$

$$-F_{US} - F_{SQ} \cdot \cos 45° = 0$$

$$-F_{US} - \left(-40\cancel{\sqrt{2}} \cdot \frac{1}{\cancel{\sqrt{2}}}\right) = 0$$

$$\boxed{F_{US} = 40\,KN\ (T)}$$

$\Sigma f_x = 0 \Rightarrow \Sigma H = 0$

$$-F_{SR} + F_{SQ} \cdot \sin 45° = 0$$

$$-F_{SR} + \left(-40\cancel{\sqrt{2}} \cdot \frac{1}{\cancel{\sqrt{2}}}\right) = 0$$

$$\boxed{F_{SR} = -40\,KN}$$

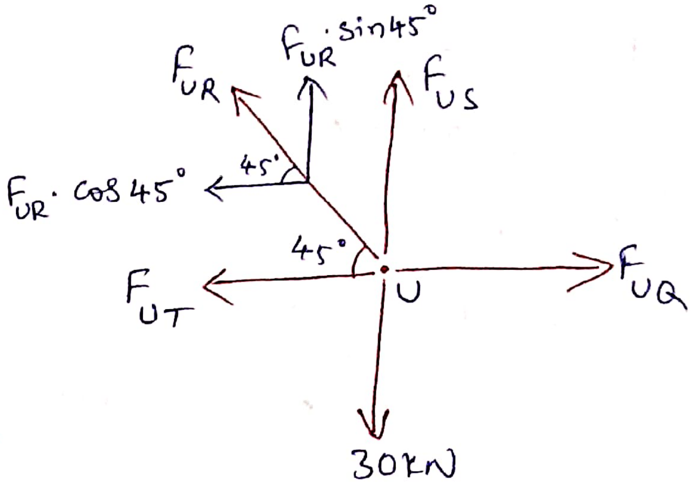
Joint U :—

$\Sigma V = 0$

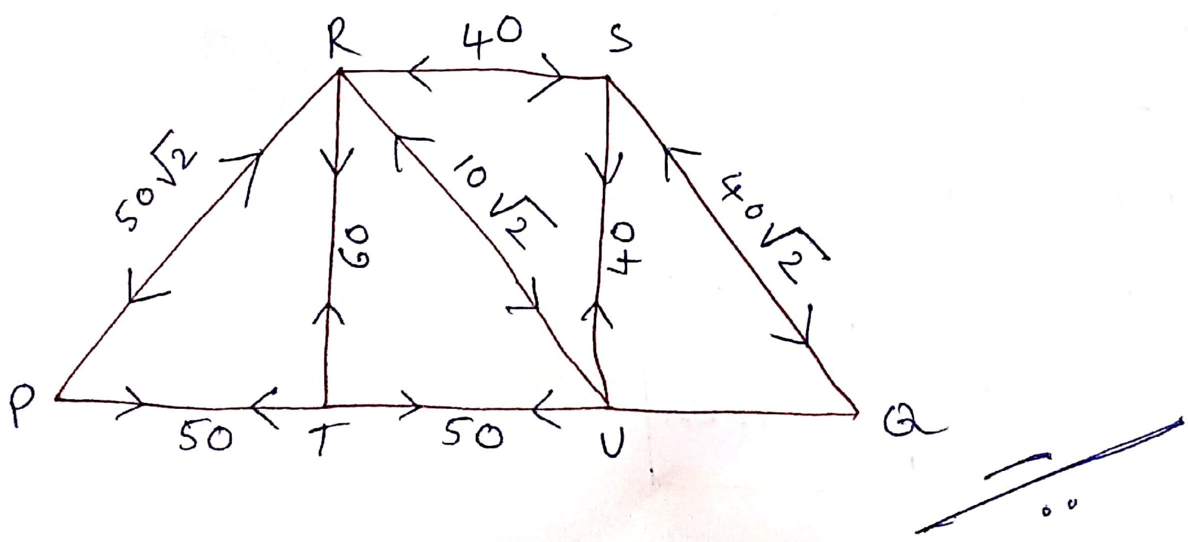
$F_{US} - 30 + F_{UR} \cdot \sin 45^\circ = 0$

$40 - 30 + F_{UR} \cdot \frac{1}{\sqrt{2}} = 0$

$\boxed{F_{UR} = -10\sqrt{2}\ \text{KN} \cdot \ (C)}$



Members	magnitude(KN)	Nature
PR	$-50\sqrt{2}$	C
PT	50	T
TR	60	T
TU	50	T
QS	$-40\sqrt{2}$	C
QU	40	T
US	40	T
SR	−40	C
UR	$-10\sqrt{2}$	C



Method of Sections :—

(1) Replace the supports with reactions

(2) ~~Ead~~ Identify the loads, if any convert all them to perfectly vertical and horizontal.

(3) Use equilibrium eq'ls for the whole truss. $\Sigma H = 0$, $\Sigma V = 0$, $\Sigma M = 0$

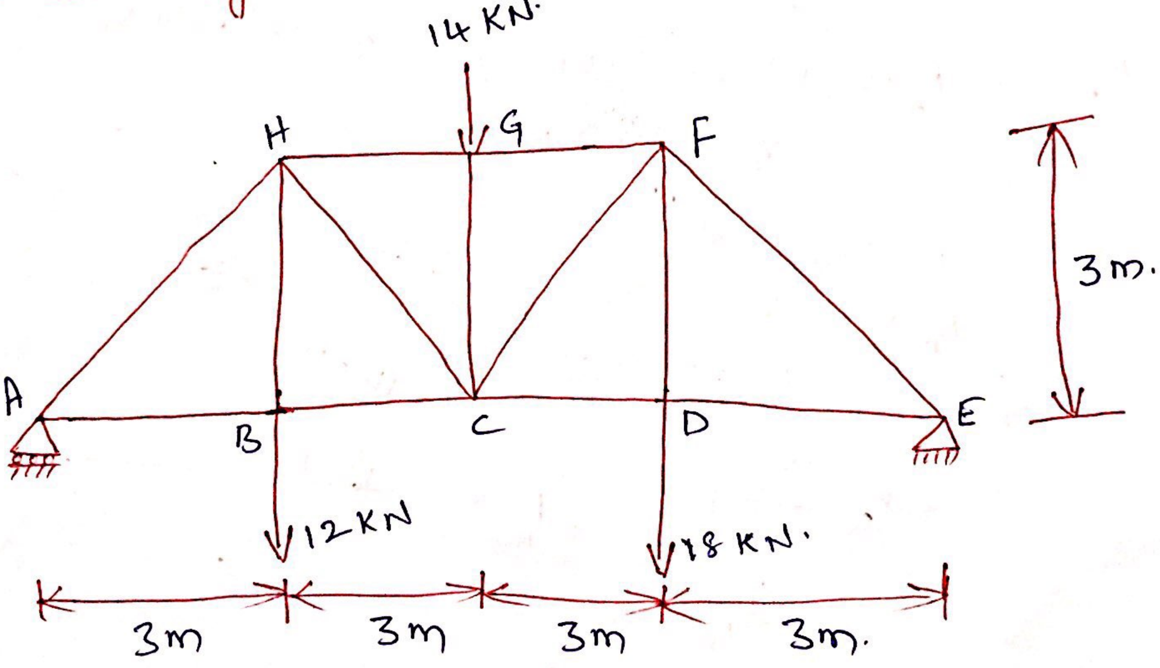
(4) After using equilibrium eq'ls, we get all the reactions.

(5) cut the truss in such a way that the section should not pass through more than 3 members.

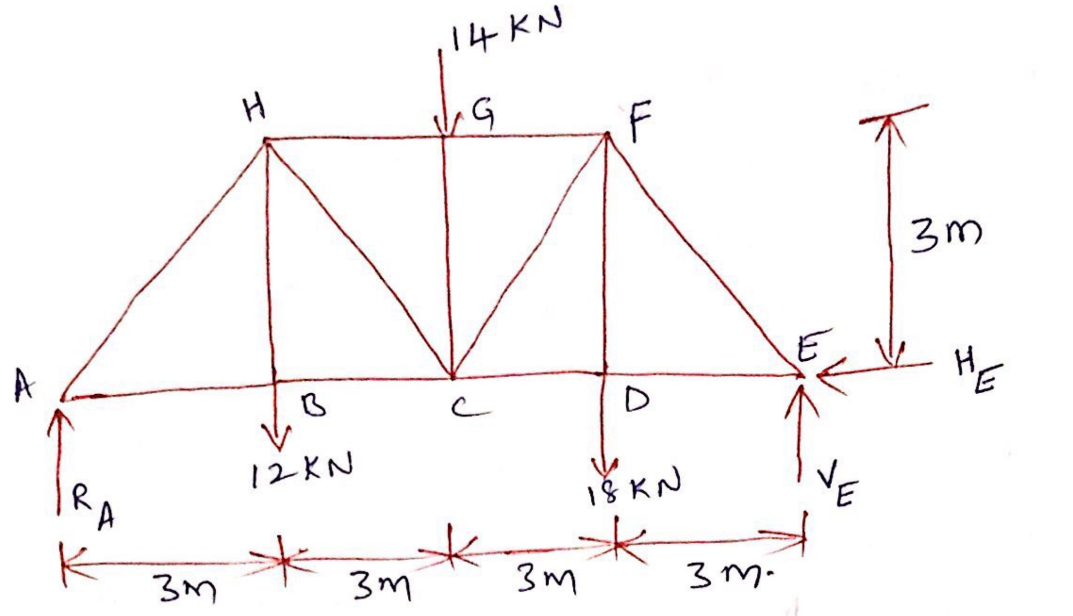
(6) Again use the equilibrium eq'ls i.e. $\Sigma H = 0$, $\Sigma V = 0$, $\Sigma M = 0$.

(7) we will get other unknowns.

(1) Determine the magnitude and nature of forces in the members HG, HC & BC of the ~~forces~~ truss loaded & supported as shown in below fig.



(A)



$$\tan^{-1}\left(\frac{3}{3}\right) = 45^\circ$$

$\uparrow$ +ve

$\downarrow$ -ve.

$$\Sigma H = 0 \;;\quad -H_E = 0 \Rightarrow H_E = 0$$

$$\Sigma V = 0 \;;\quad R_A - 12 - 18 + V_E - 14 = 0$$

$$R_A + V_E = 44 \rightarrow (1)$$

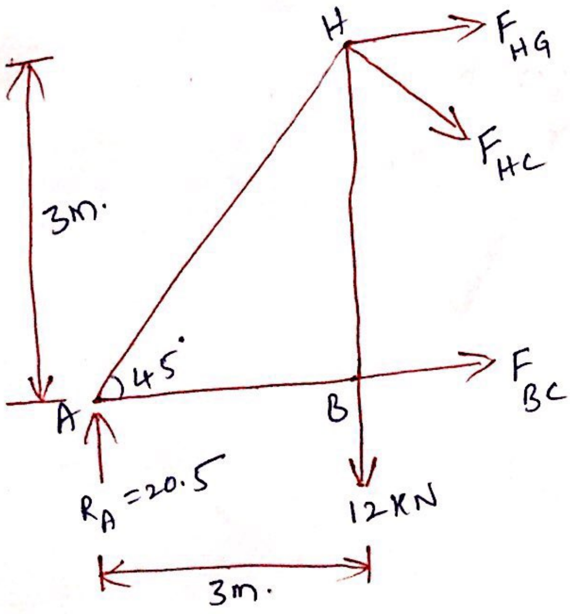
$$\Sigma M_A = 0 ; (R_A \times 0) + (12 \times 3) + (18 \times 9) - (V_E \times 12) + (14 \times 6) = 0$$

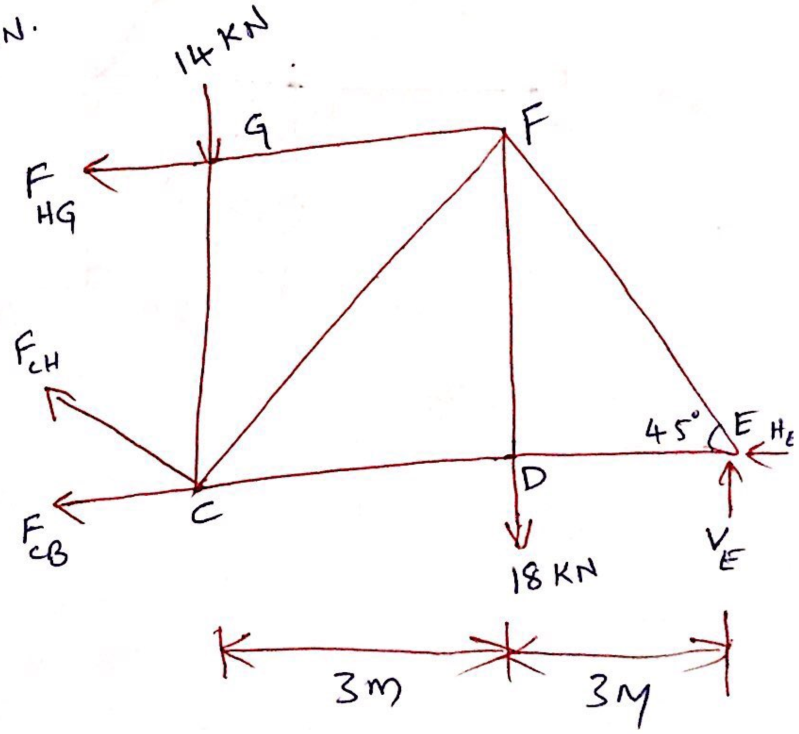
$$V_E = 23.5 \text{ KN.}$$

V_E sub. in (1)

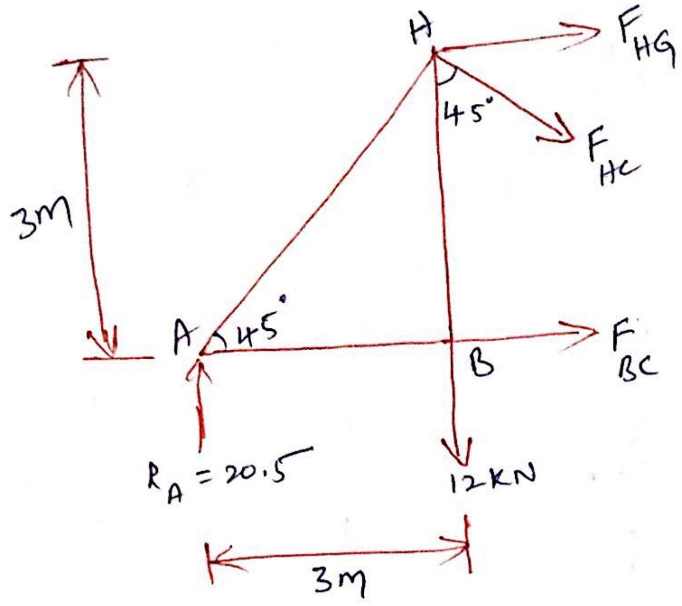
$$R_A + 23.5 = 44$$

$$R_A = 20.5 \text{ KN.}$$





Consider LHS part :-



$\Sigma V = 0;\quad 20.5 - 12 - F_{HC} \cdot \cos 45^\circ = 0$

$F_{HC} = 12.02\,KN.$

$\Sigma H = 0;\quad F_{BC} + F_{HC} \cdot \sin 45^\circ + F_{HG} = 0$

$F_{BC} + F_{HG} = -8.5 \rightarrow (2)$

$\Sigma M_H = 0;\quad (20.5 \times 3) - (F_{BC} \times 3) = 0$

$F_{BC} = 20.5\,KN.$

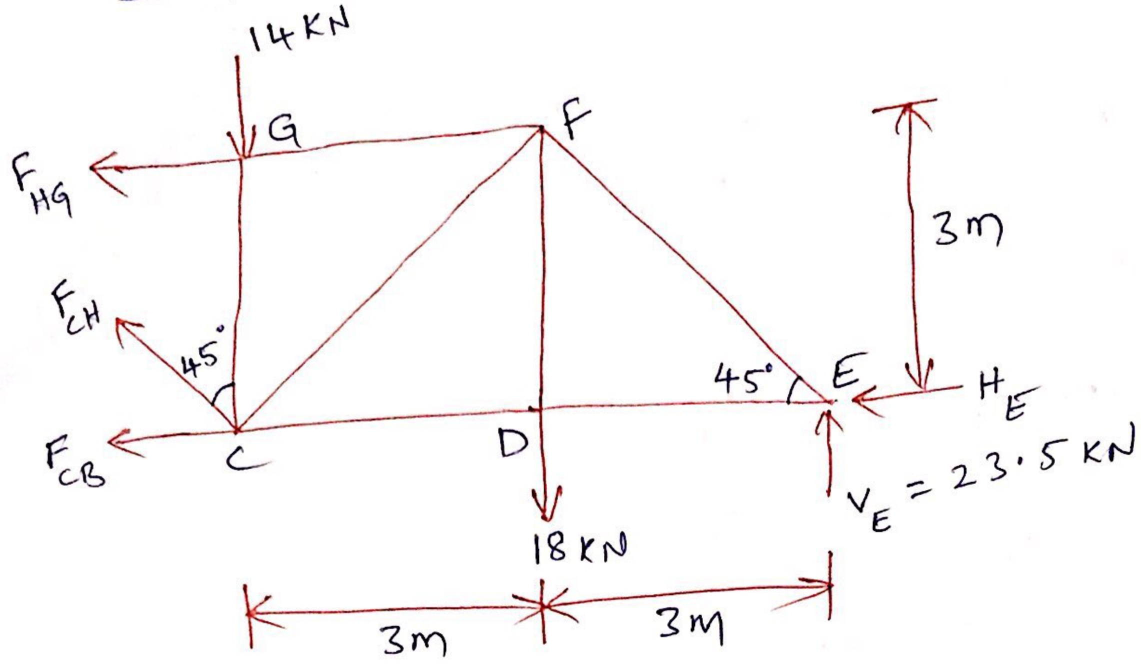
F_{BC} sub in (2)

$20.5 + F_{HG} = -8.5$

$F_{HG} = -29\,KN.$

Member	magnitude of force	Nature of force
F_{HC}	12.02 KN	Tension
F_{BC}	20.5 KN	Tension
F_{HG}	29 KN	compression

consider RHS Part :-



$\Sigma V = 0;\ F_{CH} \cdot \cos 45° - 18 + V_E - 14 = 0$

$F_{CH} = 12.02$ KN.

$\Sigma H = 0;\quad -F_{CB} - F_{CH} \cdot \sin 45° - F_{HG} = 0 \rightarrow (3)$

$\Sigma M_c = 0;\ (18 \times 3) - (V_E \times 6) - (F_{HG} \times 3) = 0$

$F_{HG} = -29$ KN.

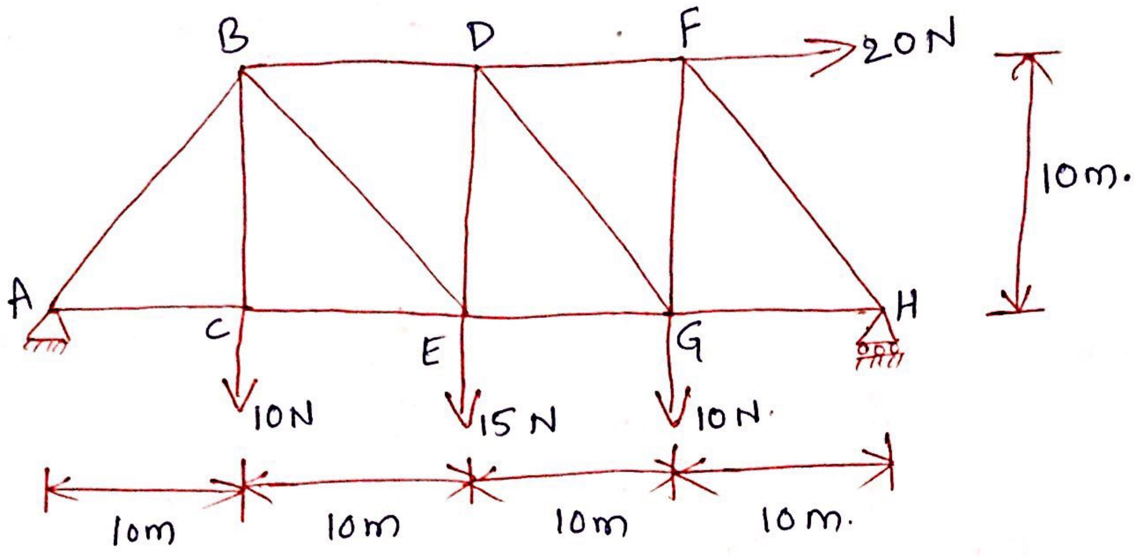
F_{HG} sub. in (3)

$-F_{CB} - 12.02 \cdot \sin 45° - (-29) = 0$

$F_{CB} = 20.5$ KN.

Member	Magnitude of Force	Nature of force
F_{CH}	12.02 KN	Tension
F_{CB}	20.5 KN	Tension
F_{GH}	29 KN	compression

(2) Find the forces in members DF, DG, EG.



(A)

Support Reactions :—

$\Sigma f_x = 0$; $H_A + 20 = 0 \Rightarrow H_A = -20N.$

$\Sigma f_y = 0$; $R_A + R_H - 10 - 15 - 10 = 0$

$R_A + R_H = +35 \rightarrow (1)$

$\Sigma M_A = 0$; $(10 \times 10) + (15 \times 20) + (10 \times 30) + (20 \times 10) - (R_H \times 40) = 0$

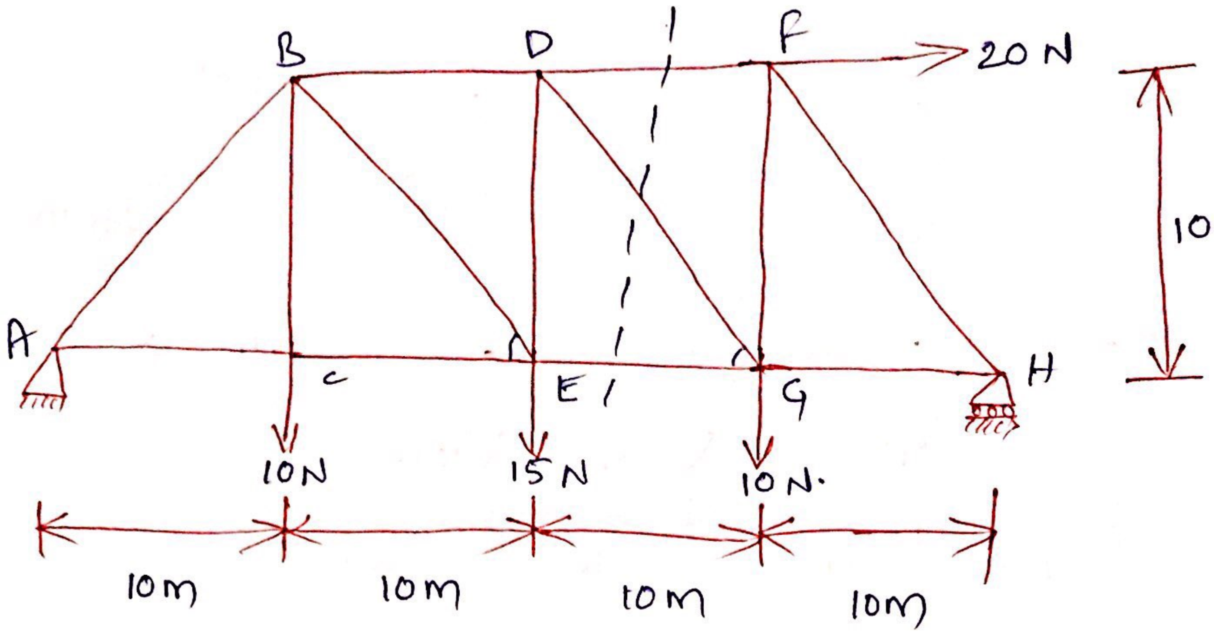
$R_H = 22.5N.$

R_H sub. in (1)

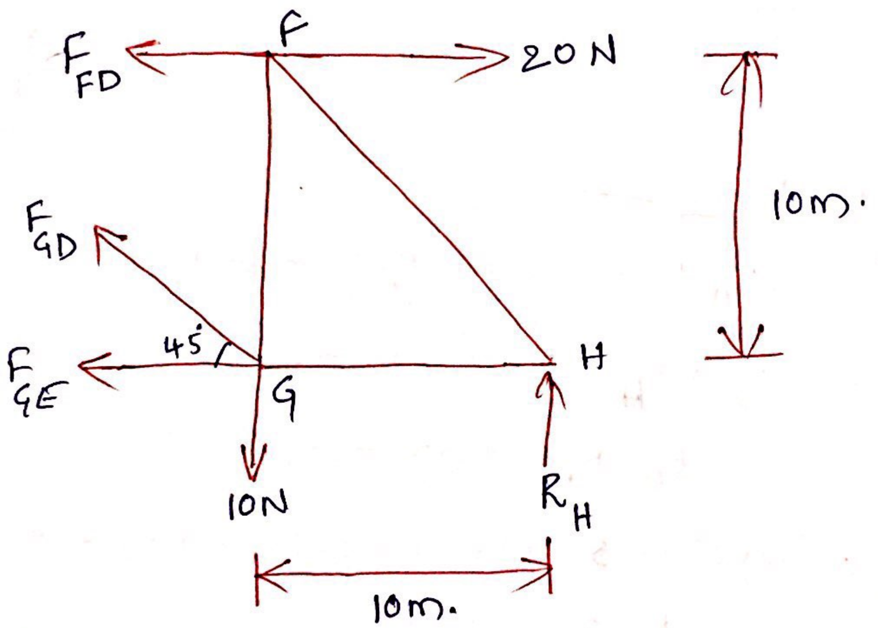
$R_A + 22.5 = +35$

$R_A = 12.5N.$

$$\boxed{\begin{aligned} \therefore R_A &= 12.5N. \\ R_H &= 22.5N \\ H_A &= -20N. \end{aligned}}$$



Consider RHS Part :—



$$\tan^{-1}\left(\frac{10}{10}\right) = 45^\circ$$

$\Sigma f_y = 0;\quad -10 + R_H + F_{DG} \cdot \sin 45^\circ = 0$

$F_{DG} = -17.677\ N.\quad (C)$

$\Sigma M_G = 0;\ (-F_{DF} \times 10) + (20 \times 10) - (R_H \times 10) = 0$

$F_{DF} = -2.5\ N.\quad (C)$

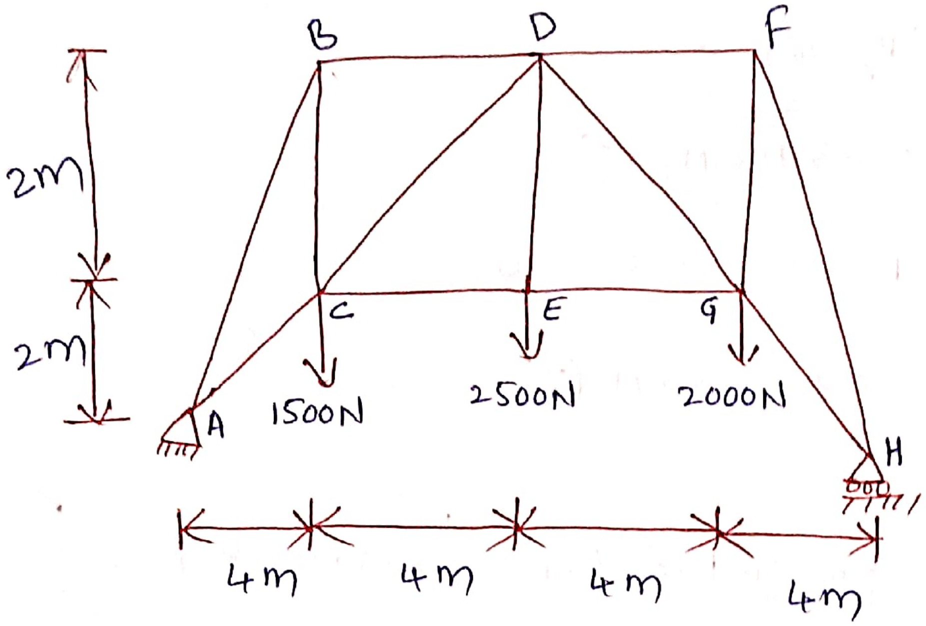
$\Sigma f_x = 0; \quad -F_{EG} - F_{DF} + 20 - F_{DG} \cdot \cos 45^\circ = 0$

$$-F_{EG} - (-2.5) + 20 - (-17.677 \cdot \cos 45^\circ) = 0$$

$$F_{EG} = 35\,N. \ (T)$$

S.NO.	Force in Member	Magnitude	Nature
1	F_{DF}	2.5 N	C
2	F_{DG}	17.677 N.	C
3	F_{EG}	35 N.	T

① Find the force in members BD, CD & CE by using method of sections. ①



Ⓐ $\Sigma H = 0$; $\boxed{H_A = 0}$

$\Sigma V = 0$; $R_A + R_H - 1500 - 2500 - 2000 = 0$

$R_A + R_H = 6000\,N \rightarrow ①$

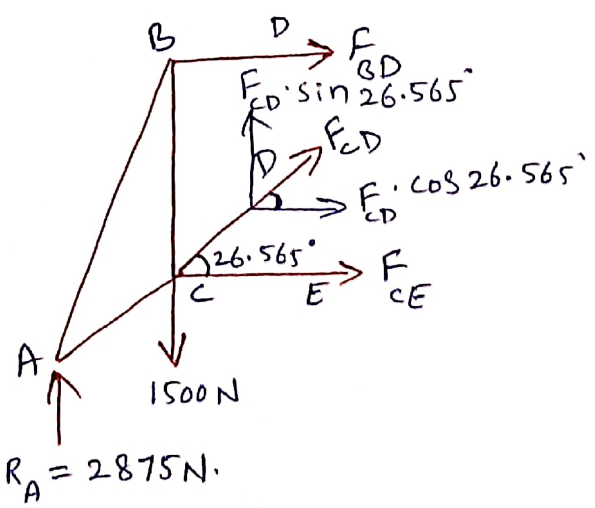
$\Sigma M_A = 0$; $(1500 \times 4) + (2500 \times 8) + (2000 \times 12) - (R_H \times 16) = 0$

$\boxed{R_H = 3125\ N.}$

$R_A + 3125 = 6000$

$\boxed{R_A = 2875\ N.}$

Consider LHS PART :-



$\tan\theta = \frac{2}{4}$

$\theta = \tan^{-1}\left(\frac{2}{4}\right)$

$\theta = 26.565°$

$\Sigma V = 0;$ $R_A + F_{CD} \cdot \sin 26.565^\circ = 1500$

$F_{CD} = -3074.598\ N.$ (C)

$\Sigma M_C = 0;$ $(F_{BD} \times 2) + (R_A \times 4) = 0$

$F_{BD} = -5750 N.$ (C)

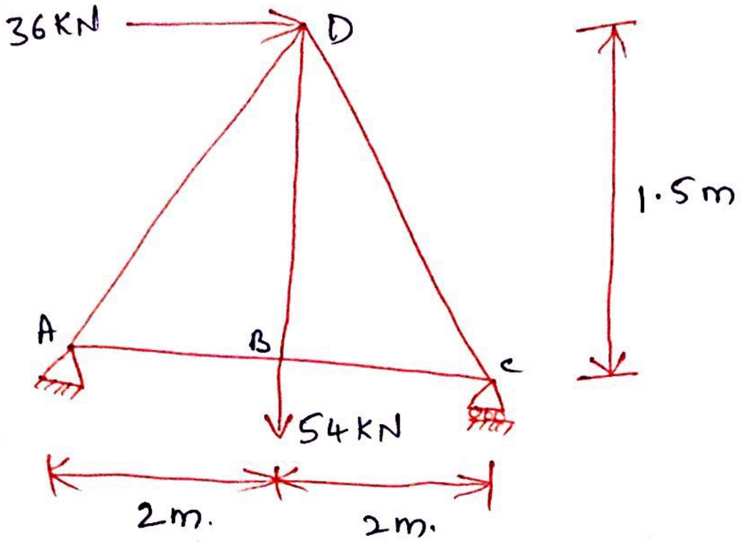
$\Sigma H = 0;$ $F_{CE} + F_{BD} + F_{CD} \cdot \cos 26.565^\circ = 0$

$F_{CE} = 8500.005 N$ (T)

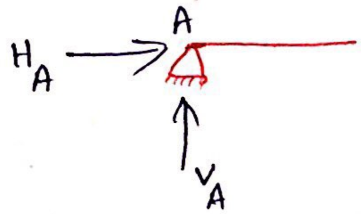
S.No.	members	magnitude (KN)	Nature of force
1	CD	-3074.598	C
2	BD	-5750	C
3	CE	8500	T

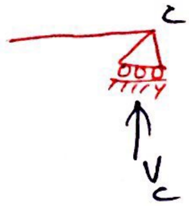
Tension coefficient Method :—

1. Using method of tension coefficient analyse the plane truss shown in fig and find the forces in the members.



(A) Co-ordinates : $A = (0,0)$, $B = (2,0)$, $C = (4,0)$, $D = (2, 1.5)$.
x, y (x, y) (x, y) (x, y)





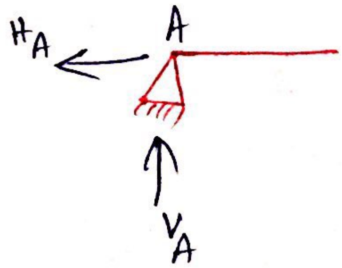
Support Reactions :—

$\Sigma f_y = 0$; $V_A + V_C - 54 = 0$

$V_A + V_C = 54 \rightarrow (1)$

$\Sigma f_x = 0$; $H_A + 36 = 0$

$H_A = -36$



$\Sigma M_A = 0$; $(54 \times 2) - (V_C \times 4) + (36 \times 1.5) = 0$

$V_C = 40.5$ KN.

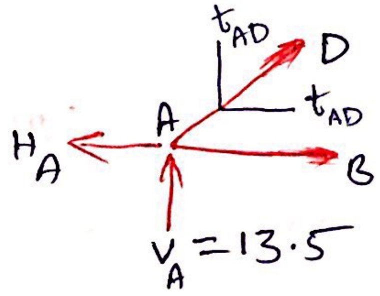
V_C sub. in (1)

$V_A + 40.5 = 54$

$V_A = 13.5$ KN.

S.No.	Member	x_i	x_j	$x_{ij}=x_j-x_i$	y_i	y_j	$y_{ij}=y_j-y_i$	$l_{ij}=\sqrt{x_{ij}^2+y_{ij}^2}$
1	AB	0	2	2	0	0	0	2
2	BC	2	4	2	0	0	0	2
3	CD	4	2	-2	0	1.5	1.5	2.5
4	BD	2	2	0	0	1.5	1.5	1.5
5	AD	0	2	2	0	1.5	1.5	2.5

Joint A :-



$\Sigma H = 0$

$x_{AB} \cdot t_{AB} + x_{AD} \cdot t_{AD} - H_A = 0$

$2 \cdot t_{AB} + 2 \cdot t_{AD} - 36 = 0 \Rightarrow (2)$

$\Sigma V = 0$

$V_A + y_{AD} \cdot t_{AD} = 0$

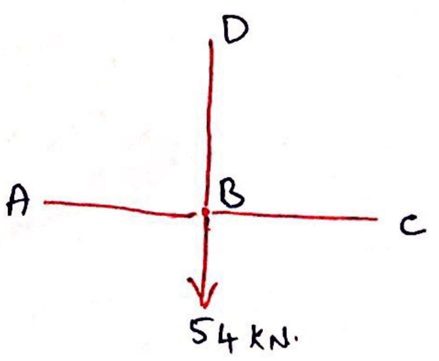
$13.5 + 1.5 \cdot t_{AD} = 0$

$\boxed{t_{AD} = -9 \text{ kN/m.}}$

t_{AD} sub. in (2)

$\boxed{t_{AB} = 27 \text{ kN/m.}}$

Joint B :-



$\Sigma H = 0$

$x_{BA} \cdot t_{BA} + x_{BC} \cdot t_{BC} = 0$

$-2 \cdot t_{BA} + 2 \cdot t_{BC} = 0$

$-2(27) + 2 t_{BC} = 0$

$\boxed{t_{BC} = 27 \text{ kN/m.}}$

$\Sigma V = 0$

$y_{BD} \cdot t_{BD} - 54 = 0$

$1.5 (t_{BD}) - 54 = 0$

$$\boxed{t_{BD} = 36\ kN/m.}$$

Joint c :—

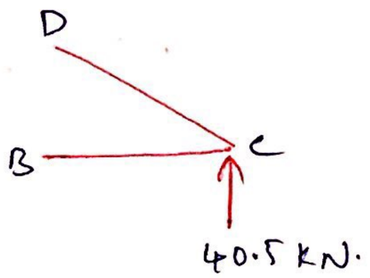
$\Sigma H = 0$

$x_{CB} \cdot t_{CB} + x_{CD} \cdot t_{CD} = 0$

$-2 \cdot t_{CB} - 2 \cdot t_{CD} = 0$

$-2(27) - 2 \cdot t_{CD} = 0$

$$\boxed{t_{CD} = -27\ kN/m}$$



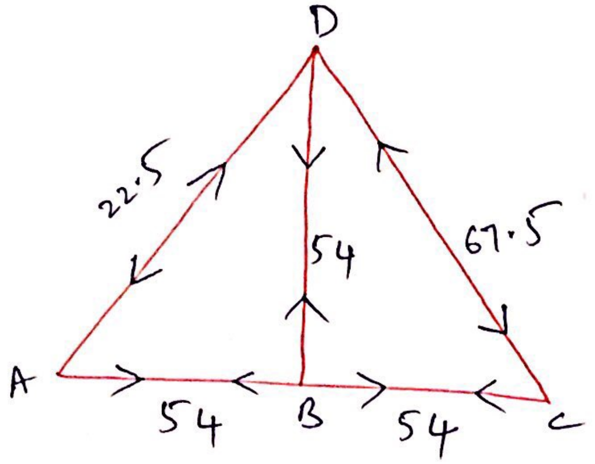
$\Sigma V = 0$

$y_{CD} \cdot t_{CD} + 40.5 = 0$

$1.5 \times (-27) = -40.5$

$$\boxed{-40.5 = -40.5}$$

S.No.	Member	t_{ij} (kN/m)	l_{ij} (m)	$T = t_{ij} \times l_{ij}$	Nature of force
1	AB	27	2	54	T
2	BC	27	2	54	T
3	CD	-27	2.5	-67.5	C
4	DA	-9	2.5	-22.5	C
5	BD	36	1.5	54	T



Tension co-efficient :-

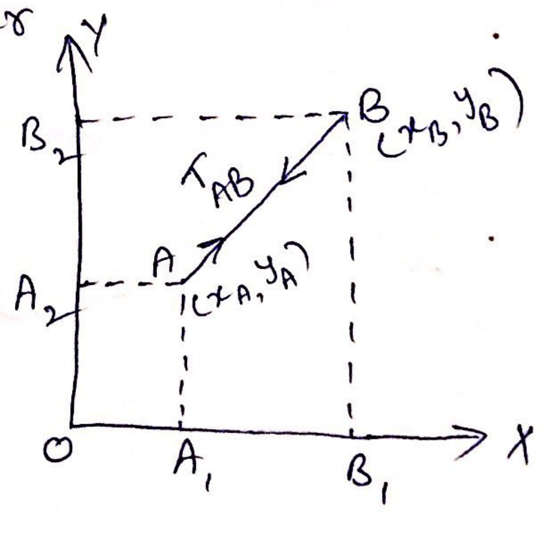
The tension coefficient for a member of a frame is defined as the pull (or) tension in that member divided by its length.

$$t = \frac{T}{L}$$

where, t = Tension coefficient for the member

T = pull in the member

L = Length of the member

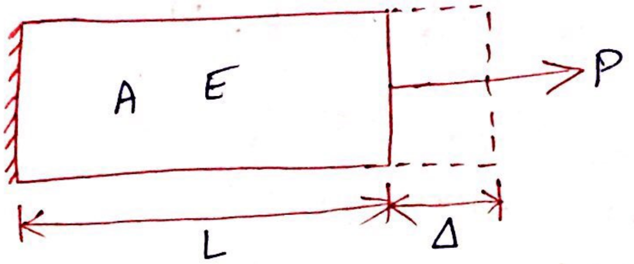


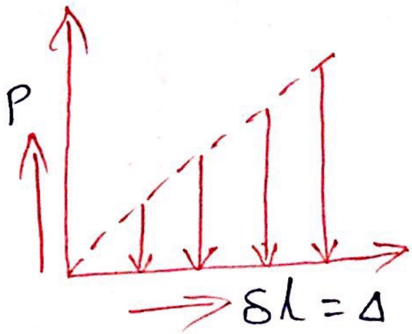
Energy Theorems

Strain Energy:— Strain energy is stored with an elastic solid when the solid is deformed under load. In the absence of energy losses, such as from friction, damping (or) yielding, the energy is equal to the workdone on the solid by external loads. strain energy is a type of potential energy.

Expression of strain energy due to Axial load :—

Strain Energy:— Internal workdone to deform a body ~~of~~ by action of applied force (or) energy stored in elastic body under loading.





workdone, $W = \frac{1}{2} \times P \times \Delta \rightarrow (1)$

Modulus of elasticity, $E = \frac{\sigma}{\varepsilon}$ $\left(\frac{\text{stress}}{\text{strain}}\right)$

$$\varepsilon = \frac{\sigma}{E} \rightarrow (2)$$

$$\text{stress, } \sigma = \frac{P}{A} \quad \left(\frac{\text{Load}}{\text{unit area}}\right) \rightarrow (3)$$

sub. (3) in eqn (2)

$$\varepsilon = \frac{P}{AE} \rightarrow (4)$$

Generally strain, $\varepsilon = \frac{\text{change in Length}}{\text{original Length}}$

$$\varepsilon = \frac{\Delta}{L} \rightarrow (5)$$

sub. (5) in eqn (4)

$$\frac{\Delta}{L} = \frac{P}{AE}$$

$$\Delta = \frac{PL}{AE} \rightarrow (6)$$

sub. (6) in eqn (1)

workdone, W = strain energy U.

So, $U = \frac{1}{2} \times P \times \Delta$

$$= \frac{1}{2} \times P \times \frac{PL}{AE}$$

$$\boxed{U = \frac{1}{2} \cdot \frac{P^2 L}{AE}}$$

$\therefore$ strain energy due to axial load, $U = \frac{1}{2} \cdot \frac{P^2 L}{AE}$

Strain energy per unit volume :-

volume, V = Area (A) x Length (L)

$$V = A \times L.$$

$$U = \frac{1}{2} \cdot \frac{P^2 L}{AE} \Big/ A \cdot L$$

$$= \frac{1}{2} \cdot \frac{P^2 \cancel{L}}{AE} \times \frac{1}{A \cdot \cancel{L}}$$

$$= \frac{1}{2} \cdot \frac{P^2}{A^2 \cdot E}$$

$$= \frac{1}{2} \cdot \left(\frac{P}{A}\right)^2 \cdot \frac{1}{E}$$

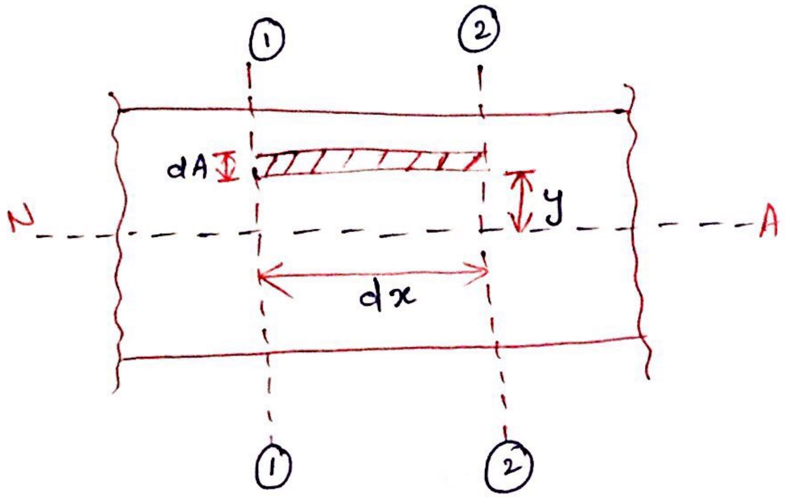
$$\left[\because \frac{P}{A} = \text{stress}, \quad \frac{P}{A} = \sigma \right]$$

$$= \frac{1}{2} \cdot \sigma^2 \cdot \frac{1}{E}$$

$$\boxed{U = \frac{1}{2} \cdot \frac{\sigma^2}{E}}$$

$\therefore$ strain energy per unit volume, $U = \frac{\sigma^2}{2E}$

Strain energy due to bending :—



$$\frac{M}{I} = \frac{\sigma}{y}$$

$$\sigma = \frac{M}{I} \cdot y$$

strain energy per unit volume $= \frac{1}{2} \cdot \frac{\sigma^2}{E}$

$$= \frac{1}{2E} \left(\frac{M}{I} \cdot y \right)^2$$

Total strain in element $= \frac{1}{2E} \left(\frac{M}{I} \cdot y \right)^2 \times dA \times dx$

$$= \frac{1}{2E} \cdot \frac{M^2}{I^2} \cdot dA \cdot dx \cdot y^2$$

Total $= \Sigma \frac{1}{2E} \cdot \frac{M^2}{I^2} \cdot dA \cdot dx \cdot y^2$

$$= \frac{M^2}{I^2 \cdot 2E} \cdot dx \cdot \Sigma \, dA \cdot y^2$$

$$= \frac{M^2}{I^{\cancel{2}} \cdot 2E} \cdot dx \cdot \cancel{I}$$

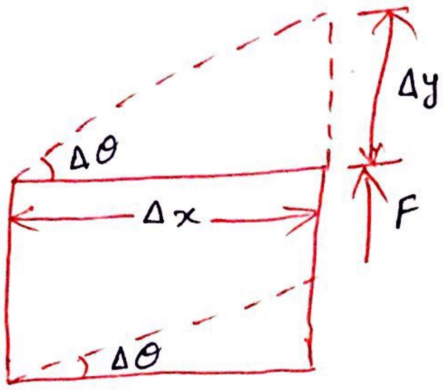
$$= \frac{M^2}{2EI} \cdot dx$$

Total strain energy stored in whole body

$$= \int \frac{M^2}{2EI} \cdot dx$$

$$= \frac{1}{2} \int \frac{M^2}{EI} \cdot dx.$$

Strain energy due to shear force :—



small element,

$$\left[\Delta\theta = \frac{\Delta y}{\Delta x} \qquad \Delta y = \Delta x \cdot \Delta\theta \right]$$

$$\Delta U = \frac{1}{2} \times F \times \Delta y$$

$$= \frac{1}{2} \times F \times \Delta x \cdot \Delta\theta$$

$$= \frac{1}{2} F \cdot \Delta\theta \cdot \Delta x.$$

$$\left[\frac{\tau}{\Delta\theta} = G \qquad \Delta\theta = \frac{\tau}{G} = \frac{F}{AG} \right]$$

$$\left[\frac{\text{Shear stress}}{\text{Shear strain}} = \text{modulus of rigidity} \right]$$

$$\left[\text{shear stress} = \frac{\text{Force}}{\text{Area}} \qquad \tau = \frac{F}{A} \right]$$

$$= \frac{1}{2} \cdot F \cdot \frac{F}{AG} \cdot \Delta x$$

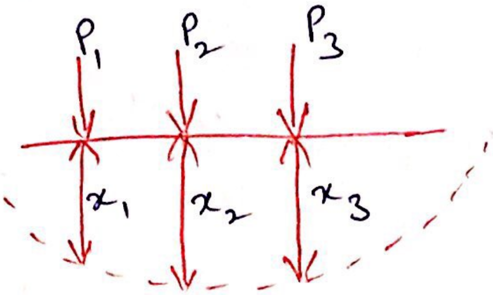
whole body, $U = \int \frac{F^2}{2AG} \cdot \Delta x = \int \Delta U$

$$U = \int \frac{F^2 \cdot dx}{2AG}$$

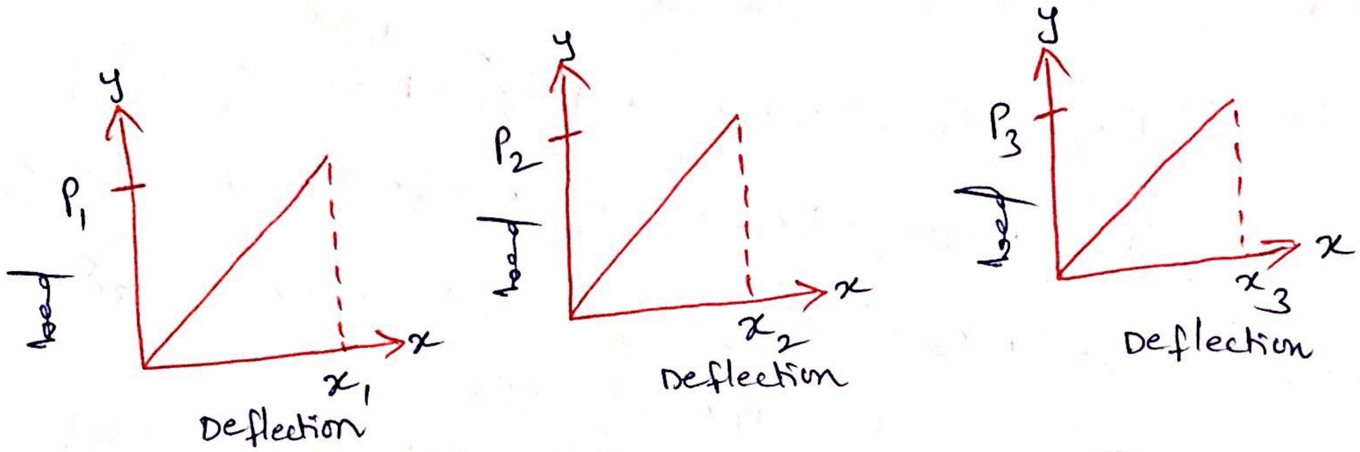
Castigliano's Theorem :—

$$\boxed{\frac{\partial u}{\partial P} = x}$$

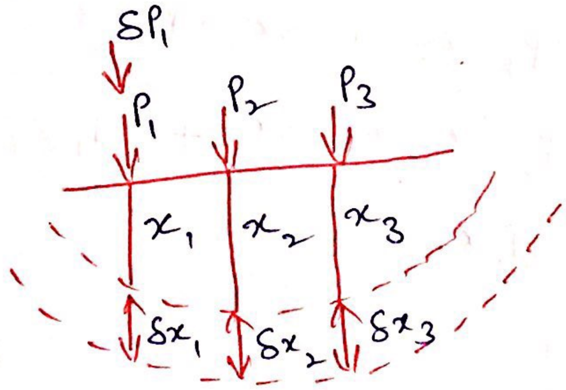
According to castigliano's theorem, rate of change of strain energy w.r.t. statitically independent force gives deflection in the direction of applied force.

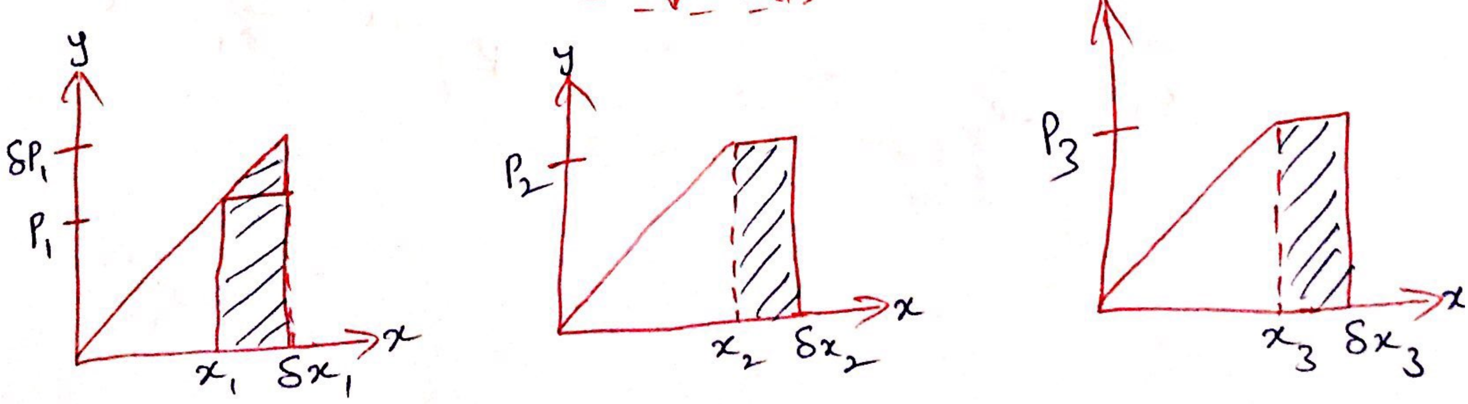


[Gradually applied load]



$$U = \frac{1}{2}P_1x_1 + \frac{1}{2}P_2x_2 + \frac{1}{2}P_3x_3 + \text{----} \Rightarrow (1)$$

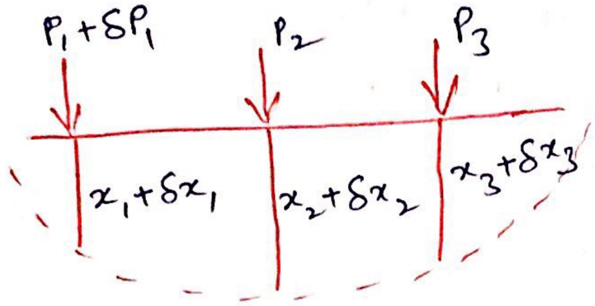




Increase in strain energy

$$\delta U = P_1 \delta x_1 + \underbrace{\tfrac{1}{2}\delta P_1 \delta x_1}_{\text{Neglect (Small quantity)}} + \cancel{\tfrac{1}{2}} P_2 \delta x_2 + \cancel{\tfrac{1}{2}} P_3 \delta x_3 + \cdots \rightarrow (2)$$

$$\delta u = P_1 \delta x_1 + P_2 \delta x_2 + P_3 \delta x_3 + \cdots \quad (3)$$



Total strain energy is $U+\delta U$ i.e. ~~(1)+(2)~~

$$\left[\cancel{U+\delta u} = \left(\tfrac{1}{2}P_1 x_1 + \tfrac{1}{2}P_2 x_2 + \tfrac{1}{2}P_3 x_3\right) + \left(P_1 \delta x_1 + \tfrac{1}{2}\delta P_1 \delta x_1 + P_2 \delta x_2 + P_3 \delta x_3\right)\right]$$

$$U+\delta U = \tfrac{1}{2}(P_1+\delta P_1)(x_1+\delta x_1) + \tfrac{1}{2}P_2(x_2+\delta x_2) + \tfrac{1}{2}P_3(x_3+\delta x_3)$$

$$U+\delta U = \tfrac{1}{2}P_1 x_1 + \tfrac{1}{2}P_2 x_2 + \tfrac{1}{2}P_3 x_3 + \tfrac{1}{2}P_1\delta x_1 + \tfrac{1}{2}P_2 \delta x_2 + \tfrac{1}{2}P_3 \delta x_3 + \tfrac{1}{2}\delta P_1 x_1 + \underbrace{\tfrac{1}{2}\delta P_1 \delta x_1}_{\text{Neglect (Small quantity)}}$$

$$\cancel{U} + \delta U = \cancel{U} + \tfrac{1}{2}(P_1 \delta x_1 + P_2 \delta x_2 + P_3 \delta x_3) + \tfrac{1}{2}\delta P_1 x_1$$

$$2\delta U = P_1 \delta x_1 + P_2 \delta x_2 + P_3 \delta x_3 + \delta P_1 x_1$$

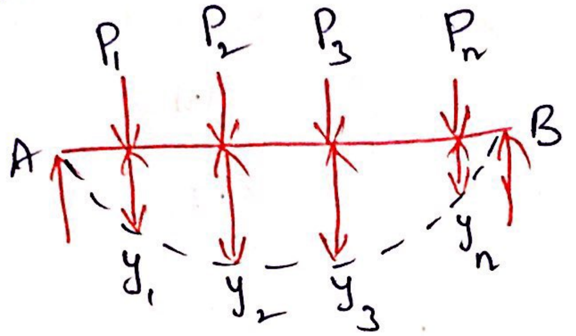
$$2\delta u = \delta u + \delta P_1 x_1$$

$$\delta U = \delta P_1 x_1$$

$$\boxed{x_1 = \frac{\delta U}{\delta P_1}} \Rightarrow \boxed{x = \frac{\delta u}{\delta P}} \text{ (or) } \boxed{\frac{\partial U}{\partial P} = x}$$

Castigliano's Theorem :—

Castigliano's theorem states that any member subjected to any system of forces, then deflection at any point is equal to the partial derivative of the strain energy w.r.t. that force.



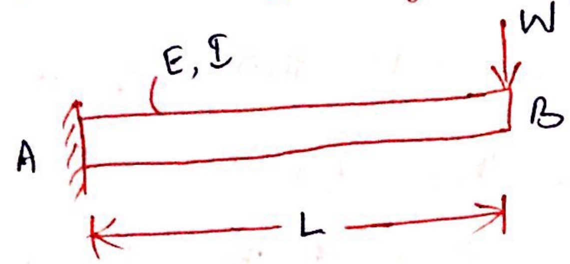
$$\boxed{y = \frac{\partial u}{\partial P}}$$

(1) Member AB carries a load P_1, P_2, P_3 - - - - P_n.

(2) $y_1 = \frac{\partial u}{\partial P_1}$, $y_2 = \frac{\partial u}{\partial P_2}$, $y_3 = \frac{\partial u}{\partial P_3}$ - - - - $y_n = \frac{\partial u}{\partial P_n}$

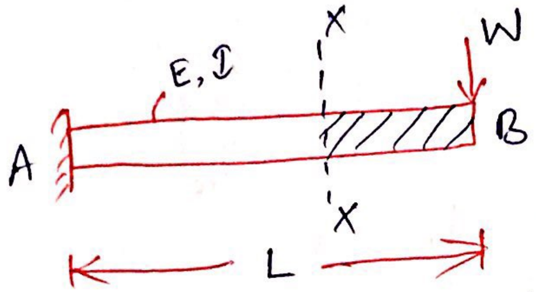
u = strain energy

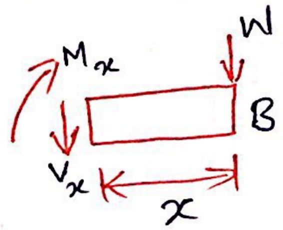
① Deflection of beam by using castigliano's theorem.



Find deflection at point 'B' by using castigliano's theorem.

Ⓐ Consider a Section X – X at a distance 'x' from end 'B'.





Moment balance about X – X

$$\Sigma M_x = 0$$

$$M_x + W \cdot x = 0$$

$$M_x = -Wx$$

we know that,
$$U = \int \frac{M_x^2}{2EI} \cdot dx$$

$$= \int_0^L \frac{M_x^2}{2EI} \cdot dx$$

$$= \int_0^L \frac{(-Wx)^2}{2EI} \cdot dx$$

$$= \int_0^L \frac{W^2 x^2}{2EI} \cdot dx$$

$$= \frac{W^2}{2EI} \int_0^L x^2 \cdot dx$$

$$= \frac{W^2}{2EI}\left[\frac{x^3}{3}\right]_0^L$$

$$U = \frac{W^2}{2EI}\cdot\frac{L^3}{3}$$

According to castigliano's theorem, we have

$$y = \frac{\partial U}{\partial P} \quad (\text{or}) \quad x = \frac{\partial U}{\partial P}$$

Deflection at point 'B'

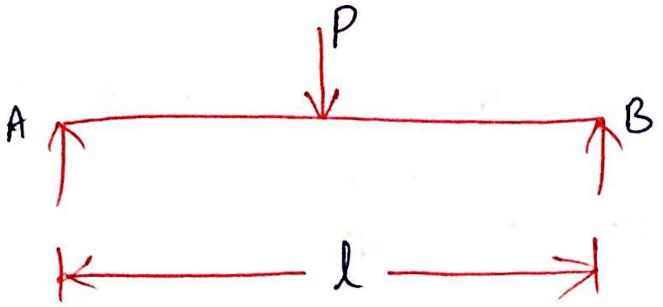
$$y_B \ (\text{or}) \ \delta_B = \frac{\partial U}{\partial W}$$

$$= \frac{\partial}{\partial W}\left[\frac{W^2}{2EI}\cdot\frac{L^3}{3}\right]$$

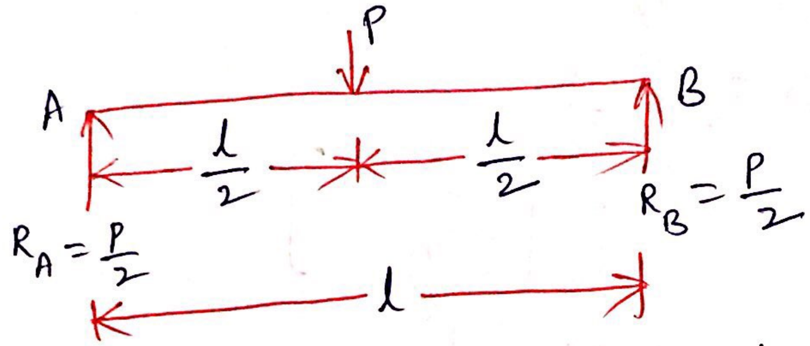
$$= \frac{\cancel{2}W}{\cancel{2}EI}\cdot\frac{L^3}{3}$$

$$\boxed{y_B \ (\text{or}) \ \delta_B = \frac{WL^3}{3EI}}$$

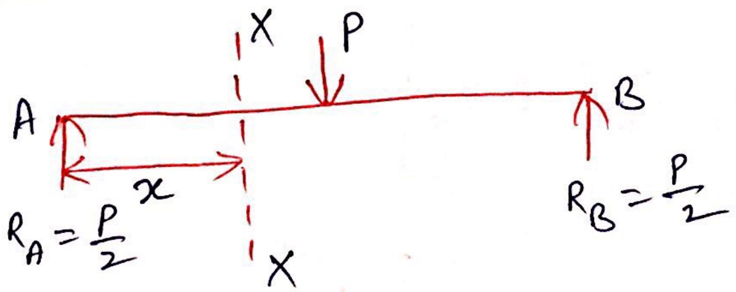
(2) Find the central deflection of a simply supported beam carrying a concentrated load at a mid span. Assume uniform flexural rigidity (EI) is constant.



(A)



consider a section X-X at a distance 'x' from 'A'.



$$M_x = \frac{P}{2} \times x = \frac{Px}{2}$$

strain energy stored in the beam, $U = \int \frac{M_x^2}{2EI} \cdot dx$

$$= 2\int_0^{l/2} \frac{\left(\frac{Px}{2}\right)^2}{2EI} \cdot dx$$

$$= \cancel{2}\int_0^{l/2} \frac{P^2x^2}{4} \times \frac{1}{\cancel{2}EI} \cdot dx$$

$$= \frac{P^2}{4EI} \int_0^{l/2} x^2 \cdot dx$$

$$= \frac{P^2}{4EI}\left[\frac{x^3}{3}\right]_0^{l/2}$$

$$= \frac{P^2}{4EI}\left[\frac{(l/2)^3}{3}\right]$$

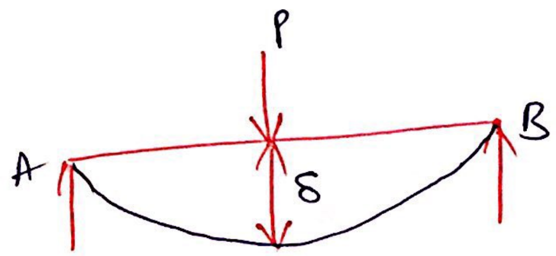
$$= \frac{P^2}{4EI}\left[\frac{l^3}{2^3} \times \frac{1}{3}\right]$$

$$= \frac{P^2}{4EI}\left[\frac{l^3}{8} \times \frac{1}{3}\right]$$

$$= \frac{P^2}{4EI}\left[\frac{l^3}{24}\right]$$

$$U = \frac{P^2 l^3}{96EI}$$

Deflection at mid-span



$$y \text{ (or) } \delta = \frac{\partial U}{\partial P}$$

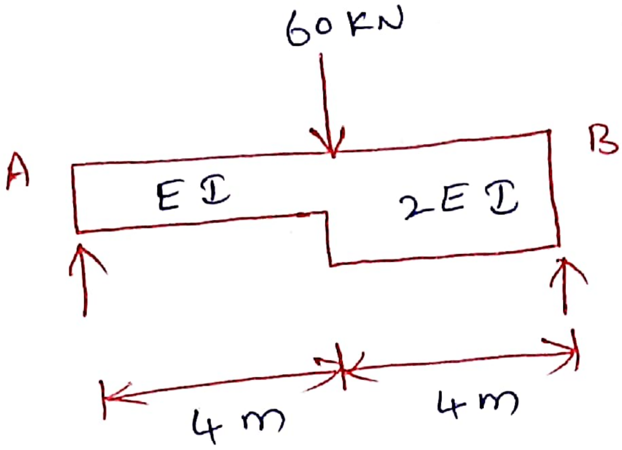
$$= \frac{\partial}{\partial P}\left(\frac{P^2 l^3}{96EI}\right)$$

$$= \frac{l^3}{96EI} \cdot \frac{\partial}{\partial P}(P^2)$$

$$= \frac{l^3}{\cancel{96}_{48} EI}(\cancel{2}P)$$

$$\boxed{y \text{ (or) } \delta = \frac{Pl^3}{48EI}}$$

(1) Find the deflection under the load by using (1) of strain energy method.



(A)

$$R_A + R_B - 60 = 0$$

$$R_A + R_B = 60 \text{ KN} \rightarrow (1)$$

$$\Sigma M_A = 0 ; \quad (60 \times 4) - (R_B \times 8) = 0$$

$$240 = 8 R_B$$

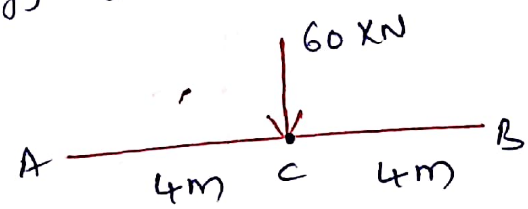
$$\boxed{R_B = 30 \text{ KN}}$$

'R_B' sub. in eqn (1)

$$R_A + 30 = 60 \text{ KN}$$

$$\boxed{R_A = 30 \text{ KN}}$$

strain energy due to bending, $U = \int_0^l \frac{M_x^2}{2EI} \cdot dx$



$$U = \int_A^C \frac{M_x^2}{2EI} \cdot dx + \int_B^C \frac{M_x^2}{2EI} \cdot dx$$

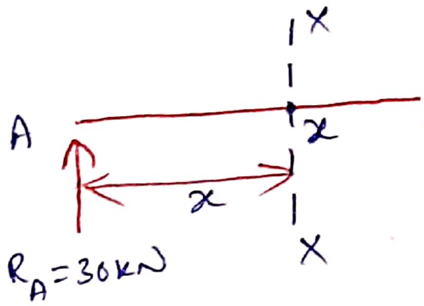
For AC portion

$EI = EI$

For BC portion

$EI = 2EI$.

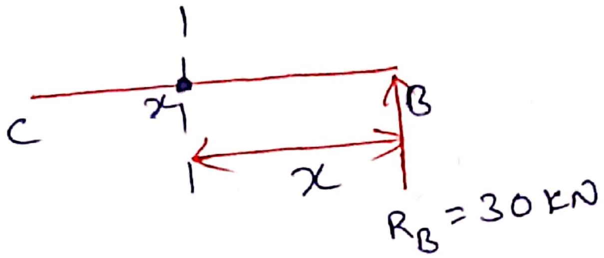
consider AC portion



$$M_x = R_A \times x$$

$$M_x = 30x$$

consider BC portion



$$M_x = R_B \times x$$

$$M_x = 30x$$

where: $$U = \int_A^C \frac{M_x^2}{2EI} \cdot dx + \int_B^C \frac{M_x^2}{2EI} \cdot dx$$

$$= \int_0^4 \frac{(30 \cdot x)^2}{2EI} dx + \int_0^4 \frac{(30 \cdot x)^2}{2 \cdot 2EI} \cdot dx$$

$$= \int_0^4 \frac{900 \cdot x^2}{2EI} \cdot dx + \int_0^4 \frac{900 \cdot x^2}{4EI} \cdot dx$$

$$= \frac{900}{2EI} \int_0^4 x^2 \cdot dx + \frac{900}{4EI} \int_0^4 x^2 \cdot dx$$

$$= \frac{900}{2EI} \left[\frac{x^3}{3} \right]_0^4 + \frac{900}{4EI} \left[\frac{x^3}{3} \right]_0^4$$

$$= \frac{900}{2EI} \cdot \frac{(4)^3}{3} + \frac{900}{4EI} \cdot \frac{(4)^3}{3}$$

$$= \frac{900 \times 64}{6EI} + \frac{900 \times 64}{12EI}$$

$$= \frac{57600}{6EI} + \frac{57600}{12EI}$$

$$= \frac{115200 + 57600}{12EI}$$

$$= \frac{172800}{12EI}$$

$$\boxed{U = \frac{14,400}{EI}}$$

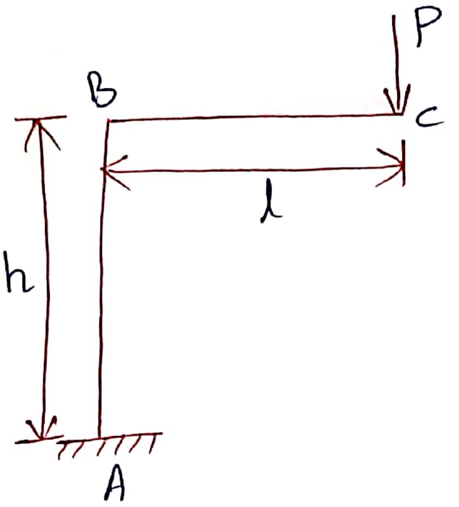
strain energy, U = workdone, w

$$U = \frac{1}{2} \times P \times \Delta_c$$

$$\frac{\overset{480}{\cancel{14,400}}}{EI} = \frac{1}{\cancel{2}} \times \overset{30}{\cancel{60}} \times \Delta_c$$

$$\boxed{\Delta_c = \frac{480}{EI}}$$

① Find deflection at 'c' by using of castigliano's theorem ①
and find strain energy at 'c'.

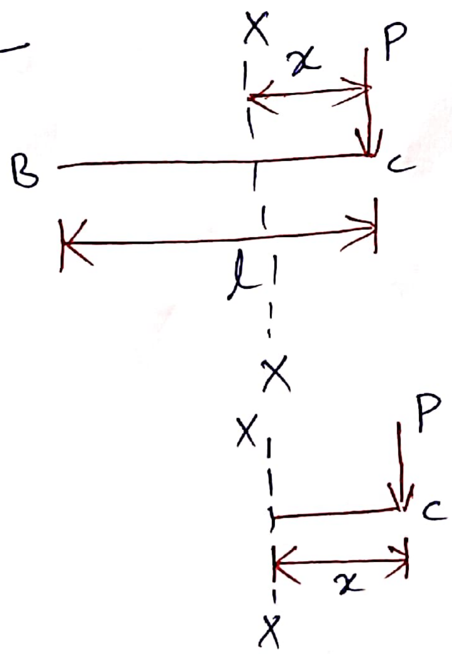


Ⓐ castigliano's theorem: $\Delta = \frac{\partial U}{\partial P}$

S.E. due to bending : $U = \int_0^l \frac{M_x^2}{2EI} \cdot dx.$

workdone (or) S.E. : $U = \frac{1}{2} \times P \times \Delta.$

<u>span</u> <u>BC</u> :-



$M_x = -P \cdot x$

where : $U = \int_0^l \frac{M_x^2}{2EI} \cdot dx$

$= \int_0^l \frac{(-P \cdot x)^2}{2EI} \cdot dx$

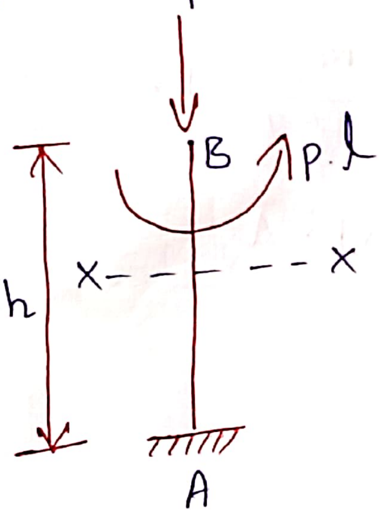
$$U = \int_0^l \frac{P^2 . x^2}{2EI} . dx$$

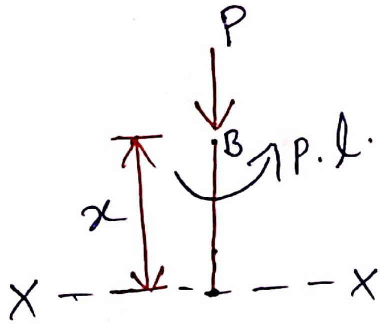
$$= \frac{P^2}{2EI} \int_0^l x^2 . dx$$

$$= \frac{P^2}{2EI} \left[\frac{x^3}{3} \right]_0^l$$

$$U = \frac{P^2 l^3}{6EI} \rightarrow (1)$$

Span AB :-





$$M_x = P.l.$$

$$U = \int_0^l \frac{M_x^2}{2EI} . dx .$$

$$U = \int_0^h \frac{(P.l)^2}{2EI} . dx$$

$$U = \int_0^h \frac{P^2 . L^2}{2EI} . dx$$

$$= \frac{P^2 l^2}{2EI} \int_0^h dx$$

$$= \frac{P^2 l^2}{2EI} [x]_0^h$$

$$U = \frac{P^2 . l^2 . h}{2EI} \longrightarrow (2)$$

$$\Delta = \left[\frac{\partial U}{\partial P}\right]_{BC} + \left[\frac{\partial U}{\partial P}\right]_{AB}$$

$$\left[\frac{\partial U}{\partial P}\right]_{BC} = \frac{\partial}{\partial P}\left[\frac{P^2 l^3}{6EI}\right]$$

$$= \frac{\cancel{2} P . l^3}{\cancel{6}_3 EI}$$

$$= \frac{P l^3}{3EI}$$

$$\left[\frac{\partial U}{\partial P}\right]_{AB} = \frac{\partial}{\partial P}\left[\frac{P^2 . l^2 . h}{2EI}\right]$$

$$= \frac{\cancel{2} P . l^2 . h}{\cancel{2} EI}$$

$$= \frac{P l^2 h}{EI}$$

$$\therefore \Delta_c = \frac{P l^3}{3EI} + \frac{P l^2 h}{EI}$$

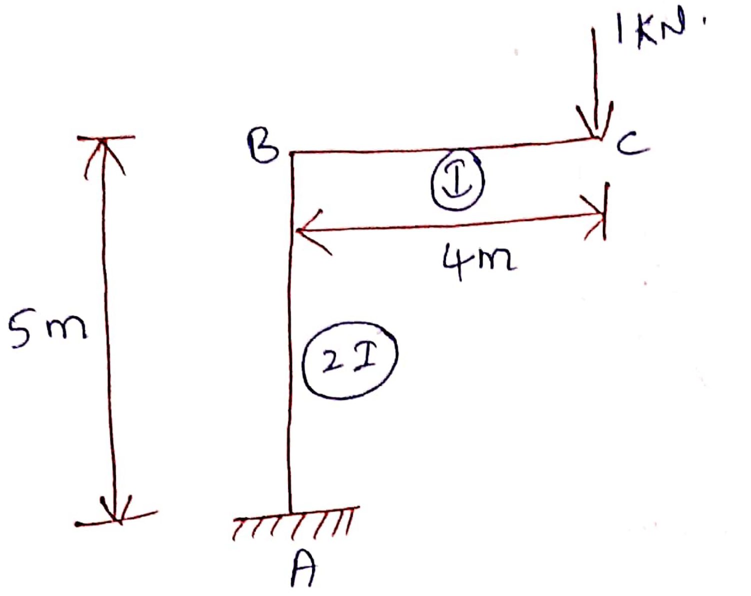
strain energy, $U = \frac{1}{2} \times P \times \Delta_c$

$$U = \frac{1}{2} \times P \times \left[\frac{Pl^3}{3EI} + \frac{Pl^2 h}{EI} \right]$$

$$U = \frac{P^2 l^3}{6EI} + \frac{P^2 l^2 h}{2EI}$$

① In the frame shown in fig. calculate the vertical deflection under the point load using strain energy. ①

Take $E = 200 \times 10^6\ kN/m^2$ and $I = 14 \times 10^{-6}\ m^4$.

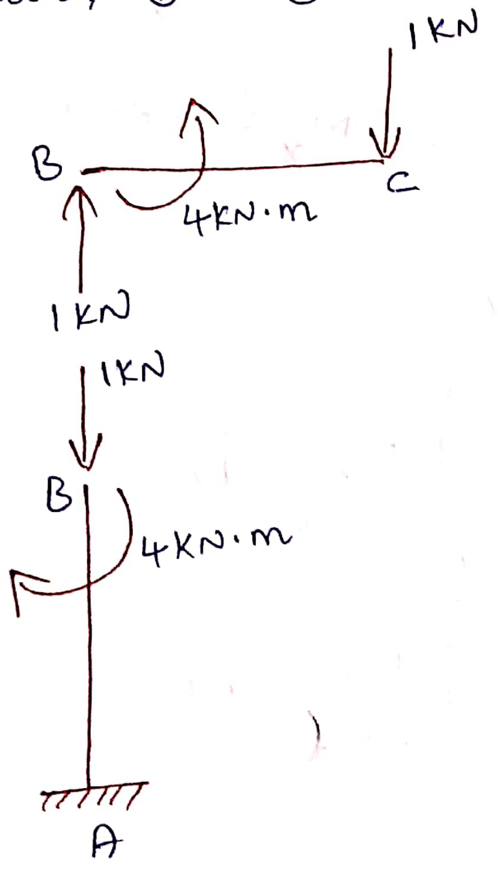


(A) strain energy due to bending, $U = \int_0^l \frac{M_x^2}{2EI} \cdot dx$

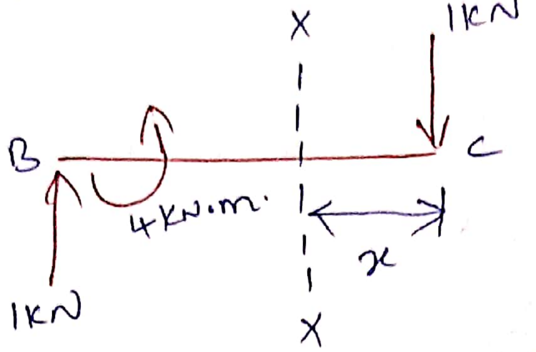
strain energy, $U = \frac{1}{2} \times P \times \Delta_c$

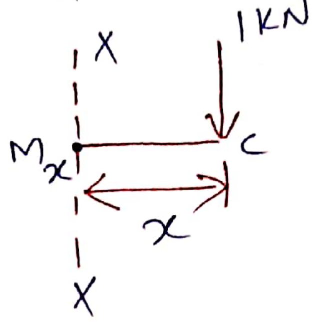
For AB span, $I = 2I$

For BC span, $I = I$



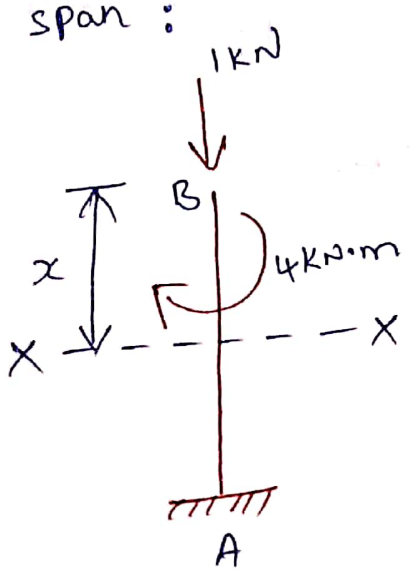
For BC span :

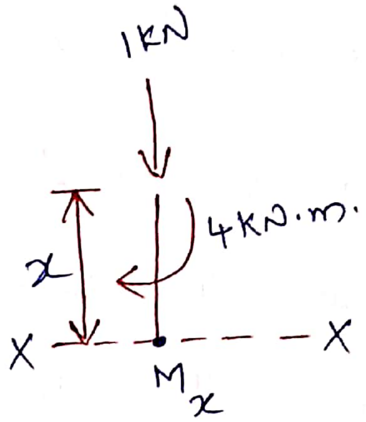




$$M_x = -1 \times x = -x \text{ KN.m.}$$

For AB span :





$$M_x = 4 \text{ KN.m.}$$

$$U = \int_0^l \frac{M_x^2}{2EI} \cdot dx.$$

$$U = \int_B^C \frac{M_x^2}{2EI} \cdot dx + \int_A^B \frac{M_x^2}{2EI} \cdot dx$$

$$= \int_0^4 \frac{M_x^2}{2EI} \cdot dx + \int_0^5 \frac{M_x^2}{2E(2I)} \cdot dx$$

$$= \int_0^4 \frac{M_x^2}{2EI} \cdot dx + \int_0^5 \frac{M_x^2}{4EI} \cdot dx$$

$$= \int_0^4 \frac{(-x)^2}{2EI} \cdot dx + \int_0^5 \frac{(4)^2}{4EI} \cdot dx$$

$$= \int_0^4 \frac{x^2}{2EI} \cdot dx + \int_0^5 \frac{16}{4EI} \cdot dx$$

$$= \frac{1}{2EI} \int_0^4 x^2 \cdot dx + \frac{\cancel{16}\,4}{\cancel{4}EI} \int_0^5 dx$$

$$= \frac{1}{2EI} \left[\frac{x^3}{3}\right]_0^4 + \frac{4}{EI} \left[x\right]_0^5$$

$$= \frac{1}{2EI} \left[\frac{(4)^3}{3}\right] + \frac{4}{EI} [5]$$

$$= \frac{\cancel{64}\,32}{\cancel{6}\,3\,EI} + \frac{20}{EI}$$

$$= \frac{32 + 60}{3EI}$$

$$U = \frac{92}{3EI}$$

workdone by the load $= \frac{1}{2} \times P \times \Delta_c$

strain energy, U = workdone by the load

$$\frac{92}{3EI} = \frac{1}{2} \times 1 \times \Delta_c$$

$$\Delta_c = \frac{184}{3EI}$$

where : $E = 200 \times 10^6 \; kN/m^2$

$I = 14 \times 10^{-6} \; m^4$

$$\Delta_c = \frac{184}{3 \times 200 \times 10^6 \times 14 \times 10^{-6}}$$

$$\boxed{\Delta_c = 0.0219 \; m}$$

(OR)

$$\boxed{\Delta_c = 21.9 \; mm}$$

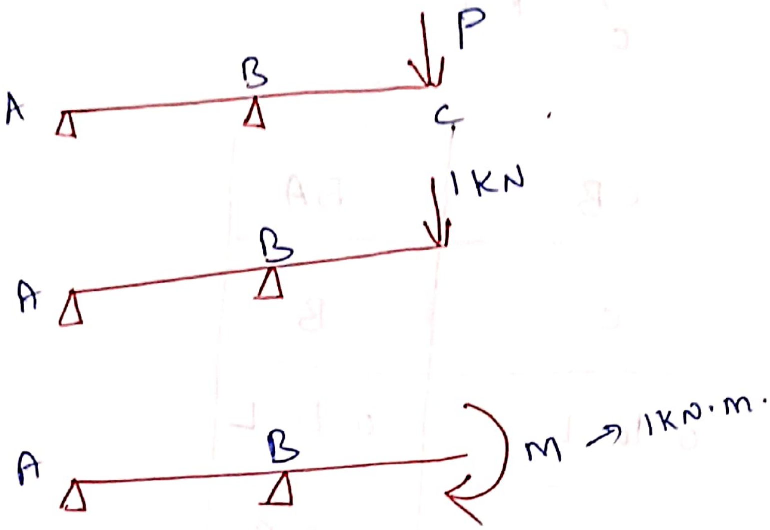
Unit Load Method :-

(1) It is a force method.

(2) It is used for both determinate and indeterminate structures.

(3) It is modification of castigliano's II theorem.

(4) In this method, we apply a virtual load/moment of unit magnitude in the direction of deflection/rotation ~~and~~ at that particular point.



For deflection, $\Delta = \int_0^l \frac{M.m_1}{EI}.dx$

For rotation, $\theta = \int_0^l \frac{M.m_2}{EI}.dx$

M ⇒ Bending moment due to the external applied load (P).

m_1 ⇒ Bending point due to unit load applied at that point of deflection.

m_2 ⇒ Bending moment at any point due to unit moment applied at ~~that~~ the point where rotation is asked.

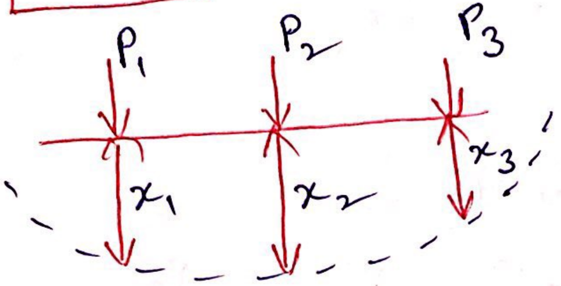
Unit load Method :-

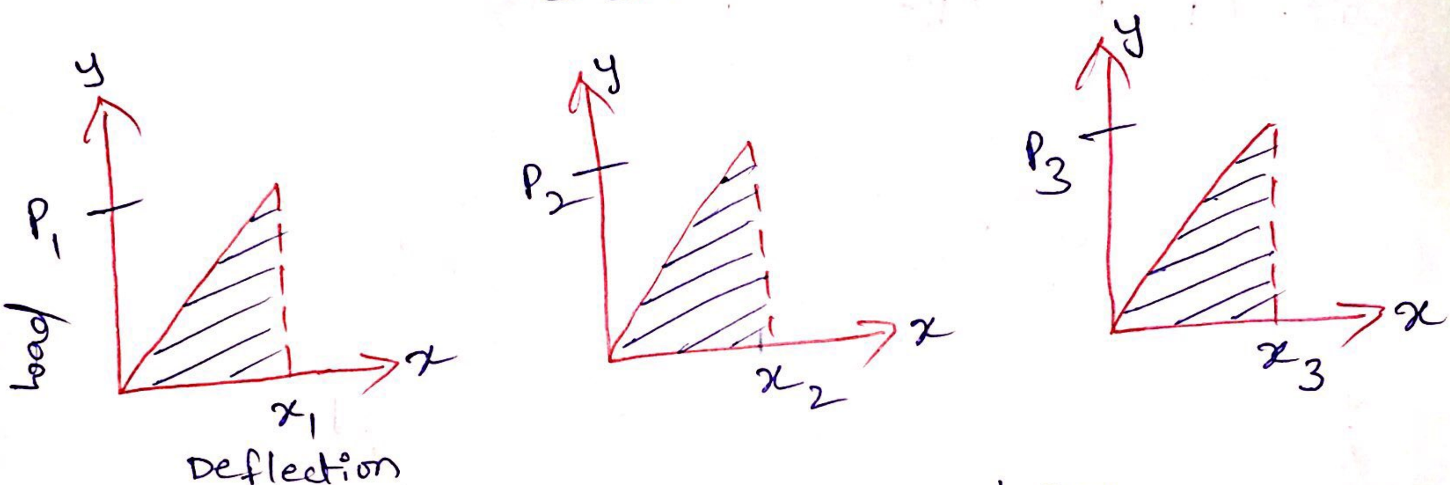
$$x = \partial U \quad \text{(OR)}$$

$$\boxed{x = \delta U}$$

case (I) :
Apply load

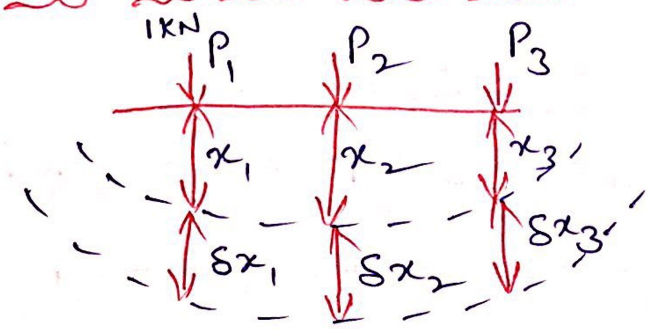
[Gradually applied load]

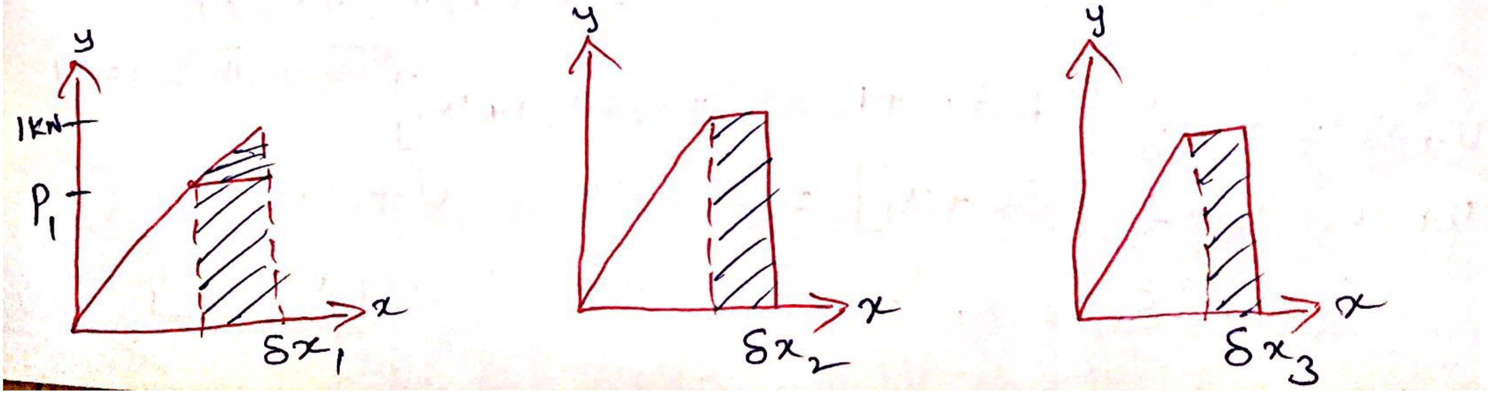




strain energy, $U = \frac{1}{2}P_1 x_1 + \frac{1}{2}P_2 x_2 + \frac{1}{2}P_3 x_3$

case (II) :
Apply extra load i.e. unit load (1KN) :





Increase in strain energy

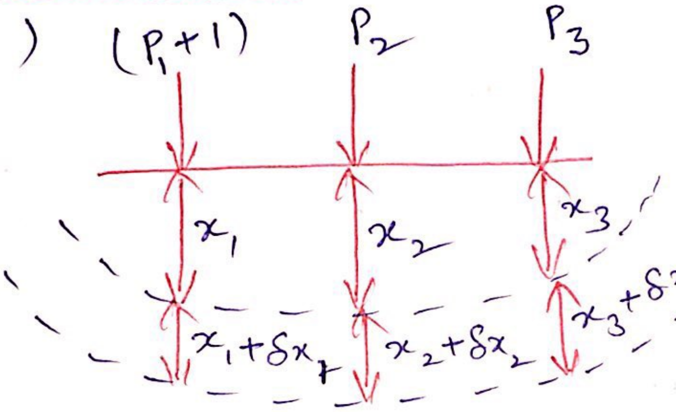
$$\delta U = \underbrace{\frac{1}{2}\times 1\times \delta x_1}_{\text{Small quantity, so neglect.}} + P_1\,\delta x_1 + P_2\,\delta x_2 + P_3\,\delta x_3$$

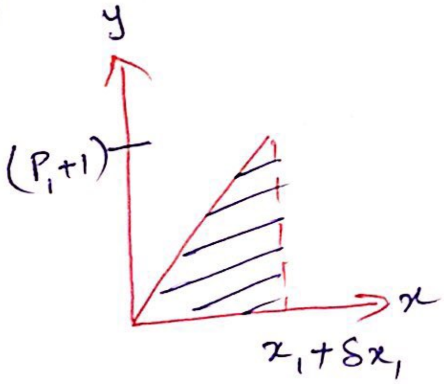
$$\delta U = P_1\,\delta x_1 + P_2\,\delta x_2 + P_3\,\delta x_3$$

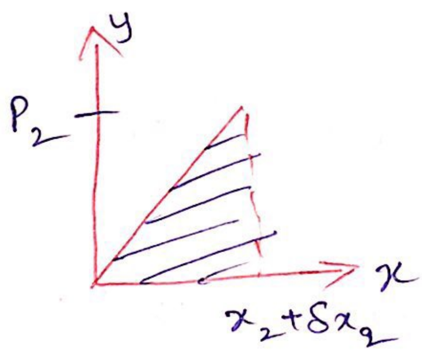
Case (III) :

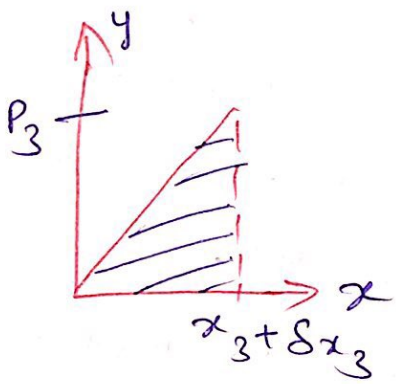
Apply loads simultaneously :—

i.e. (P_1+1)









Total strain energy : $(U+\delta U)$

$$U+\delta U = \frac{1}{2}(P_1+1)(x_1+\delta x_1) + \frac{1}{2}P_2(x_2+\delta x_2) + \frac{1}{2}P_3(x_3+\delta x_3)$$

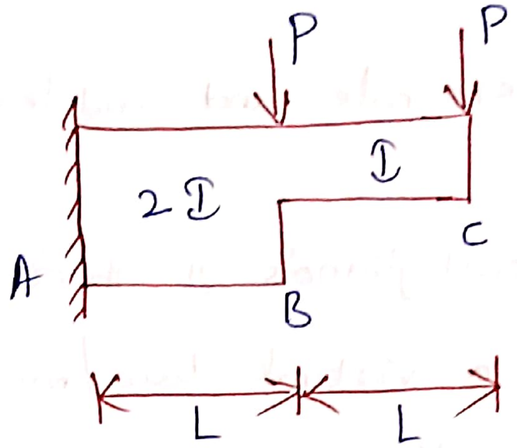
$$= \frac{1}{2}P_1x_1 + \frac{1}{2}P_2x_2 + \frac{1}{2}P_3x_3 + \frac{1}{2}P_1\,\delta x_1 + \frac{1}{2}P_2\,\delta x_2 + \frac{1}{2}P_3\,\delta x_3 + \frac{1}{2}\times1\times x_1 + \underbrace{\frac{1}{2}\times1\times\delta x_1}_{\text{Small Quantity}}$$

$$U+\delta U = U + \frac{1}{2}\left[(P_1\,\delta x_1 + P_2\,\delta x_2 + P_3\,\delta x_3) + x_1\right]$$

$$\cancel{U}+\delta U = \cancel{U} + \frac{1}{2}\left[\delta U + x_1\right] \Rightarrow 2\,\delta U = \delta U + x_1 .$$

$$2\,\delta U - \delta U = x_1 \Rightarrow x_1 = \delta U \quad \text{i.e.} \quad \boxed{x = \delta U}$$

① Determine the deflection ~~and rotation~~ at the free end.

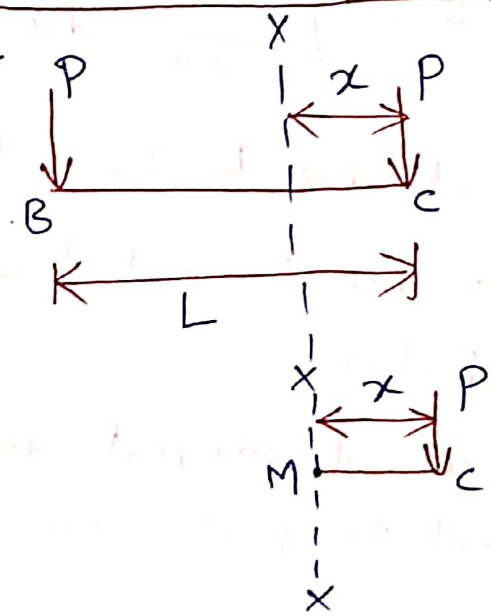


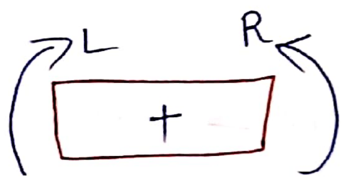
By using of unit load method.

Ⓐ Deflection, $\Delta = \int_0^l \frac{M \cdot m_1}{EI} \cdot dx$

Span	CB	BA
origin	C	B
Limits	0 to L	0 to L
I	I	2I
M	$-Px$	$-P(L+x) - Px$
m_1	$-1 \cdot x$	$-1(x+L)$

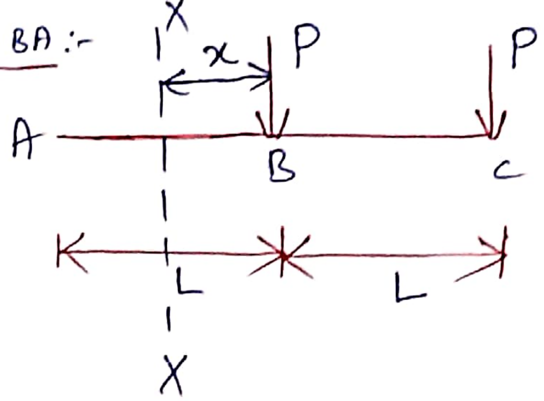
For span CB :-

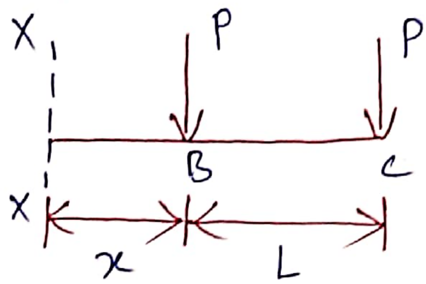




$$\boxed{M = -P \cdot x}$$

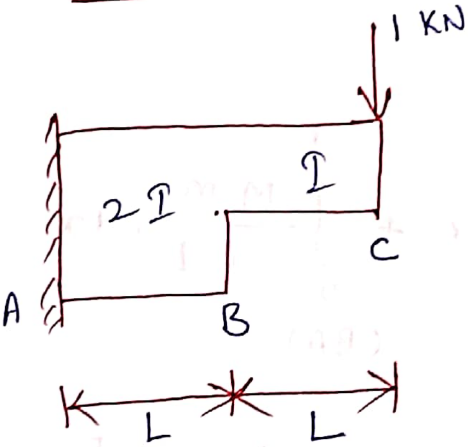
For span BA :-



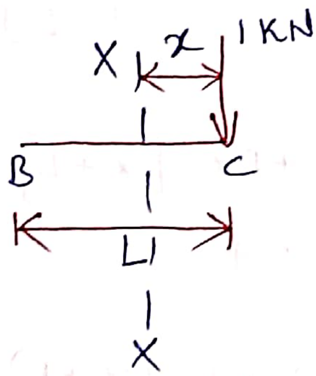


$$\boxed{M = -P(L+x) - Px}$$

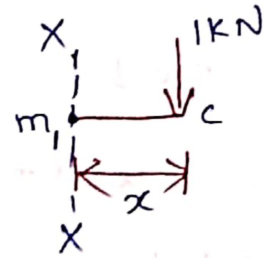
Now calculate m_1 :-



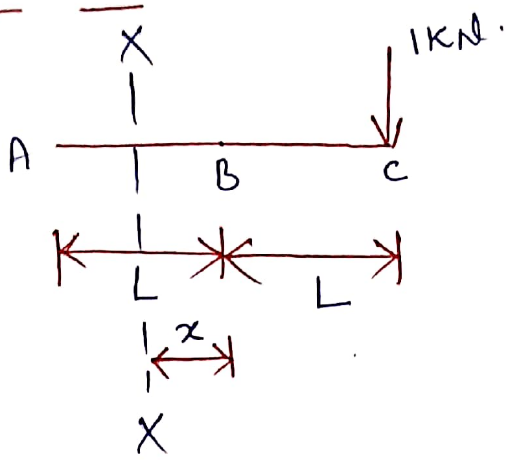
For span CB :-

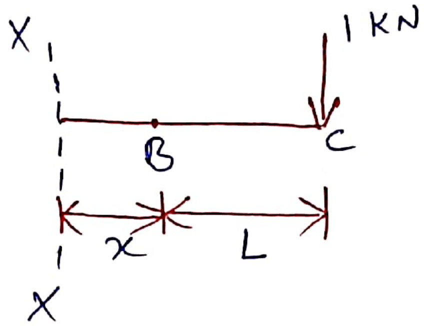


$$\boxed{m_1 = -1x}$$



For span BA :-





$$\boxed{m_1 = -1(x+L)}$$

$$\text{Deflection, } \Delta = \underset{(CB)}{\int_0^l \frac{M \cdot m_1}{EI} \cdot dx} + \underset{(BA)}{\int_0^l \frac{M \cdot m_1}{EI} \cdot dx}$$

$$= \int_0^L \frac{(-P \cdot x)(-x)}{EI} \cdot dx + \int_0^L \frac{[-P(x+L) - Px] \cdot [-1(x+L)]}{E(2I)} \cdot dx$$

$$= \int_0^L \frac{Px^2}{EI} \cdot dx + \int_0^L \frac{[P(x+L) + Px][x+L]}{2EI} \cdot dx$$

$$= \int_0^L \frac{Px^2}{EI} \cdot dx + \int_0^L \frac{P(x+L)^2 + Px(x+L)}{2EI} \cdot dx$$

$$= \int_0^L \frac{Px^2}{EI} \cdot dx + \int_0^L \frac{P[(x+L)^2 + x(x+L)]}{2EI} \cdot dx$$

$$= \int_0^L \frac{Px^2}{EI} \cdot dx + \int_0^L \frac{P}{2EI}\left[x^2 + L^2 + 2xL + x^2 + xL\right] dx$$

$$= \int_0^L \frac{2Px^2}{2EI} \cdot dx + \int_0^L \frac{P}{2EI}\left[x^2 + L^2 + 2xL + x^2 + xL\right] dx$$

$$= \frac{P}{2EI}\left[\int_0^L 2x^2 \cdot dx + \int_0^L (x^2 + L^2 + 2xL + x^2 + xL)\, dx\right]$$

$$= \frac{P}{2EI}\left\{\left[\frac{2x^3}{3}\right]_0^L + \left[\frac{x^3}{3} + L^2 \cdot x + \frac{\cancel{2}Lx^2}{\cancel{2}} + \frac{x^3}{3} + \frac{x^2L}{2}\right]_0^L\right\}$$

$$= \frac{P}{2EI}\left[\frac{2L^3}{3} + \frac{L^3}{3} + L^2 \cdot L + L \cdot L^2 + \frac{L^3}{3} + \frac{L^2 \cdot L}{2}\right]$$

$$= \frac{P}{2EI}\left[\frac{2L^3}{3} + \frac{L^3}{3} + L^3 + L^3 + \frac{L^3}{3} + \frac{L^3}{2}\right]$$

$$= \frac{P}{2EI}\left[\frac{4L^3}{3} + 2L^3 + \frac{L^3}{2}\right]$$

$$= \frac{P}{2EI}\left[\frac{8L^3 + 12L^3 + 3L^3}{6}\right]$$

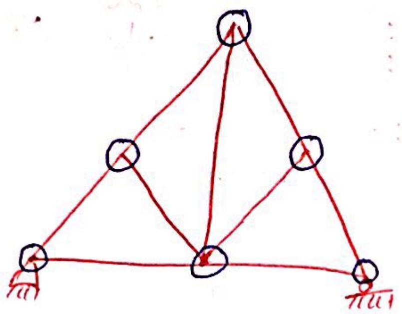
$$\boxed{\Delta = \frac{23PL^3}{12EI}}$$

Unit load Method :-

Application of unit load method for finding out deflection in Pin-jointed frames (Trusses) :-

A truss is a structure made up of member of bars connected with each other with frictionless pins.

said to be statically determinate member are connected at end. so no member is continuous through joints.



(1) support reactions

(2) 'P' analysis

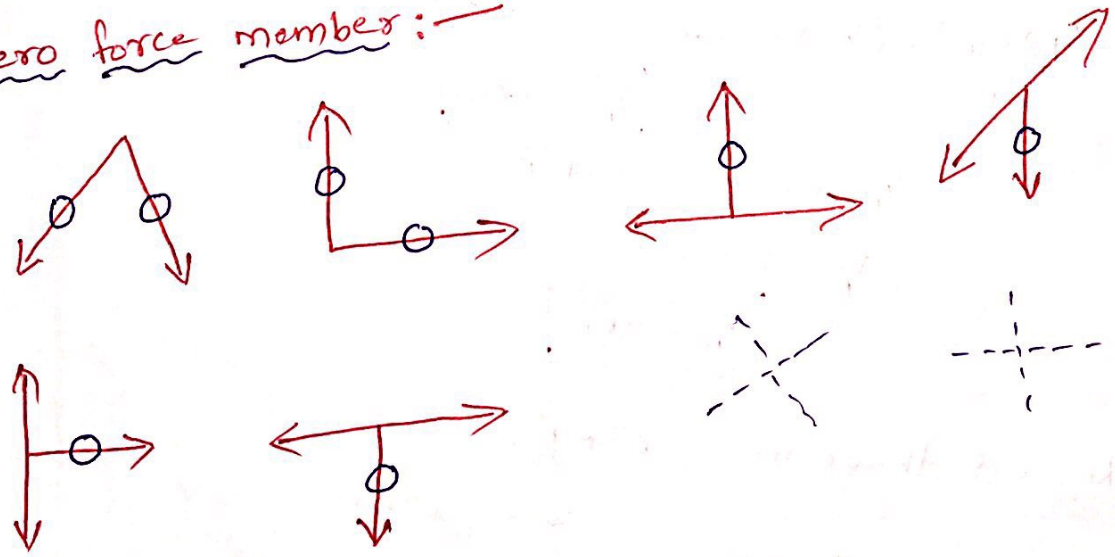
(3) 'K' analysis (unit load)

↓

support reactions

(4) Table

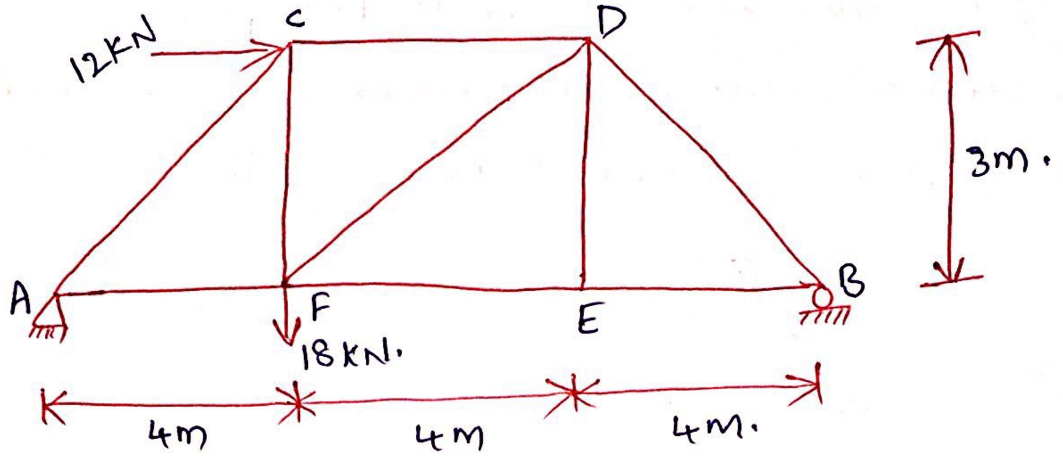
Zero force member :-



Problem :-

① Using unit load method find vertical deflection of joint 'E' of a pin jointed truss loaded as shown in fig.

Take EI is constant in all member.



Ⓐ

Support Reactions :-

$\Sigma f_x = 0; \quad -H_A + 12 = 0 \Rightarrow \boxed{H_A = 12\,KN.}$

$\Sigma f_y = 0; \quad V_A + V_B - 18 = 0 \Rightarrow V_A + V_B = 18 \rightarrow ①$

$\Sigma M_A = 0; \ (18 \times 4) - (V_B \times 12) + (12 \times 3) = 0$

$\boxed{V_B = 9\,KN.}$

V_B sub. in eqn ①

$V_A + 9 = 18$

$\boxed{V_A = 9\,KN.}$

From Pythagoras theorem: $x^2 + y^2 = z^2$

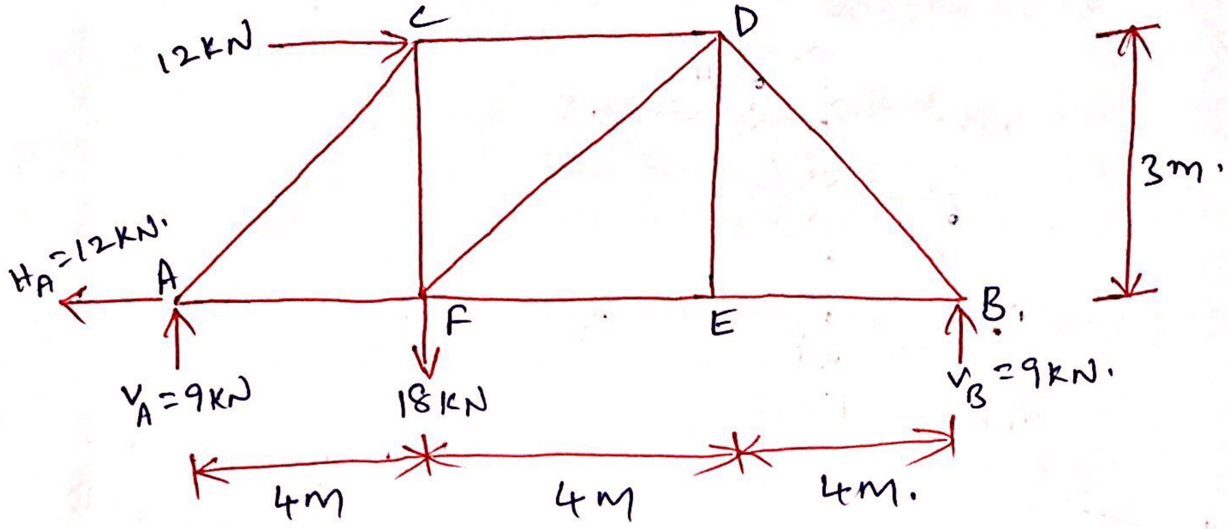
$z = \sqrt{x^2 + y^2}$

Here : $x = 4m, \ y = 3m.$

$z = \sqrt{4^2 + 3^2}$

$z = 5m.$ (AC = 5m, FD = 5m, BD = 5m.

'P' analysis :-



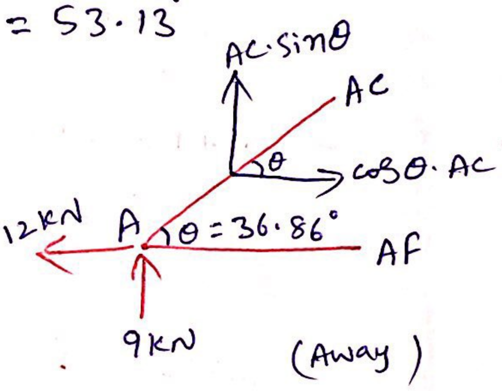
$\theta = \tan^{-1}\left(\frac{3}{4}\right) \left(\frac{opp}{Adj}\right)$

$= 36.86°$

$\theta = \tan^{-1}\left(\frac{4}{3}\right) \left(\frac{opp}{Adj}\right)$

$= 53.13°$

Joint A :-

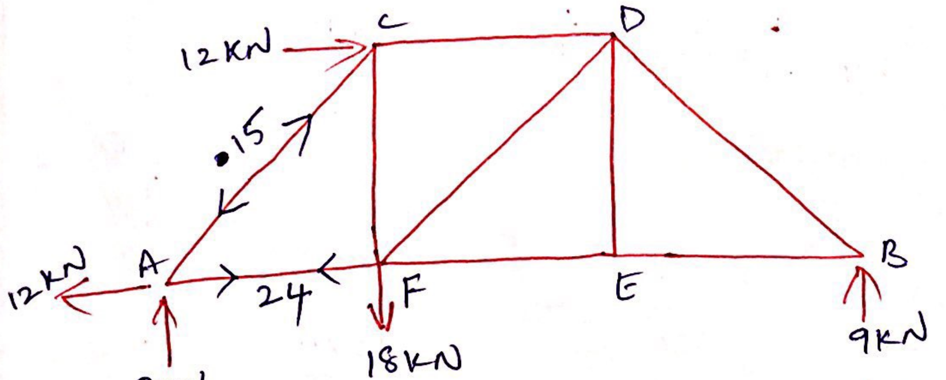


$\Sigma F_y = 0;\ 9 + AC.\sin(36.86) = 0$

$\boxed{AC = -15\,KN.\ (C)}$

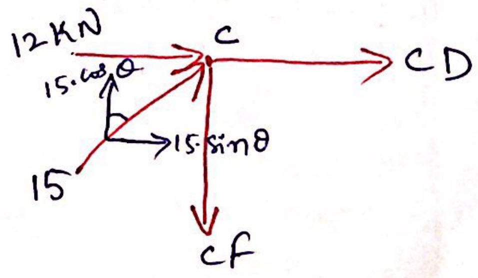
$\Sigma F_x = 0;\ -12 + AF + AC.\cos(36.86) = 0$

$\boxed{AF = 24\,KN.\ (T)}$



Joint C :-

$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$



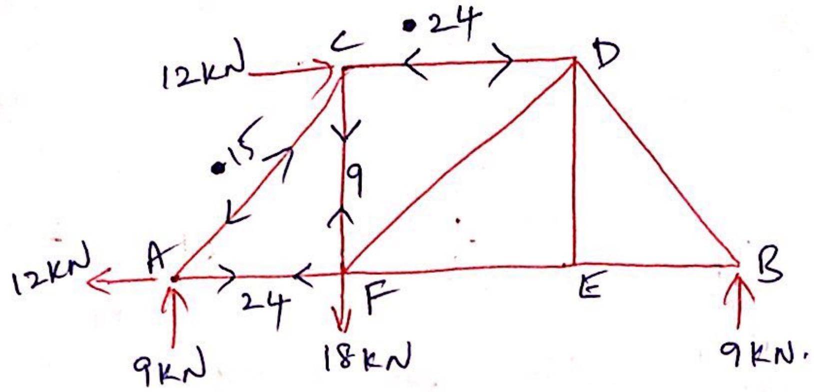
$\Sigma F_y = 0 \Rightarrow 15.\cos\theta - CF = 0$

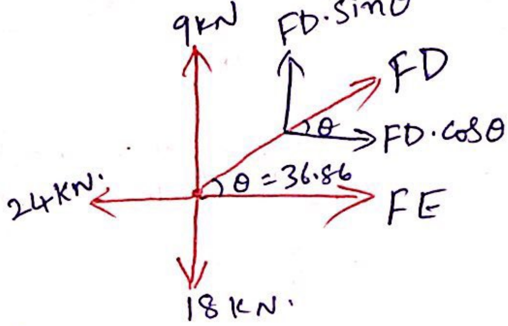
$15.\cos(53.13) - CF = 0$

$\boxed{CF = 9\,KN\,(T)}$

$\Sigma f_x = 0; \quad 12 + CD + 15 \cdot \sin(53.13) = 0$

$\boxed{CD = -24\,KN\ (C)}$





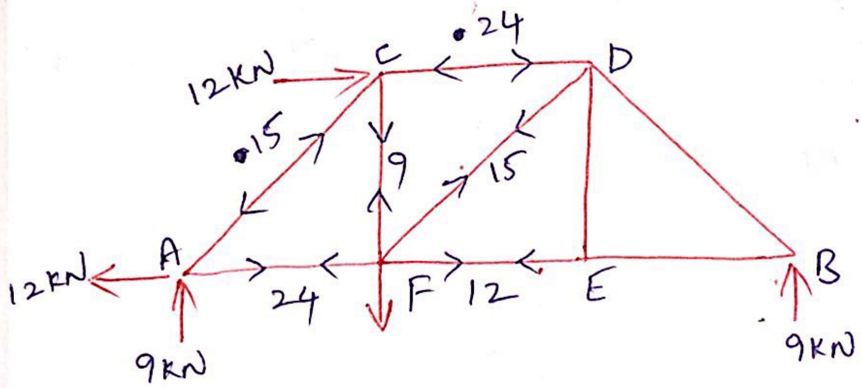
Joint F :—

$\Sigma f_y = 0; \quad -18 + 9 + FD \cdot \sin(36.86) = 0$

$\boxed{FD = 15\,KN.\ (T)}$

$\Sigma f_x = 0; \quad -24 + FE + FD \cdot \cos(36.86) = 0$

$\boxed{FE = 12\,KN\ (T)}$

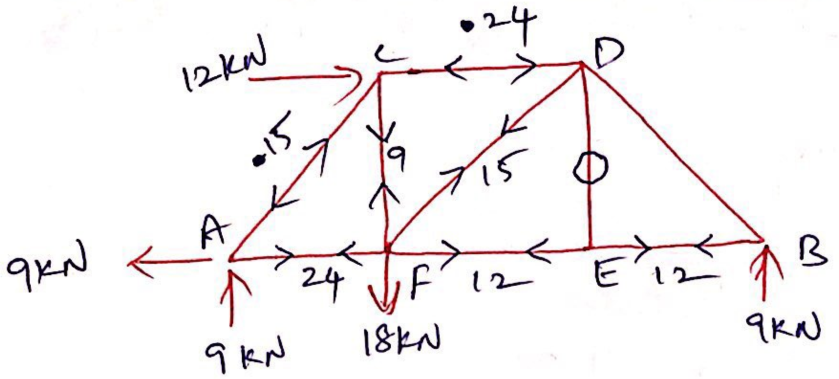


Joint E :-

$\Sigma f_y = 0; \quad \boxed{ED = 0}$

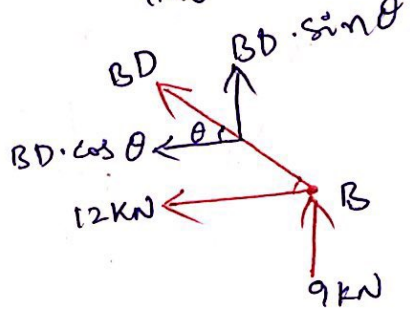
$\Sigma f_x = 0; \quad -12 + EB = 0$

$\boxed{EB = 12\,KN\ (T)}$



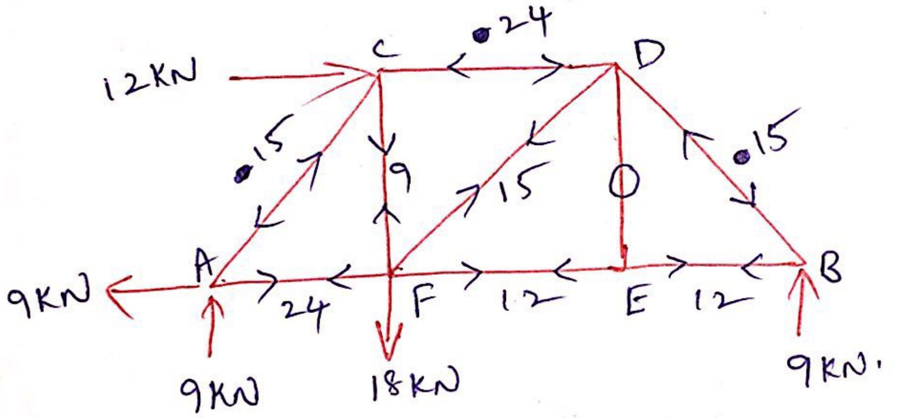
Joint B :- $\theta = Tan^{-1}\left(\frac{3}{4}\right)$

$= 36.86^{\circ}$

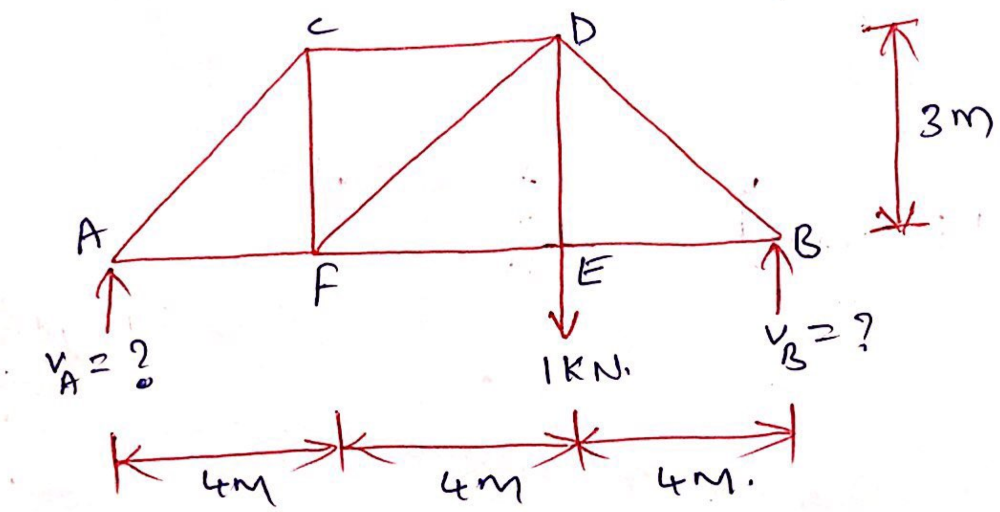


$$\Sigma f_y = 0;\ 9 + BD \cdot \sin(36.86) = 0$$

$$\boxed{BD = -15\,KN\ (C)}$$



'K' analysis :-



$\Sigma f_y = 0;\quad V_A + V_B - 1 = 0 \Rightarrow V_A + V_B = 1\ \text{KN.} \rightarrow (2)$

$\Sigma M_A = 0;\quad (1 \times 8) - (V_B \times 12) = 0$

$$\boxed{V_B = 0.67\ \text{KN.}}$$

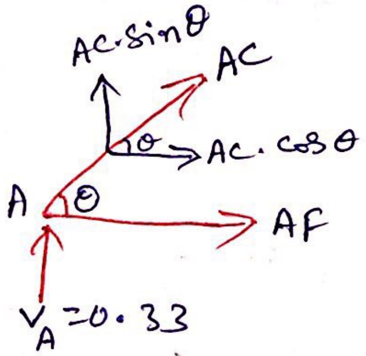
'V_B' sub. in eqn (2)

$V_A + 0.67 = 1\ \text{KN.}$

$$\boxed{V_A = 0.33\ \text{KN}}$$

Joint A :—

$\tan^{-1}\left(\frac{3}{4}\right) = 36.86^{\circ}$



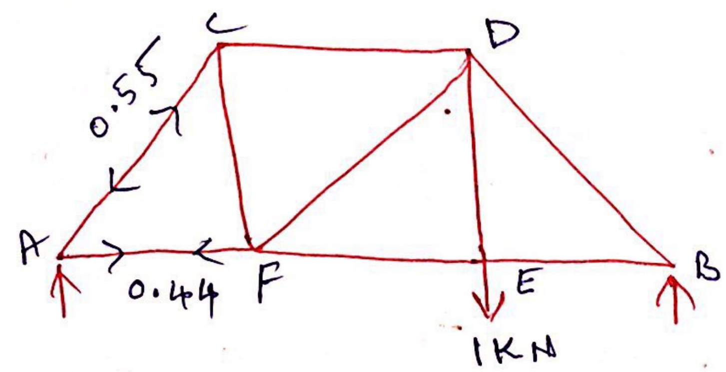
$\Sigma f_y = 0;\quad 0.33 + AC \cdot \sin(36.86^{\circ}) = 0$

$$\boxed{AC = -0.55\ (C)}$$

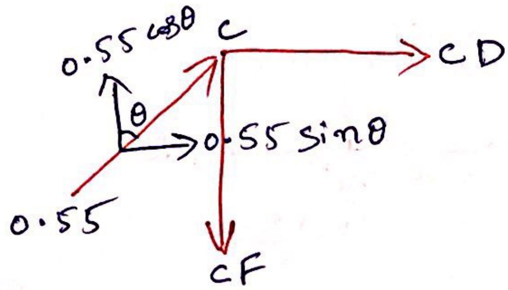
$\Sigma f_x = 0;\quad AF + AC \cdot \cos(36.86) = 0$

$AF + (-0.55)\cos(36.86) = 0$

$$\boxed{AF = 0.44\ \text{KN}(T)}$$



Joint C :-



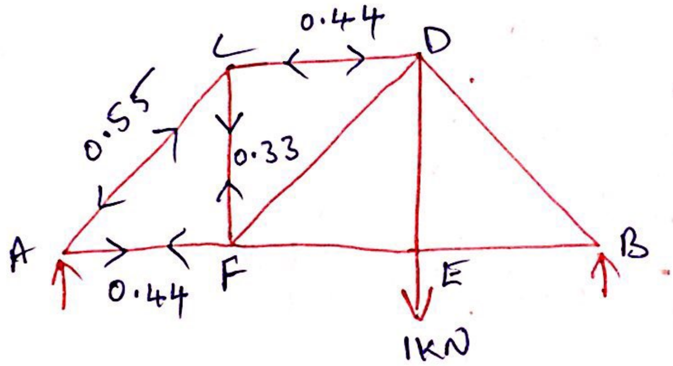
$\theta = \tan^{-1}\left(\frac{4}{3}\right)$
$= 53.13^\circ$

$\Sigma f_y = 0;\quad -CF + 0.55.\cos(53.13^\circ) = 0$

$$\boxed{CF = 0.33\,KN\ (T)}$$

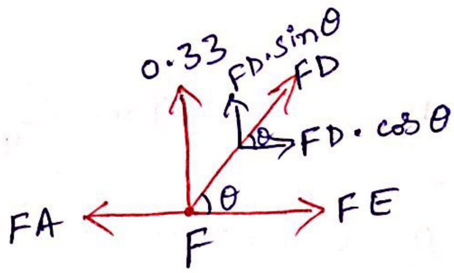
$\Sigma f_x = 0;\quad CD + 0.55.\sin(53.13^\circ) = 0$

$$\boxed{CD = -0.44\,KN\ (C)}$$



Joint F :-

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$
$= 36.86^\circ$



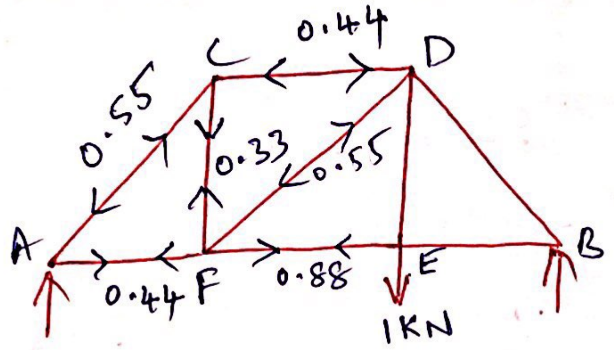
$\Sigma f_y = 0;\quad 0.33 + FD.\sin(36.86^\circ) = 0$

$$\boxed{FD = -0.55\,KN\ (C)}$$

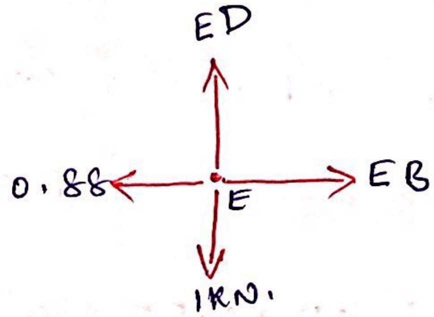
$\Sigma f_x = 0;\quad FE - FA + FD.\cos(36.86^\circ) = 0$

$FE - 0.44 + (-0.55)\cos(36.86^\circ) = 0$

$$\boxed{FE = 0.88\,KN\ (T)}$$



Joint E :-

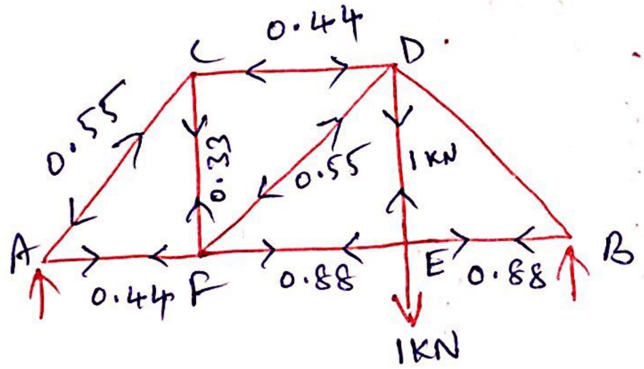


$\Sigma f_y = 0; \quad ED - 1 = 0$

$$\boxed{ED = 1\,KN\ (T)}$$

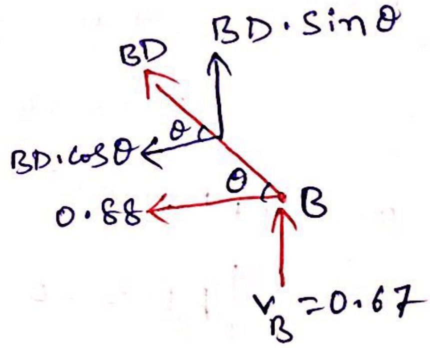
$\Sigma f_x = 0; \quad EB - 0.88 = 0$

$$\boxed{EB = 0.88\,KN\ (T)}$$



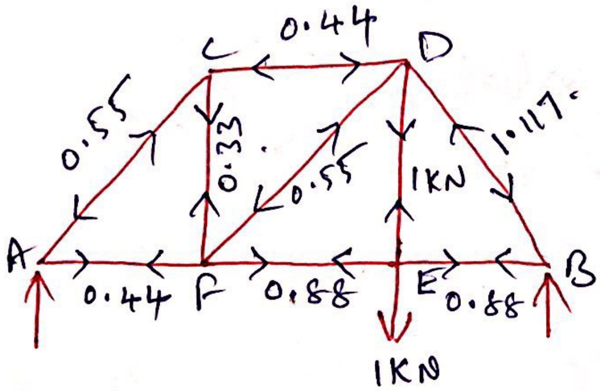
Joint B :-

$\theta = Tan^{-1}\left(\frac{3}{4}\right) = 36.86^\circ$



$\Sigma f_y = 0; \quad 0.67 + BD.\sin(36.86^\circ) = 0$

$$\boxed{BD = -1.117\,KN\ (C)}$$



Member	P (KN)	K (KN)	l (m)	PKl
AF	24	0.44	4	42.24
AC	-15	-0.55	5	41.25
CF	9	0.33	3	8.91
CD	-24	-0.44	4	42.24
FD	15	-0.55	5	-41.25
FE	12	0.88	4	42.24
DE	0	1	3	0
BD	-15	-1.117	5	83.775
BE	12	0.88	4	42.24

$\Sigma PKl = 261.645 \text{ KN}^2.\text{m}.$

EI is constant (KN^2).

$$y_E = \frac{261.645 \cancel{KN^2}.m}{EI \cancel{KN^2}}$$

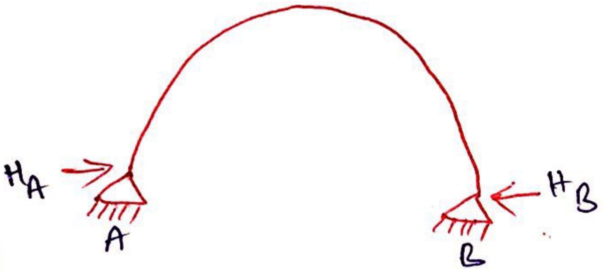
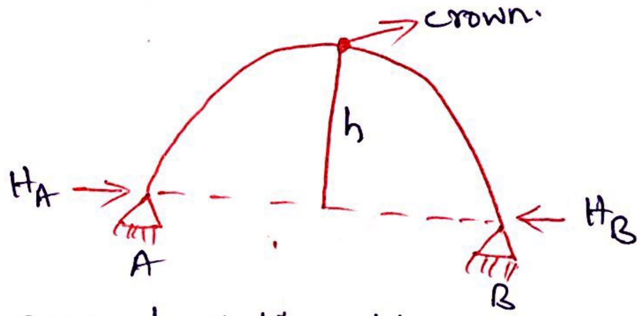
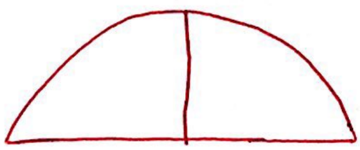
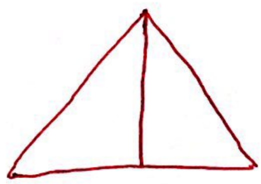
$$\boxed{y_E = 261.645 \text{ m}.}$$

Three Hinged Arches :-

Introduction :-

Arch :- Arch is a curved symmetrical structure spanning an opening and typically supporting the weight of a bridge, roof (or) wall above it.

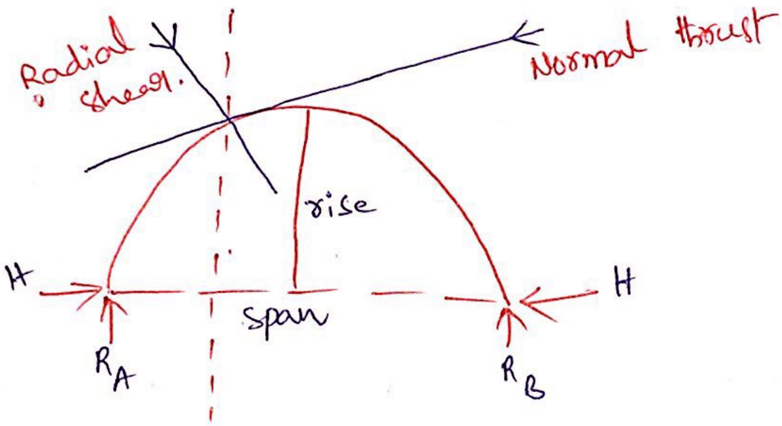
Difference between two hinged arch and three hinged arch:-

Two Hinged Arch	Three Hinged Arch.
	
Degree of static indeterminacy $D_S=1$, structure is indeterminate upto 1 degree.	Degree of static indeterminacy $D_S=0$, structure is statically determinate.
with increase in temparature, horizontal thrust also increases.	with increase in temparature, horizontal thrust decreases.
Influence line diagram (ILD) is parabolic. 	Influence line diagram (ILD) is triangular. 

It will develop stresses due to sinking of supports.	It will develop no stress due to sinking of supports as structure is statically determinate i.e. $D_s = 0$.
It is more efficient and cost effective.	It is easy to analyse but more cost is required.

Horizontal Thrust :— The thrust is the resultant of two forces. The horizontal thrust is applied on both springers, but it is also found on top of the arch, as it represents the balance of the second half of the arch.

Radial Shear :— The total force acting along the radial direction is called radial shear.



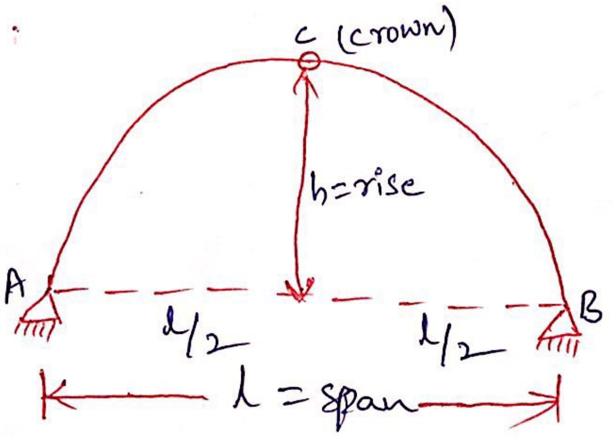
concept of three hinged arch :—

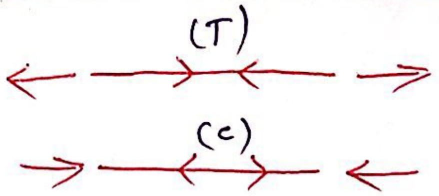
(1) 3-Hing parabolic arch at same level. i.e. supported

(2) 3-Hing parabolic arch at different level. i.e. supported

(3) circular arch.

(1) 3-Hing parabolic arch at same level i.e. supported :—





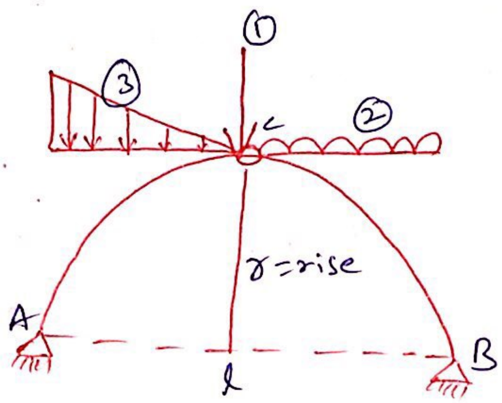
In this chapter only come compression, because it is a 3-Hing arches.
'H' is always inward.

three types of loads comes in 3-Hing arches.

i.e. (1) point load

(2) UDL

(3) UVL

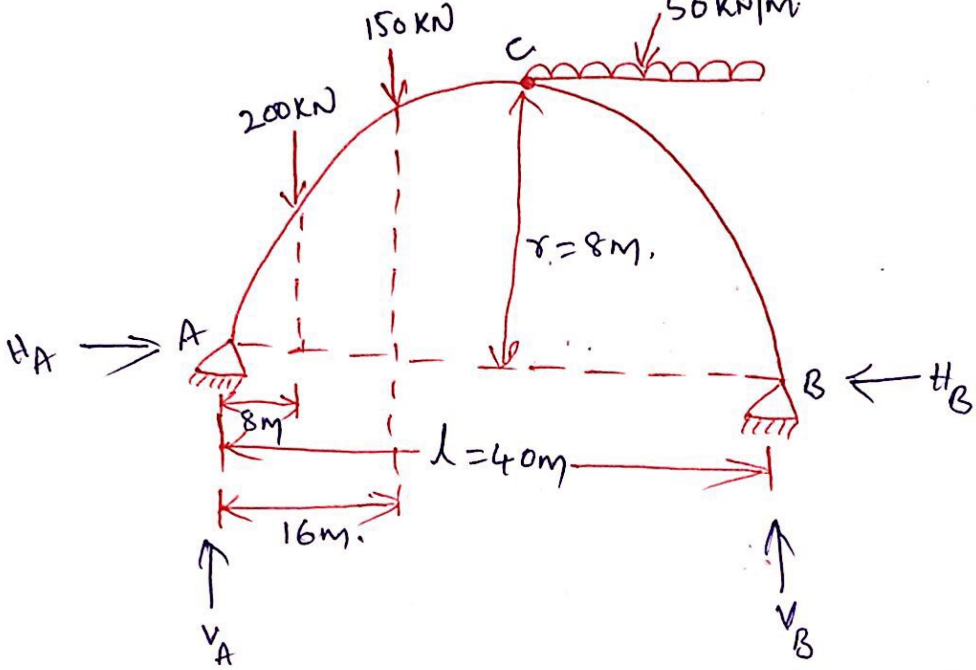


$$\boxed{y = \frac{4hx}{l^2}(l - x) \quad \text{(OR)} \quad y = \frac{4h}{l^2}(lx - x^2)}$$

Problems :-

① A 3-hinged arch of span 40m. and rise 8m. carries concentrated loads of 200KN and 150KN at a distances of 8m. and 16m. from the left end and an UDL of 50KN/M. on the right half of the span. Find horizontal thrust.

Sol :-



Horizontal thrust, $H = ?$. $(H_A = H_B = H)$

Support Reactions :-

$$\Sigma M_A = 0;\ (200 \times 8) + (150 \times 16) + (50 \times 20)\left(20 + \frac{20}{2}\right) - (V_B \times 40) = 0.$$

$$\boxed{V_B = 850\ KN}$$

$$\Sigma f_y = 0;\ V_A + V_B - 200 - 150 - (50 \times 20) = 0.$$

$$V_A + 850 - 200 - 150 - (50 \times 20) = 0$$

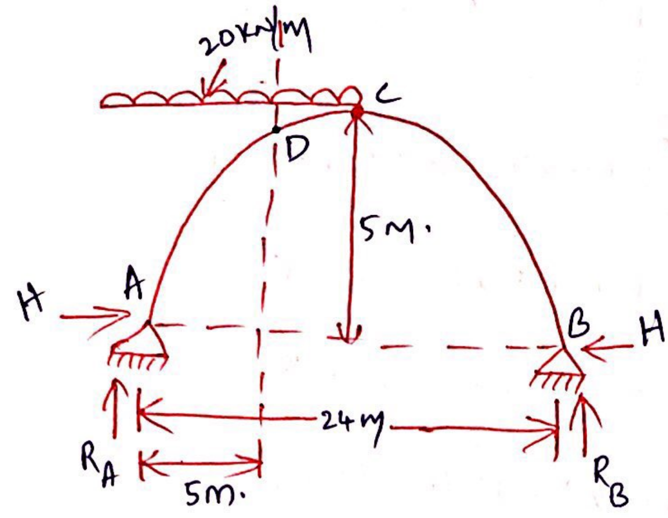
$$\boxed{V_A = 500\ KN.}$$

To find horizontal thrust (H) :- consider part CA.

$$(500 \times 20) - (200 \times 12) - (150 \times 4) - (H_A \times 8) = 0 \Rightarrow H_A = 875\ KN.$$

$$\boxed{\therefore\ H_A = H_B = H = 875\ KN}$$

(2) A three hinged symmetrical parabolic arch is loaded as shown in fig. calculate (i) support reactions (ii) BM at AC & BC (iii) Normal thrust and Radial shear force at D. (15)



(A) (i) Support Reactions :—

$\Sigma M_A = 0;\ (-R_B \times 24) + (20 \times 12) \times \frac{12}{2} = 0$

$$\boxed{R_B = 60\,KN}$$

$\Sigma f_y = 0;\ R_A + R_B - (20 \times 12) = 0$

$$\boxed{R_A = 180\,KN}$$

BM_C (consider Right part):

$(-R_B \times 12) + (H \times 5) = 0$

$$\boxed{H = 144\,KN}$$

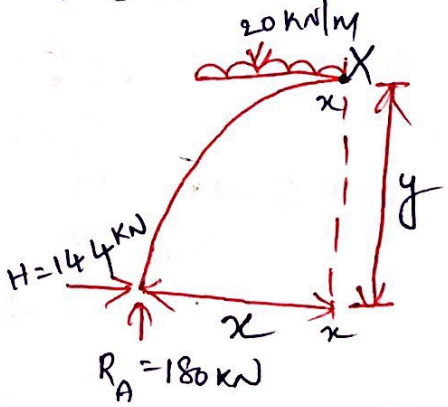
(ii) BM at AC & BC :—

consider BM at AC :—

consider a section x–x in b/w AC at a distance 'x' from 'A'

$M_x = (180 \times x) - (20 \times x) \times \frac{x}{2} - (144 \times y)$

where: $y = \frac{4hx}{L^2}(L - x)$



Scanned with CamScanner

$$= 180x - 10x^2 - 144\left[\frac{4hx}{L^2}(L-x)\right]$$

$$= 180x - 10x^2 - 144\left[\frac{4\times 5\times x}{(24)^2}(24-x)\right]$$

$$= 180x - 10x^2 - 144\left[0.03472\,x(24-x)\right]$$

$$M_x = 180x - 10x^2 - 120x + 5x^2 \Rightarrow (1)$$

$$\frac{dM_x}{dx} = 0 \Rightarrow \frac{d}{dx}\left(180x - 10x^2 - 120x + 5x^2\right) = 0$$

$$180 - (2\times 10x) - 120 + (2\times 5x) = 0$$

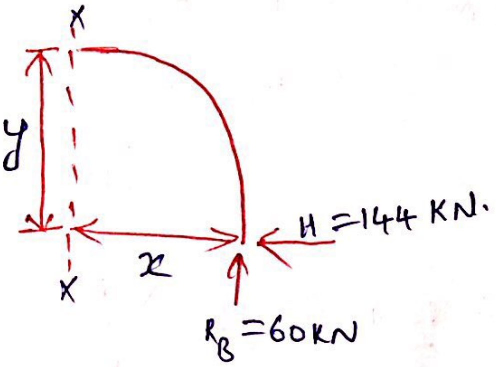
$$\boxed{x = 6\,m.}$$

Sub. $x = 6\,m.$ in eqn (1)

$$M_x = 180(6) - 10(6)^2 - 120(6) + 5(6)^2$$

$$\boxed{M_x = 180\,KN.m. \text{ for } AC.}$$

Consider BM at BC :—



$$M_x = -60\times x + 144\times y$$

$$= -60\times x + 144\left[\frac{4hx}{L^2}(L-x)\right]$$

$$= -60\times x + 144\left[\frac{4\times 5\times x}{(24)^2}(24-x)\right]$$

$$M_x = -60x + 120x \mp 5x^2 \rightarrow (2)$$

$$\frac{dM_x}{dx} = \frac{d}{dx}\left[-60x + 120x \mp 5x^2\right]$$

$$= -60 + 120 \mp (2\times 5x) = 0$$

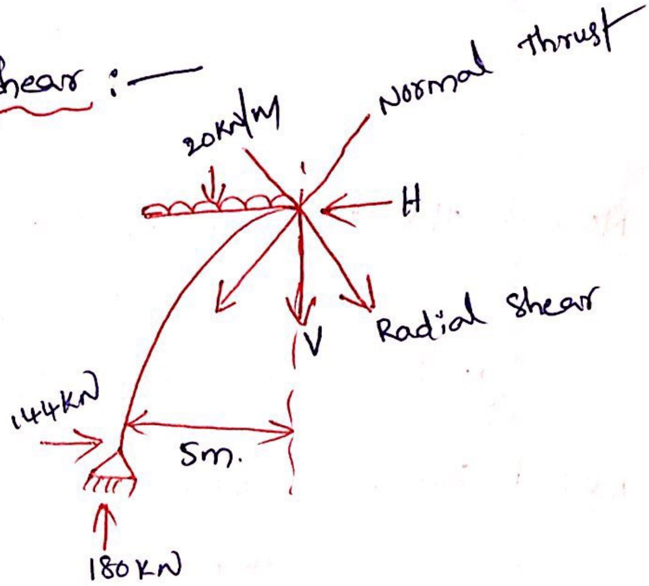
$$\boxed{x = 6\,m.}$$

Sub. $x = 6$ in eqn (2)

$$M_x = -60(6) + 120(6) - 5(6)^2$$

$$\boxed{M_x = -180 \text{ KN.m. for BC.}}$$

(iii) Normal Thrust and Radial Shear :—

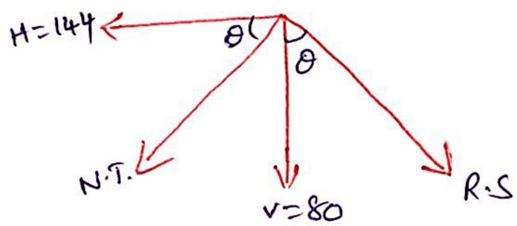


$\Sigma f_y = 0;\ 180 - V - (20 \times 5) = 0$

$$\boxed{V = 80 \text{ KN.}}$$

$\Sigma f_x = 0;\ 144 - H = 0$

$$\boxed{H = 144 \text{ KN.}}$$



$$\frac{dy}{dx} = \operatorname{Tan}\theta$$

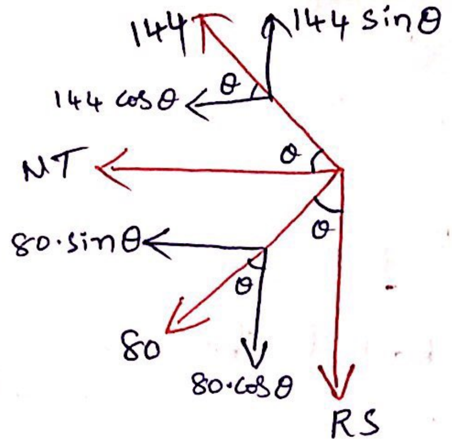
$$\operatorname{Tan}\theta = \frac{d}{dx}\left[\frac{4hx}{L^2}(L-x)\right]$$

$$= \frac{4h}{L^2} \cdot \frac{d}{dx}\left[x(L-x)\right]$$

$$= \frac{4h}{L^2}\left[L - 2x\right]$$

$$\operatorname{Tan}\theta = \frac{4 \times 5}{(24)^2}\left[24 - 2(5)\right]$$

$$\boxed{\theta = 25.92^\circ}$$



Normal thrust $= -144 \cdot \cos\theta - 80\sin\theta = 0$

$$-144 \cdot \cos(25.92^\circ) - 80 \cdot \sin(25.92^\circ) = 0$$

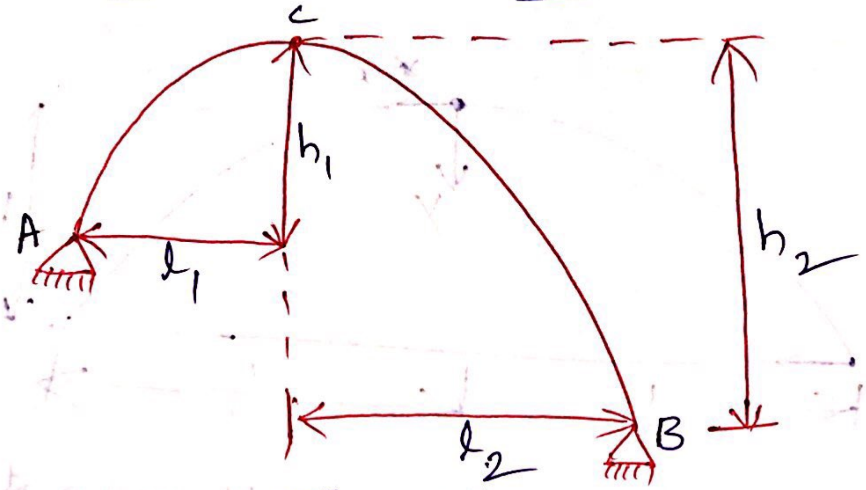
$$\boxed{N.T. = 164.48\ KN.}$$

Radial Shear $= 144 \cdot \sin\theta - 80 \cdot \cos\theta = 0$

$$144 \cdot \sin(25.92^\circ) = 80 \cdot \cos(25.92^\circ)$$

$$\boxed{R.S. = 9\ KN}$$

Three Hinged parabolic arch at different level :-

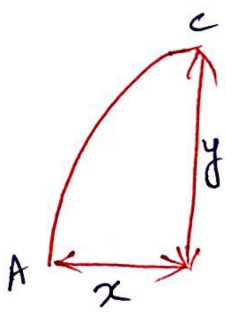


$h_1 = ?$; $l_1 = \dfrac{L\sqrt{h_1}}{\sqrt{h_1}+\sqrt{h_2}}$

$h_2 = ?$; $l_2 = \dfrac{L\sqrt{h_2}}{\sqrt{h_2}+\sqrt{h_1}}$

$$L = l_1 + l_2$$

BM at Left part i.e. AC :-



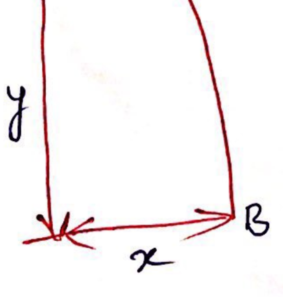
$y = \dfrac{4hx}{L^2}(L-x)$

$h = h_1$ ($\because$ Left part)

$L = 2 \times l_1$

$x = ?$

BM at Right part i.e. BC :-



$y = \dfrac{4hx(L-x)}{L^2}$

$h = h_2$ ($\because$ Right part)

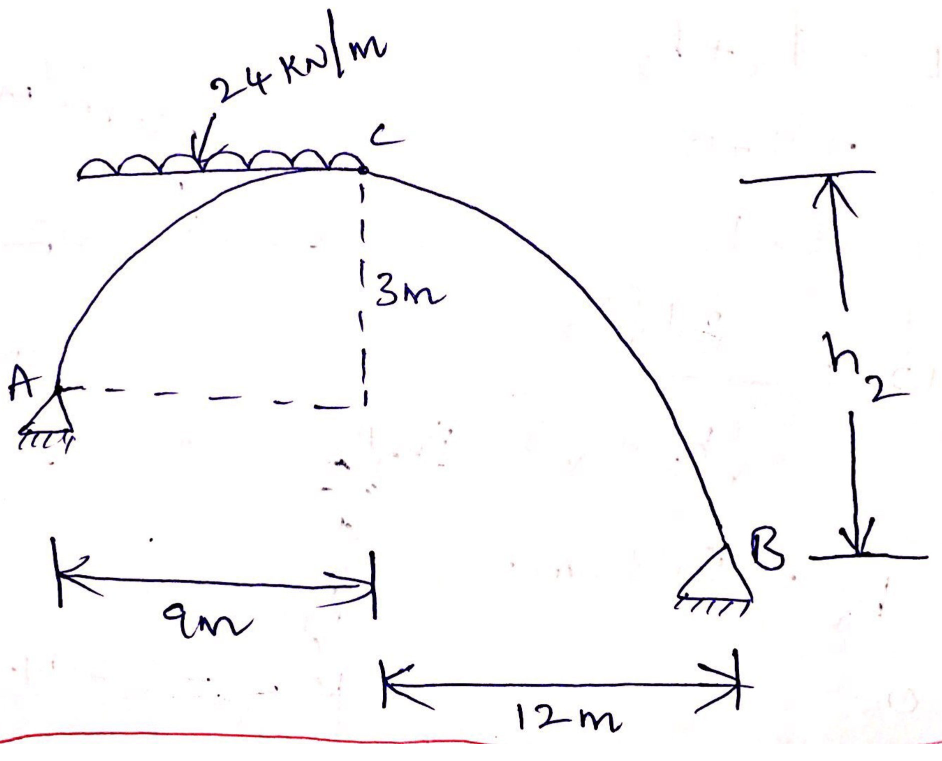
$L = 2 \times l_2$

$x = ?$

Parabolic Arch at Different Level :—

Unsymmetrical Arch :—

(1) An unsymmetrical three hinged parabolic arch is loaded as shown. Determine support reaction, BM, NT, RSF at 5m. from left support. Also find max. BM in left and right ~~support~~ part



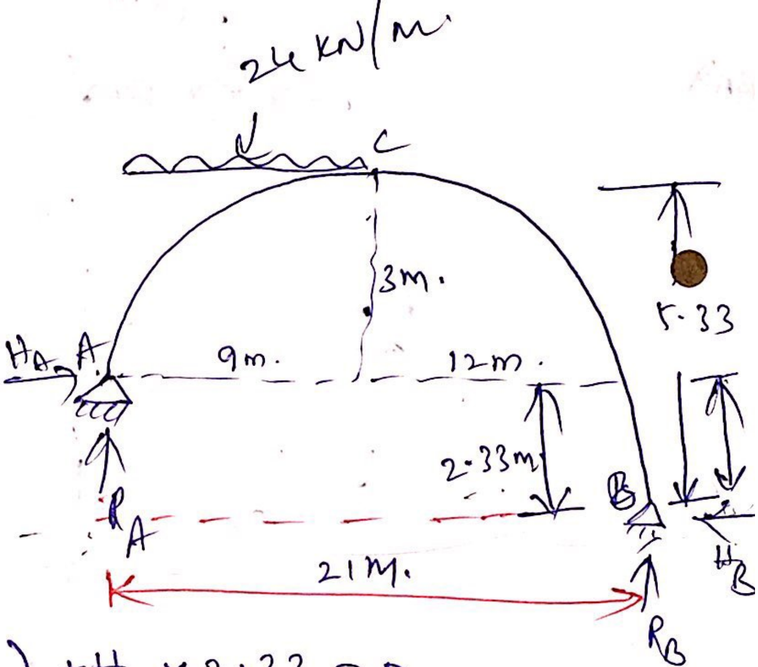
Sol:- $$l_2 = \frac{L\sqrt{h_2}}{\sqrt{h_2}+\sqrt{h_1}}$$

$$12 = \frac{L\sqrt{h_2}}{\sqrt{h_2}+\sqrt{h_1}}$$

$$L = l_1 + l_2$$
$$= 9+12$$
$$= 21$$

$$12 = \frac{21\sqrt{h_2}}{\sqrt{h_2}+\sqrt{3}}$$

$$h_2 = 5.33\,m.$$



$\Sigma M_A = 0.$

$$-R_B \times 21 + \left(24 \times 9 \times \frac{9}{2}\right) + H_B \times 2.33 = 0$$

$$-21R_B + 2.33\,H_B = -972 \rightarrow (1)$$

5.33
3.00
2.33

BM at C. (Right part)

$$-R_B \times 12 + H_B \times 5.33 = 0$$

$$-12R_B + 5.33H_B = 0 \rightarrow (2)$$

solving (1) & (2)

$$R_B = 61.69 \approx 61.7\ KN.$$

'R_B' sub in (2); $H_B = 139$ KN.

$\Sigma F_x = 0 \Rightarrow H_A = H_B = H = 139$ KN.

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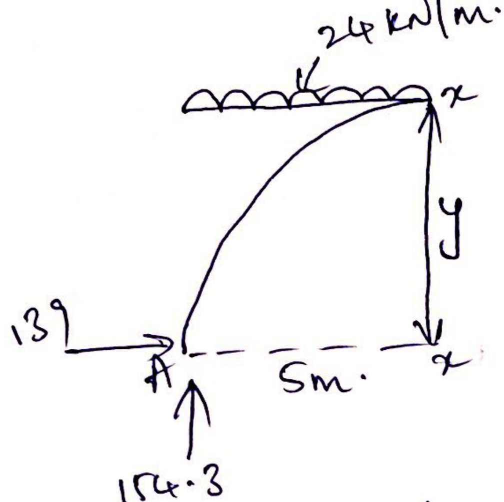
$\Sigma f_y = 0$

$R_A + 61.7 - (24 \times 9) = 0$

$R_A = 154.3 \text{ KN}.$

BM at 5m. from left support :-



$$y = \frac{4hx}{L^2}(L - x)$$

$h = h_1 = 3$ (Left part).

$L = 2 \times l_1 = 2 \times 9 = 18$ (Left part).

$x = 5\text{m}.$

$$y = \frac{4 \times 3 \times 5}{(18)^2}(18 - 5)$$

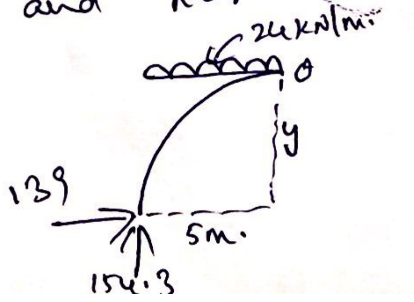
$y = 2.4 \text{ m}.$

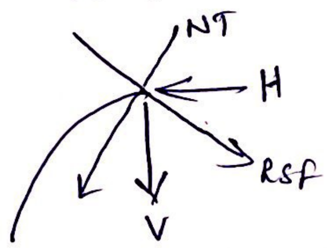
$M_x = 154.3 \times 5 - 139 \times 2.4 - (24 \times 5) \times \frac{5}{2}$

$= 138 \text{ KN.m}.$

NT and RSF at 5m. from LS.

It is not in equilibrium state.





$\Sigma f_y = 0$

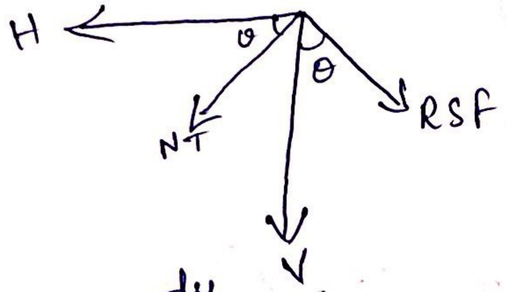
$154.3 - (24 \times 5) - V = 0$

$V = 34.3$ KN

$\Sigma F_x = 0$

$139 - H = 0$

$H = 139$ KN



$$\theta = \tan\theta = \frac{dy}{dx}$$

$$y = \frac{4hx}{L^2}(L - x)$$

$$\tan\theta = \frac{d}{dx}\left[\frac{4hx}{L^2}(L - x)\right]$$

$$= \frac{4h}{L^2}(L - 2x)$$

$$\tan\theta = \frac{4 \times 3}{(18)^2}(18 - 2 \times 5)$$

$L = 2 \times l_1$

$= 2 \times 9 = 18$m.

$$\theta = 16.5^\circ$$

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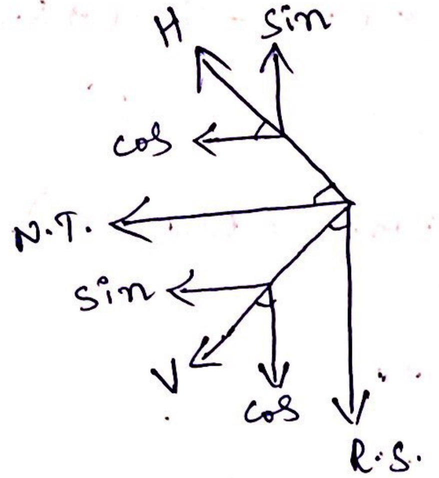


$NT = V \cdot \sin\theta + H \cdot \cos\theta$ (Horizontal Direction)

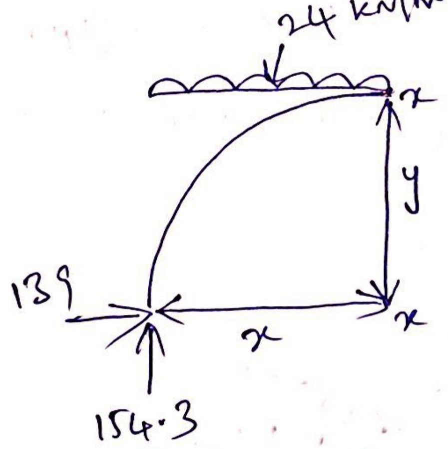
$= 143 \text{ KN}.$

$RS = V \cdot \cos\theta - H \cdot \sin\theta$ (vertical Direction)

$= -6.6 \text{ KN}.$

$R.S = +6.6 \text{ KN}.$

Maximum BM at left part :-



$M_x = 154.3 \times x - 139 \times y - (24 \times x) \times \frac{x}{2}$

$= 154.3x - 12x^2 - 139y$

$= 154.3 \cdot x - 12x^2 - 139 \left[\frac{4 \times 3 \times x}{(18)^2} (18 - x) \right]$

$M_x = 154.3x - 12x^2 - 92.6x + 5.14x^2$ → (3)

$\boxed{y = \frac{4hx}{L^2}(L-x)}$

$h = h_1 = 3m.$

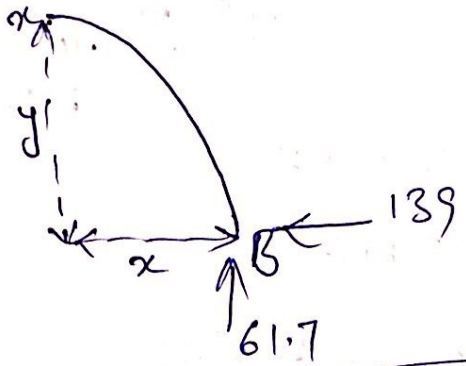
$L = 2 \times l_1$
$= 2 \times 9 = 18m$

$$\frac{dM_x}{dx} = 0 \Rightarrow 154.3 - 12 \times 2 \times x - 92.6 + 5.14 \times 2 \times x = 0.$$

$x = 4.5\,m.$ sub. in (8)

$M_x = 138.735$ KN.m. (B/W AC).

<u>Right part</u> :-



$\left[M_x = -61.7 \times x + 139 \times y.\right]$

$M_x = 61.7 \times x - 139 \times y$

$\boxed{y = \frac{4hx}{L^2}(L-x)}$

$= 61.7x - 139\left[\frac{4 \times 5.33 \times x}{(24)^2}(24-x)\right]$

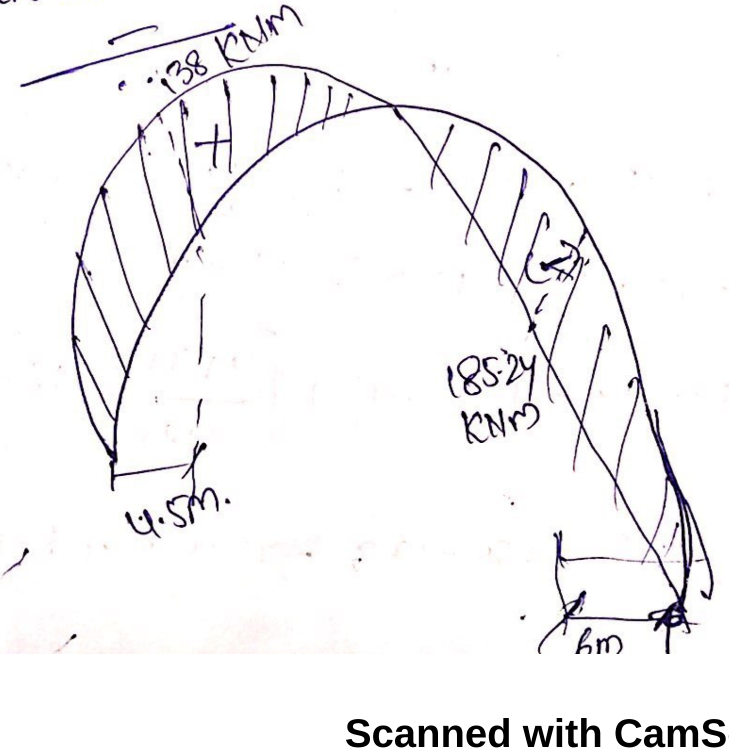
$h = h_2 = 5.33\,m.$
$L = 2 \times l_2 = 2 \times 12 = 24\,M.$

$M_x = 61.7x - 123.432x + 5.143x^2 \rightarrow$ (4)

$$\frac{dM_x}{dx} = 61.7 - 123.432 + 5.143 \times 2x = 0$$

$x = 6\,m.$ sub in (4)

$M_x = -185.244$ KN.m.



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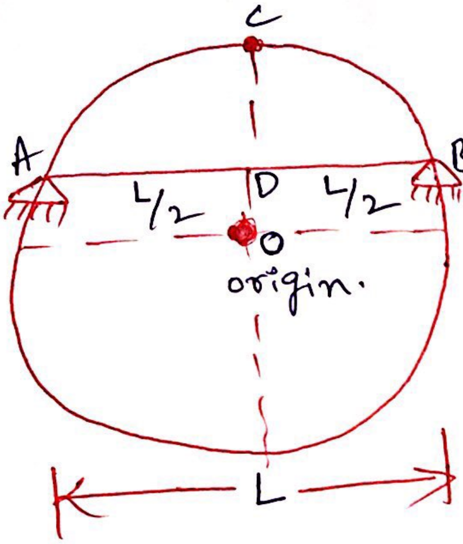
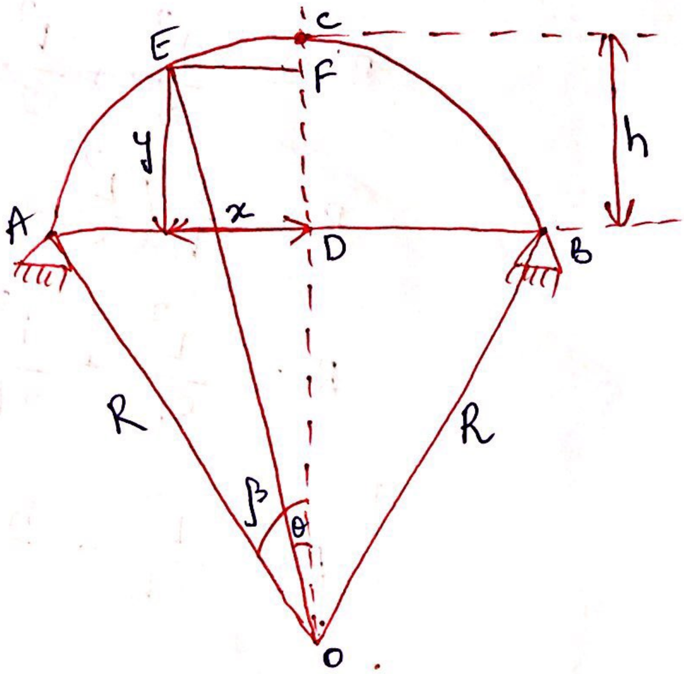
Three Hinged circular Arch :-

$$y = \sqrt{R^2 - x^2} - R + h$$

$$R = \frac{L^2}{8h} + \frac{h}{2}$$





consider ΔOEF ; (from Pythagoras Theorem)

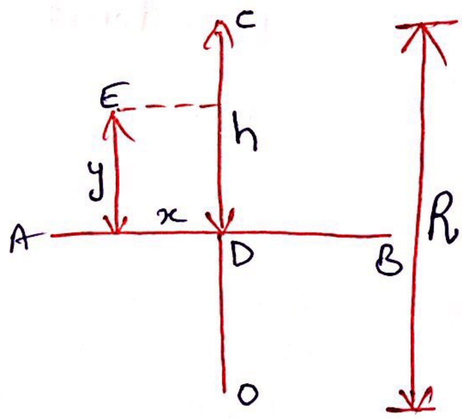
$$OE^2 = OF^2 + EF^2$$

$$R^2 = (R - h + y)^2 + x^2$$

$$R^2 - x^2 = (R - h + y)^2$$

$$\sqrt{R^2 - x^2} = R - h + y$$

$$\boxed{y = \sqrt{R^2 - x^2} - R + h}$$



consider ΔADO ;

$$OA^2 = AD^2 + OD^2$$

$$R^2 = \left(\frac{L}{2}\right)^2 + (R - h)^2$$

$$\cancel{R^2} = \frac{L^2}{4} + \cancel{R^2} + h^2 - 2Rh$$

$$2Rh = \frac{L^2}{4} + h^2$$

$$Rh = \frac{1}{2}\left(\frac{L^2}{4} + h^2\right)$$

$$Rh = \frac{L^2}{8} + \frac{h^2}{2}$$

$$R = \frac{1}{h}\left(\frac{L^2}{8} + \frac{h^2}{2}\right)$$

$$R = \frac{L^2}{8h} + \frac{h^{\cancel{2}}}{2\cancel{h}}$$

$$\boxed{R = \frac{L^2}{8h} + \frac{h}{2}}$$

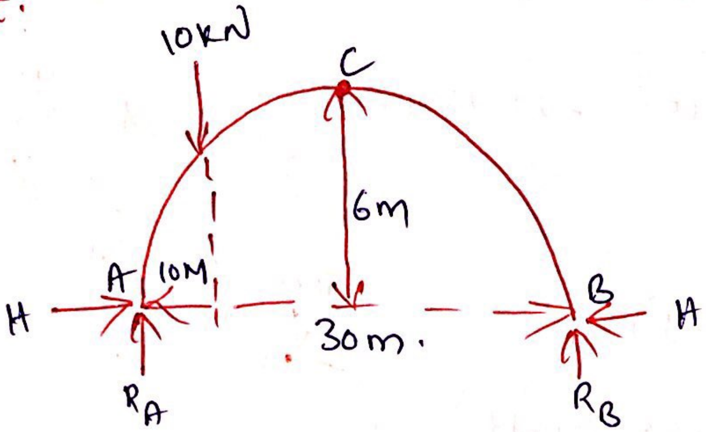
R = Radius

L = Length (or) span

h = rise.

(1) A three hinged circular arch of span 30m. with rise of 6m. carries a point load of 10KN at 10m. from left hinge. calculate support reaction and maximum +ve & −ve bending moment. (18)

(A)



Support reactions :—

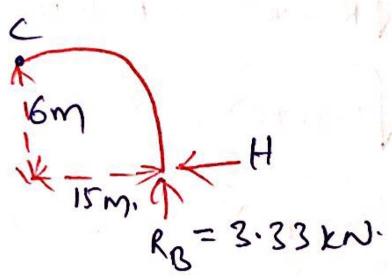
$\Sigma M_A = 0;\quad (-R_B \times 30) + (10 \times 10) = 0$

$-30R_B + 100 = 0 \Rightarrow \boxed{R_B = 3.33\,KN}$

$\Sigma f_y = 0;\quad R_A + R_B - 10 = 0$

$R_A + 3.33 = 10 \Rightarrow \boxed{R_A = 6.67\,KN.}$

$BM_C = 0;$ consider right part i.e. BC.



$(-R_B \times 15) + (H \times 6) = 0.$

$(-3.33 \times 15) + (H \times 6) = 0$

$\cancel{H = 21.75}$

$\boxed{H = 8.33\,KN}$

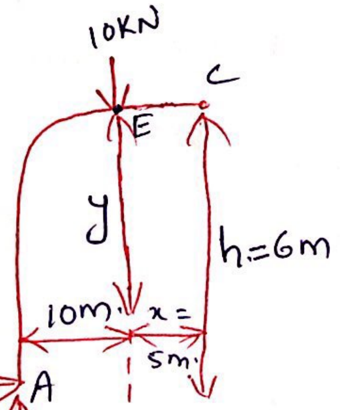
Radius: $R = \frac{L^2}{8h} + \frac{h}{2}$.

$= \frac{(30)^2}{8(6)} + \frac{6}{2}$

$\boxed{R = 21.75\,M.}$

Maximum +ve and -ve bending moment :—

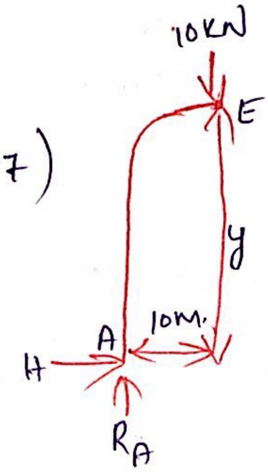
Maximum +ve BM i.e. AC :—



$$y = \sqrt{R^2 - x^2} - R + h.$$

$$y = \sqrt{(21.75)^2 - (5)^2} - 21.75 + 6$$

$$\boxed{y = 5.417\,m.}$$

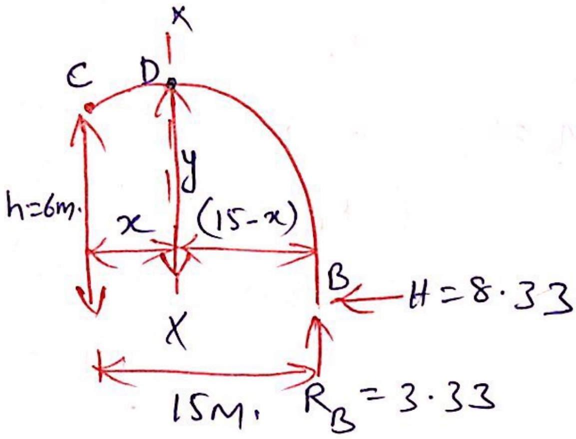


$$BM_E = (R_A \times 10) - (H \times y)$$

$$= (6.67 \times 10) - (8.33 \times 5.417)$$

$$\boxed{BM_E = 21.6\ KN.m.}$$

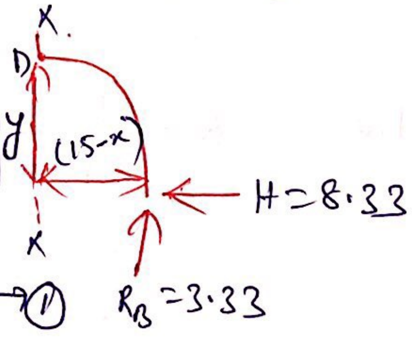
Maximum -ve BM i.e. BC :—



$$y = \sqrt{R^2 - x^2} - R + h$$

$$= \sqrt{(21.75)^2 - x^2} - R + h.$$

$$y = \sqrt{(21.75)^2 - x^2} - 21.75 + 6$$



$$BM_D = -3.33 \times (15 - x) + (8.33 \times y) = 0.$$

$$BM_D = 50 - 3.33x - 8.33\left[\sqrt{(21.75)^2 - x^2} - 21.75 + 6\right]$$

$$= 50 - 3.33x - 8.33\left(\sqrt{473 - x^2}\right) + 131.19 \rightarrow (1)$$

$$\frac{dM_D}{dx} = \frac{d}{dx}\left[50 - 3.33x - 8.33\left(\sqrt{473 - x^2}\right) + 131.19\right]$$

$$\Rightarrow 0 - 3.33 - 8.33\left(\frac{0-2x}{2\sqrt{473-x^2}}\right) = 0$$

$$\boxed{x = 8\,m.}$$

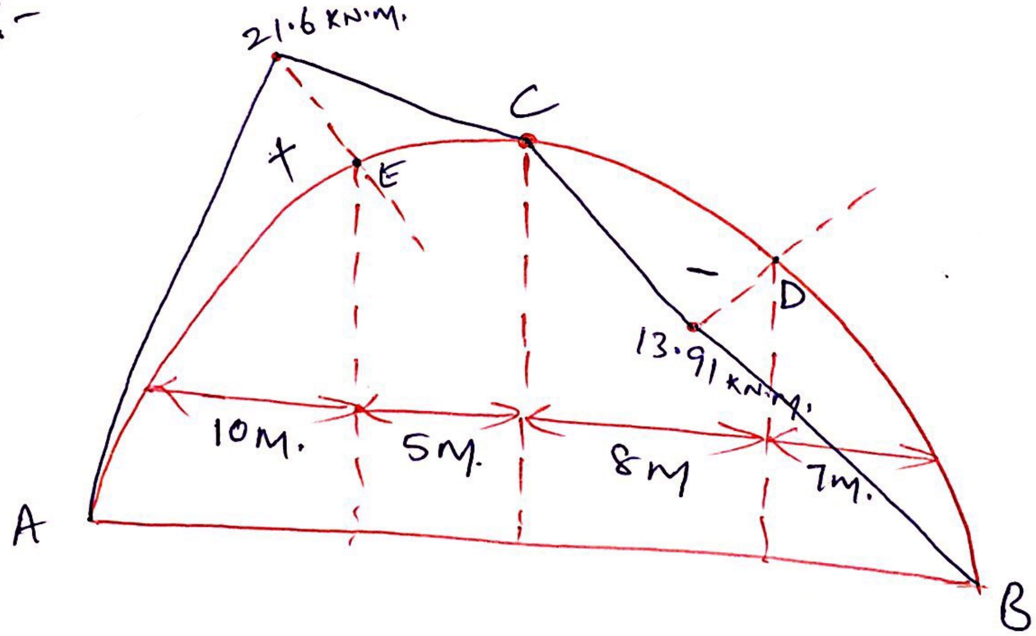
$$\left[\frac{d}{dx}\left(\sqrt{f(x)}\right) = \frac{\frac{d}{dx}.f(x)}{2\sqrt{f(x)}}\right]$$

$x = 8$ m. sub. in eqn (1)

$$BM_D = 50 - 3.33(8) - 8.33\left(\sqrt{433-(8)^2}\right) + 131.19$$

$$\boxed{BM_D = -13.91\ kN.m.}$$

BMD :-



Types of structure on their determinacy :—

Determinate structure :— If all the support reactions & member forces can be found out using equilibrium eq^ns.

Indeterminate structure :— If all the support reactions cannot be found out using equilibrium eq^ns then the structure will be called an indeterminate structure.

Indeterminacy

- **Static Indeterminacy**
  Based on external support reactions and their internal member forces (or) geometry.
  (D_S)

- **Kinematic Indeterminacy**
  Based on their degree of freedom available at all joints.
  (D_K)

Static Indeterminacy (D_S)

- **External static indeterminacy**
  (D_{se})
  Based on support reactions

- **Internal static indeterminacy**
  (D_{si})
  Based on the geometry of structure.
  [Truss
  Frames]

$$D_S = D_{se} + D_{si}$$

(i) External static indeterminacy (D_{se}) :—

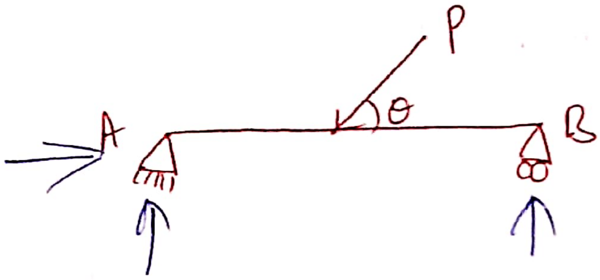
Based on support reactions.

It is defined as the support reactions which are in excess to the no. of equilibrium eqns.

D_{se} = Total no. of support reactions − no. of equilibrium eqns.

$$D_{se} = r_e - E$$

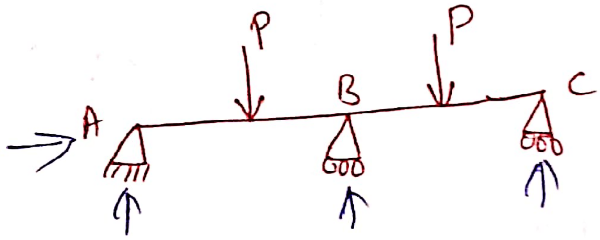
Ex :- (1)



$r_e = 3$, $E = 3$

$D_{se} = r_e - E = 3 - 3 = 0$

Ex :- (2)



$r_e = 3$, $E = 2$

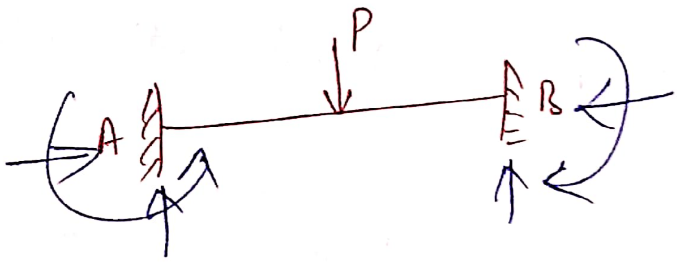
$D_{se} = r_e - E = 3 - 2 = 1$

condition :-

[Beam
vertical loading
Same level.]

General loading & vertical loading

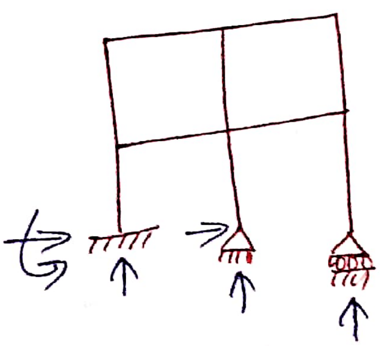
Ex :- (3)



$r_e = 4$, $E = 2$

$D_{se} = r_e - E = 4 - 2 = 2$

Ex :- (4)



$r_e = 6$, $E = 3$

$D_{se} = r_e - E = 6 - 3 = 3$

Internal static Indeterminacy :—

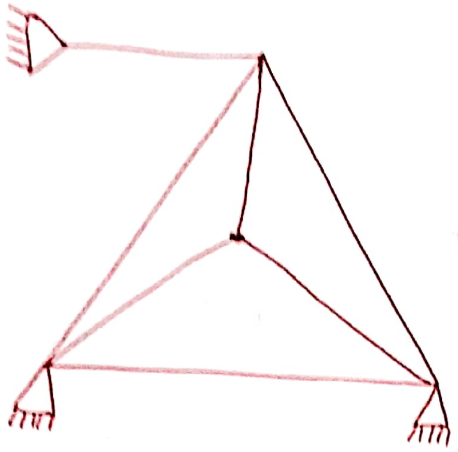
It is based on the geometry of the structure.

* Truss
* Frames (included beams also)

Truss :—

D_{si} = ~~Number of~~ $m-(2j-3)$

(1)



$D_s = ?$

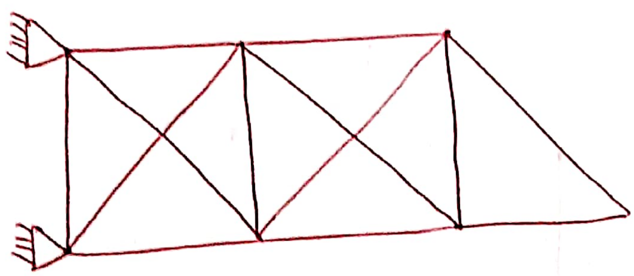
$D_s = D_{se} + D_{si}$

(A) $D_{se} = r_e - E = 6-3 = 3$

$D_{si} = m-(2j-3) = 7-(2\times5-3) = 0$

$D_s = D_{se} + D_{si} = 3+0 = 3$

(2)



$D_s = ?$

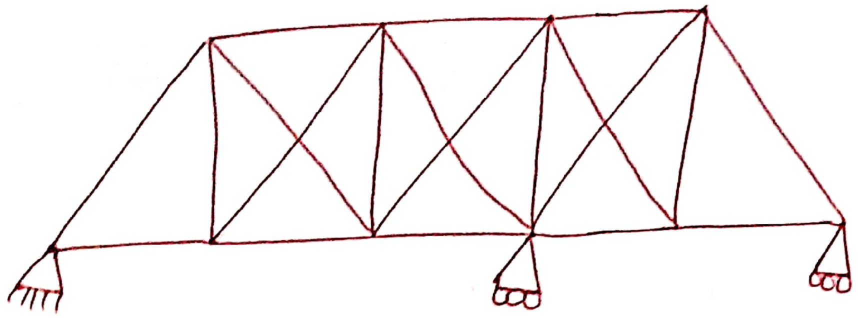
$D_s = D_{se} + D_{si}$

(A) $D_{se} = r_e - E = 4-3 = 1$

$D_{si} = m-(2j-3) = 13-(2\times7-3) = 2$

$D_s = D_{se} + D_{si} = 1+2 = 3$

(3)



$D_s = ?$

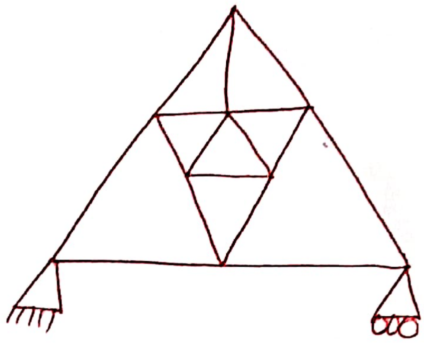
$D_s = D_{se} + D_{si}$

(A) $D_{se} = r_e - E = 4 - 3 = 1$

$D_{si} = m - (2j - 3) = 20 - (2 \times 10 - 3) = 3$

$D_s = D_{se} + D_{si} = 1 + 3 = 4$

(4)



$D_s = ?$

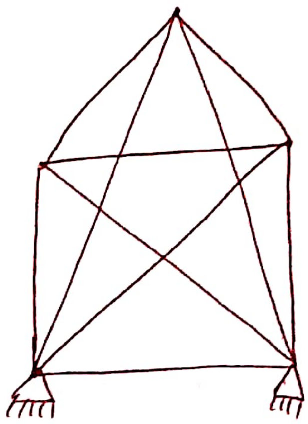
$D_s = D_{se} + D_{si}$

(A) $D_{se} = r_e - E = 3 - 3 = 0$

$D_{si} = m - (2j - 3) = 16 - (2 \times 9 - 3) = 1$

$D_s = D_{se} + D_{si} = 0 + 1 = 1$

(5)



$D_s = ?$

$D_s = D_{se} + D_{si}$

(A) $D_{se} = 4 - 3 = 1$

$D_{si} = m - (2j - 3) = 10 - (2 \times 5 - 3) = 3$

$D_s = D_{se} + D_{si} = 1 + 3 = 4$

Frames :-

For frames $\Rightarrow$ $D_{si} = 3c - r_r$

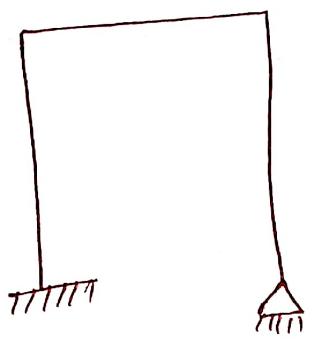
$c \rightarrow$ No. of closed loops

$r_r \rightarrow$ No. of released reactions.

$r_r = \Sigma(m-1)$

$m \rightarrow$ No. of members connected to internal hing.

(1)



$D_s = ?$

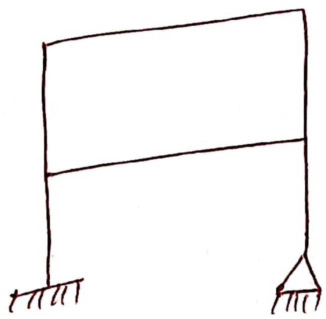
$D_s = D_{se} + D_{si}$

(A) $D_{se} = r_e - E = 5 - 3 = 2$

$D_{si} = 3c - r_r = 3(0) - 0 = 0$

$D_s = D_{se} + D_{si} = 2 + 0 = 2$

(2)



$D_s = ?$

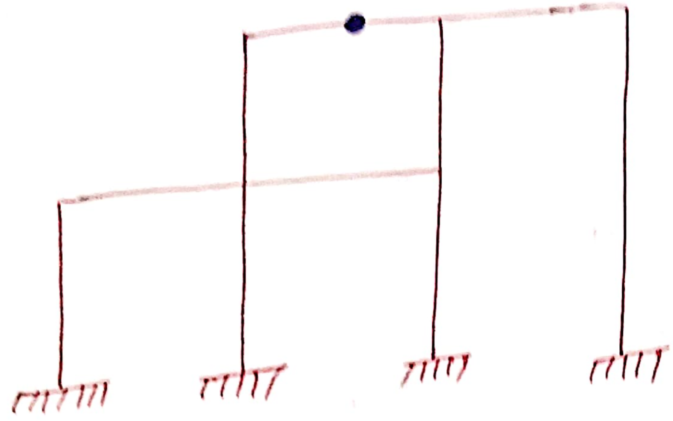
$D_s = D_{se} + D_{si}$

(A) $D_{se} = r_e - E = 5 - 3 = 2$

$D_{si} = 3c - r_r = 3(1) - 0 = 3$

$D_s = D_{se} + D_{si} = 2 + 3 = 5$

(3)



$D_s = ?$

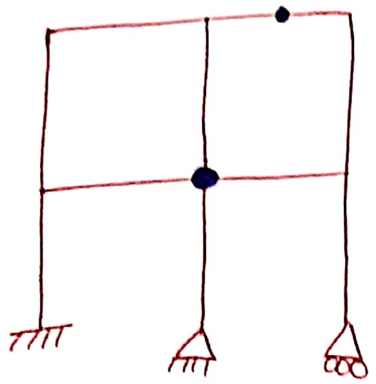
$D_s = D_{se} + D_{si}$

(A) $D_{se} = \gamma_e - E = 12 - 3 = 9$

$D_{si} = 3c - \gamma_r = 3(1) - (m-1) = 3 - (2-1) = 2$

$D_s = D_{se} + D_{si} = 9 + 2 = 11$

(4)



$D_s = ?$

$D_s = D_{se} + D_{si}$

(A) $D_{se} = \gamma_e - E = 6 - 3 = 3$

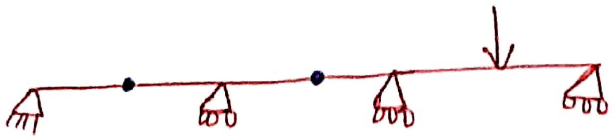
$D_{si} = 3c - \gamma_r = 3(2) - \Sigma(m-1)$

$\Sigma(m-1) = (4-1) + (2-1) = 3 + 1 = 4$

$D_{si} = 3c - \gamma_r = 3(2) - 4 = 2$

$D_s = D_{se} + D_{si} = 3 + 2 = 5$

(5)



$D_s = ?$

(A) $D_{se} = \gamma_e - E = 4 - 3 = 1$

$D_{si} = 3c - \gamma_r = 3(0) - [(2-1)+(2-1)] = -2$

$D_s = D_{se} + D_{si}$
$= 1 + (-2)$

$D_s = -1$ (unstable)

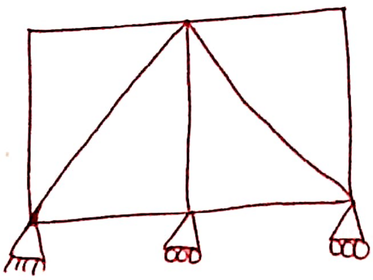
Kinematic Indeterminacy :— (D_K)

Truss $\rightarrow$ $D_K = 2j - r_e$

$j \rightarrow$ No. of joints

$r_e \rightarrow$ support reactions

1



(A)

$D_K = 2j - r_e$

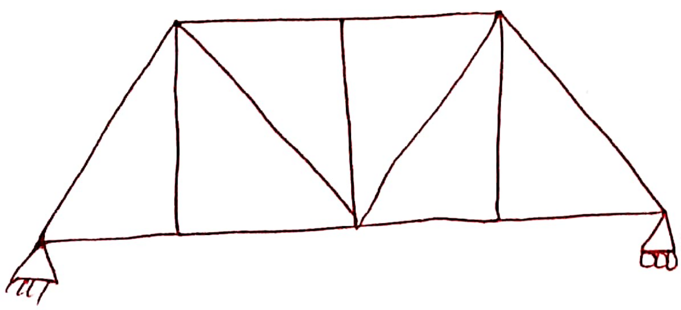
$j = 6$

$r_e = 4$

$D_K = 2(6) - 4$

$D_K = 8$

2



(A)

$D_K = 2j - r_e$

$j = 8$

$r_e = 3$

$D_K = 2(8) - 3$

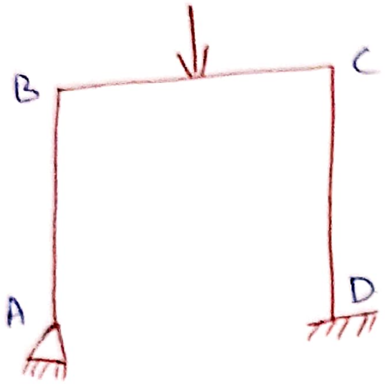
$D_K = 13$

Frame :-

Rigid jointed frame :- (Extensible members)

$$D_K = 3j - r_e$$

(1)



(A)

$D_K = 3j - r_e$

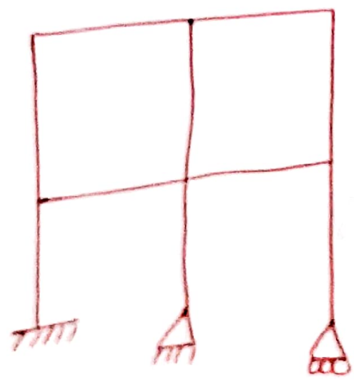
$j = 4$

$r_e = 5$

$D_K = 3(4) - 5$

$D_K = 7$

(2)



(A)

$D_K = 3j - r_e$

$j = 9$

$r_e = 6$

$D_K = 3(9) - 6$

$D_K = 21$

Rigid jointed frame :- (Inextensible members)

$D_k = 3j - r_e - n_r + r_r$

j = No. of joints

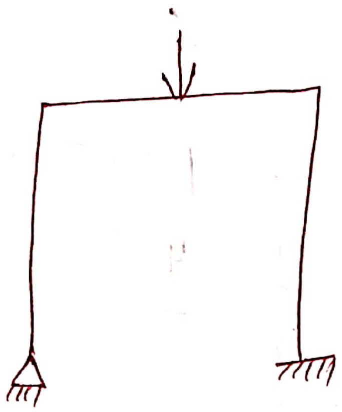
r_e = Support reactions

n_r = No. of inextensible members

r_r = Released reactions $= \Sigma(m-1)$

m → No. of members connected to the internal hinge.

(1)



(A) $D_k = 3j - r_e - n_r + r_r$

$j = 4$

$r_e = 5$

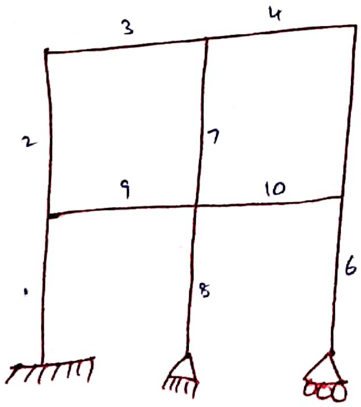
$n_r = 3$

$r_r = 0$

$D_k = 3(4) - 5 - 3 + 0$

$D_k = 4$

(2)



(A) $D_K = 3j - r_e - n_r + r_r$

$j = 9$

$r_e = 6$

$n_r = 10$

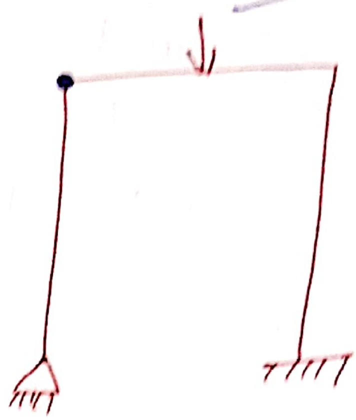
$r_r = 0$

$D_K = 3(9) - 6 - 10 + 0$

$D_K = 11$

27
16
11

(4)



(A) $D_K = 3j - r_e - n_r + r_r$

$j = 4$

$r_e = 5$

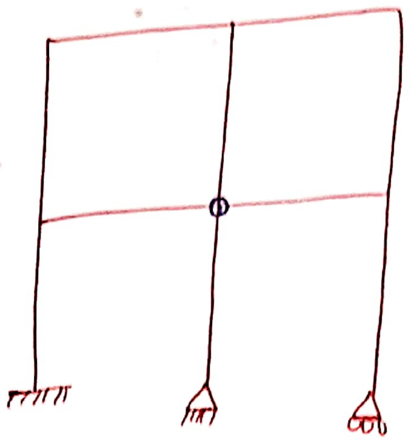
$n_r = 3$

$r_r = \Sigma(m-1) = 2-1 = 1$

$D_K = 3(4) - 5 - 3 + 1$

$D_K = 5$

(3)



(A) $D_K = 3j - r_e - n_r + r_r$

$j = 9$

$r_e = 6$

$n_r = 10$

$r_r = \Sigma(m-1) = 4-1 = 3$

$D_K = 3(9) - 6 - 10 + 3$

$D_K = 14$

30
16
14

13
08
5

Method of consistent deformation :-

Procedure :-

(7)

(1) Find D_s

(2) Remove the redundant reaction and find out the displacement (θ or Δ) in the direction of redundant direction.

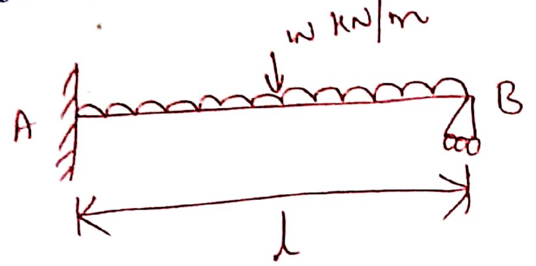
Force $\Rightarrow$ Deflection (Δ)

Moment $\Rightarrow$ slope (θ)

(3) Remove the applied loading and find out the displacement in the direction of the loading.

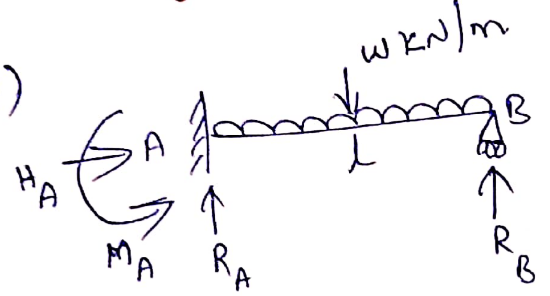
(4) obtain compatibility equation and find out the redundant reaction.

(1)



- Analyse the structure.

(A)



$D_s = D_{se} + D_{si}$

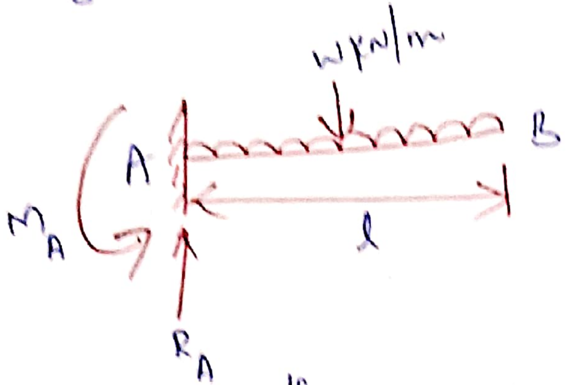
$D_{se} = r_e - E$
$= 3 - 2$
$= 1$

$D_{si} = 3C$
$= 3(0)$
$= 0$

$D_s = D_{se} + D_{si} = 1 + 0 = 1$

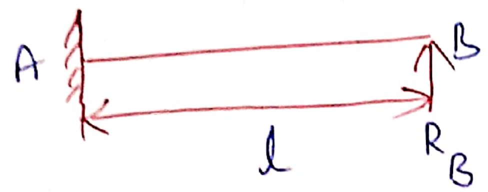
Let 'R_B' redundant

Remove 'R_B' and find 'Δ_{B1}'



$$\Delta_{B1} = \frac{wl^4}{8EI} \quad (\downarrow)_+$$

Remove the loading, apply redundant reaction and find the displacement (Δ_{B2}).



$$\Delta_{B2} = \frac{-Pl^3}{3EI}$$

$$\Delta_{B2} = \frac{-R_B \cdot l^3}{3EI} \quad (\uparrow)_-$$

obtain compatibility equation.

$$\Delta_B = 0$$

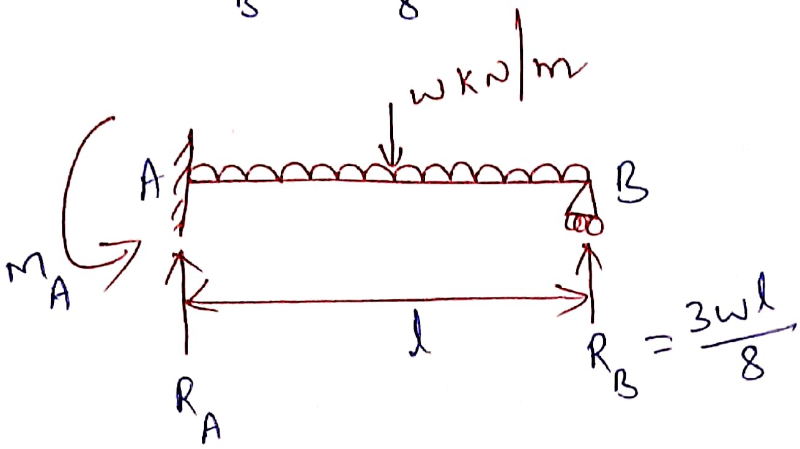
$$\Delta_{B1} + \Delta_{B2} = 0$$

$$\frac{wl^4}{8EI} + \left(\frac{-R_B \cdot l^3}{3EI}\right) = 0$$

$$\frac{wl^{4}}{8EI} = \frac{R_B \cdot l^{3}}{3EI}$$

(8)

$$R_B = \frac{3wl}{8}$$



$\Sigma V = 0;\ R_A + R_B - wl = 0$

$$R_A + R_B = wl \longrightarrow (1)$$

$$R_A + \frac{3wl}{8} = wl$$

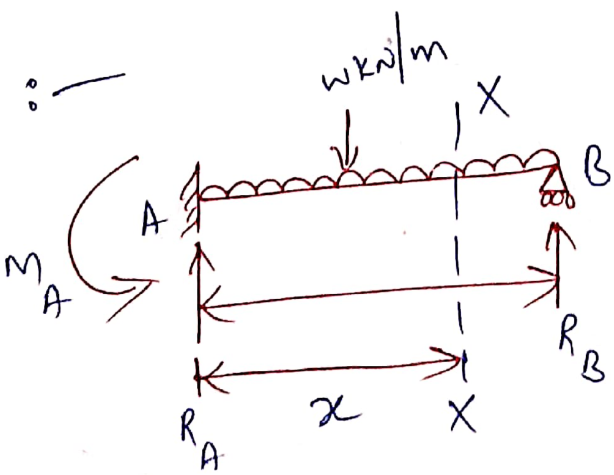
$$R_A = \frac{5wl}{8}$$

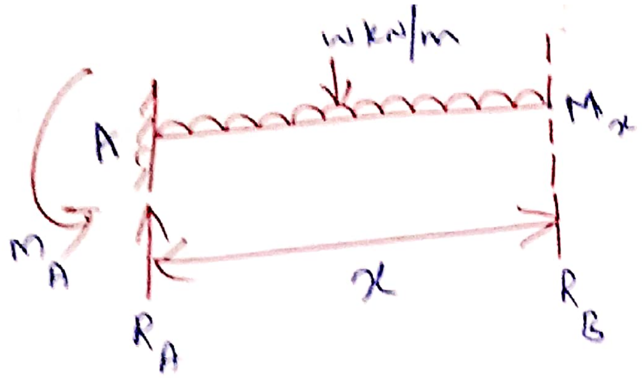
$$\Sigma M_A = 0;\ (-R_B \times l) + (w \cdot l) \cdot \frac{l}{2} - M_A = 0$$

$$\left(-\frac{3wl \times l}{8}\right) + \frac{wl^2}{2} - M_A = 0$$

$$M_A = \frac{wl^2}{8}$$

Analyse BMD :-





$$M_x = (R_A \times x) - M_A - (w \cdot x) \cdot \frac{x}{2}$$

$$M_x = \frac{5wl}{8} \times x - \frac{wl^2}{8} - \frac{wx^2}{2} \longrightarrow (2)$$

$$\frac{dM_x}{dx} = 0$$

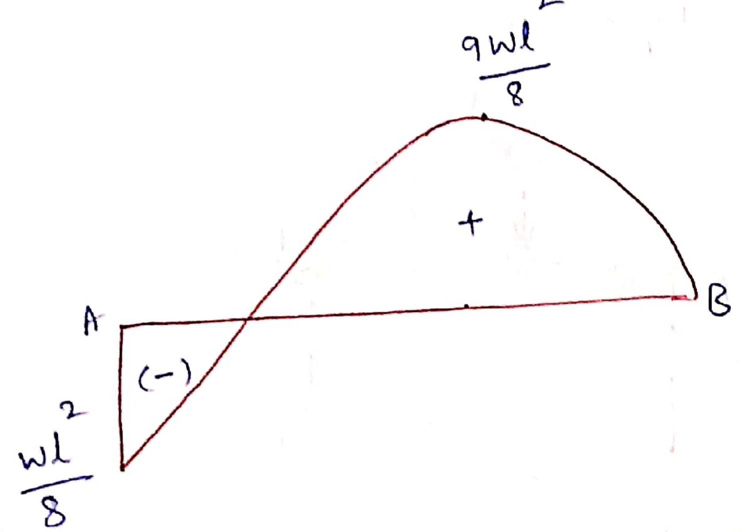
$$\frac{d}{dx}\left(\frac{5wl}{8} \cdot x - \frac{wl^2}{8} - \frac{wx^2}{2}\right) = 0$$

$$\frac{5wl}{8} - 0 - \frac{\cancel{2}wx}{\cancel{2}} = 0$$

$$x = \frac{5l}{8} \text{ (from Left)}$$

$$M_x = \frac{5wl}{8} \times \frac{5l}{8} - \frac{wl^2}{8} - \frac{w}{2} \cdot \left(\frac{5l}{8}\right)^2$$

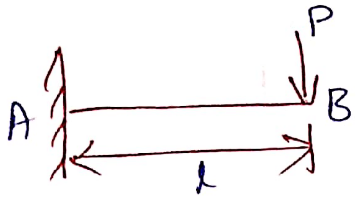
$$M_x = \frac{9wl^2}{128}$$



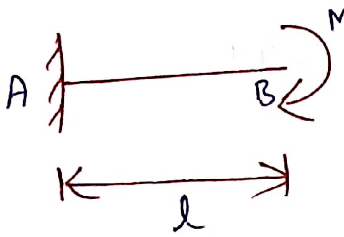
Standard Results of slope and Deflection :—

slope $\Rightarrow \theta$

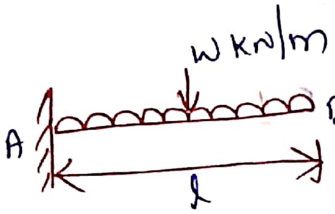
Deflection $\Rightarrow \delta$

(1)


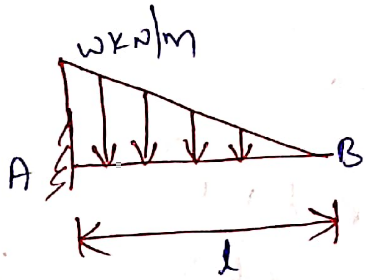
$$\theta_B = \frac{Pl^2}{2EI} \qquad \delta_B = \frac{Pl^3}{3EI}$$

(2)


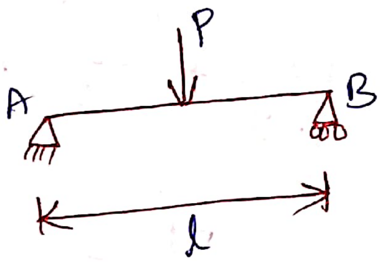
$$\theta_B = \frac{M \cdot l}{EI} \qquad \delta_B = \frac{Ml^2}{2EI}$$

(3)


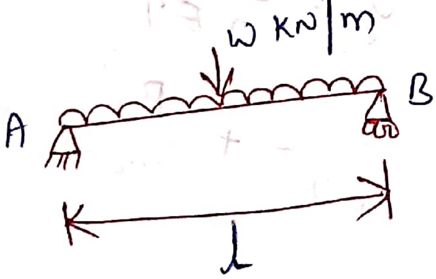
$$\theta_B = \frac{wl^3}{6EI} \qquad \delta_B = \frac{wl^4}{8EI}$$

(4)


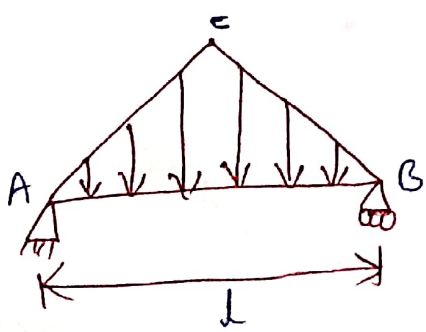
$$\theta_B = \frac{wl^3}{24EI} \qquad \delta_B = \frac{wl^4}{30EI}$$

(5)


$$\theta_A = \theta_B = \frac{Pl^2}{16EI} \qquad \delta_{max} = \frac{Pl^3}{48EI}$$

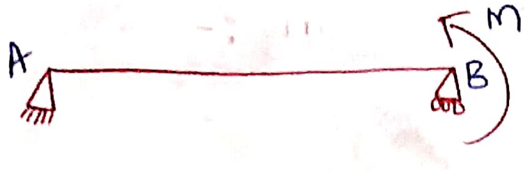
(6)


$$\theta_A = \theta_B = \frac{wl^3}{24EI} \qquad \delta_{max} = \frac{5wl^4}{384EI}$$

(7)


$$\theta_A = \theta_B = \frac{5wl^3}{192EI} \qquad \delta_c = \frac{wl^4}{120EI}$$

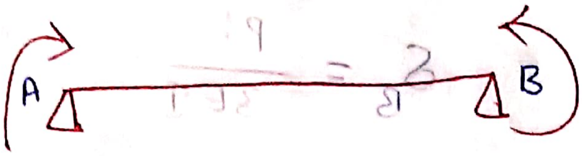
(8)



$$\theta_A = \frac{ml}{6EI}$$

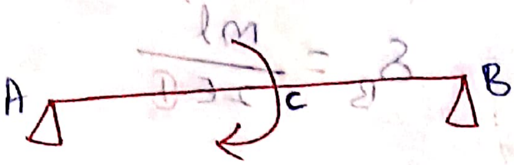
$$\theta_B = \frac{ml}{3EI}$$

(9)



$$\theta_A = \theta_B = \frac{ml}{2EI}$$

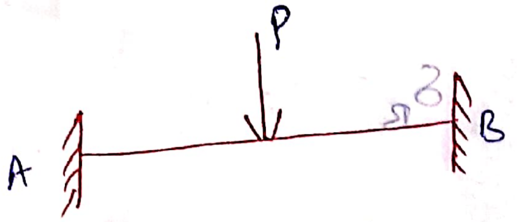
(10)



$$\theta_A = \theta_B = \frac{ml}{24EI}$$

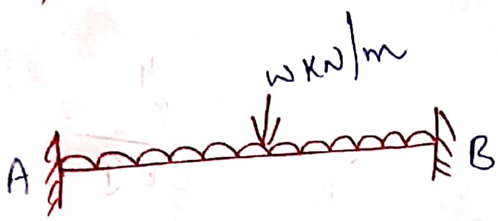
$$\theta_c = \frac{ml}{12EI}$$

(11)



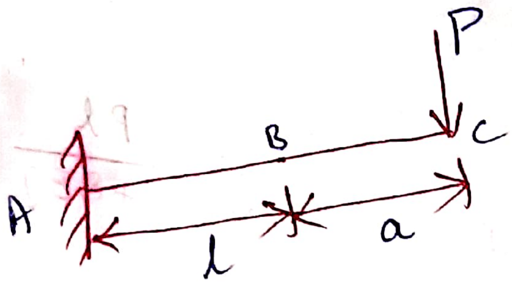
$$\delta = \frac{Pl^3}{192EI}$$

(12)



$$\delta = \frac{wl^4}{384EI}$$

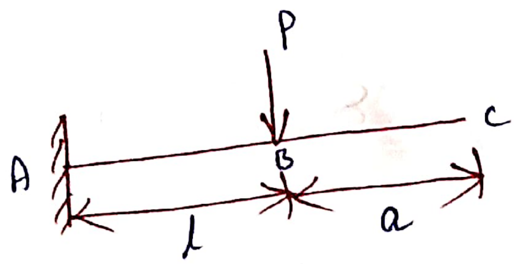
(13)



$$\delta_c = \frac{P(l+a)^3}{3EI}$$

$$\delta_B = \frac{Pl^3}{3EI} + \frac{(P.a)l^2}{2EI}$$

(14)



$$\theta_B = \theta_c = \frac{Pl^2}{2EI}$$

$$\delta_c = \frac{Pl^3}{3EI} + \frac{Pl^2}{2EI} \cdot a$$

UNIT - III

Propped cantilever & Fixed Beams

Introduction :—

Indeterminacy of structure :—

Static Indeterminacy (D_s)

Kinematic Indeterminacy (D_K)

Static :— To know the external and internal forces.

Kinematic :— To know the rotation and displacement.

Determinate structure :— Number of unknown is equal to the number of equation of equilibrium.

Ex:-

Reactions, R = 3

Equilibrium equations, E = 3

$\boxed{3 = 3}$

Indeterminate structure :— Number of unknown is greater than number of equation of equilibrium.

Ex:-

Reactions, R = 6

Equilibrium equations, E = 3

$\boxed{6 > 3}$

Degree of Static Indeterminacy :—

D_S = Total number of unknown − Total number of equation of equilibrium.

Ex:-

(Just stable / Just Rigid)

$D_S = 3 - 3 = 0$

Ex:-

(over stable / over Rigid)

$D_S = 6 - 3 = 3$

Ex:—

(unstable)

$D_S = 2 - 3 = -1$

'D_S', we can calculate to finding of stability of structure.

If D_S = −ve ⇛ unstable

If $D_S = 0$ ⇛ Just stable / Just Rigid

If $D_S > 0$ ⇛ over stable / over Rigid.

Degree of Static Indeterminacy (D_s) :-

For Beams

$$\boxed{D_s = D_{se} + D_{si}}$$

where: D_{se} = External static Indeterminacy

D_{si} = Internal " "

For Beams $D_{si} = 0$ (Because No. of closed loop $= 0$)

$D_{si} = M \times$ (No. of closed loop)

$$\boxed{D_{se} = R - E - A}$$

R = No. of unknown Reactions

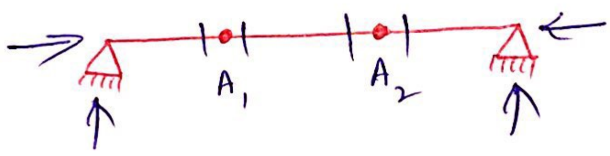
E = No. of equilibrium condition

A = No. of additional equilibrum (i.e. condition due to internal hinge)

$A = (M - 1)$ → Member

Problems :-

(1)



$D_{se} = R - E - A$

$R = 4$, $E = 3$, $A = A_1 + A_2$; $A_1 = 2-1 = 1$; $A_2 = 2-1 = 1$

$A = 1 + 1 = 2$

$D_{se} = 4 - 3 - 2 = -1$; $\therefore D_s = D_{se} + D_{si} = -1 + 0 = -1$

(2)

$R = 3$, $E = 3$, $A = 0$

$D_{se} = 3 - 3 - 0 = 0$.

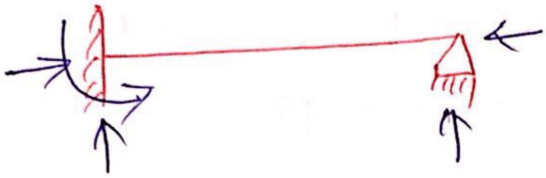
$\therefore D_s = D_{se} + D_{si} = 0 + 0 = 0$.

(3)

$R=3,\ E=3,\ A=M-1=2-1=1$

$D_{se}=3-3-1=-1\ ;\ \therefore D_s=D_{se}+D_{si}=-1+0=-1$

(4)

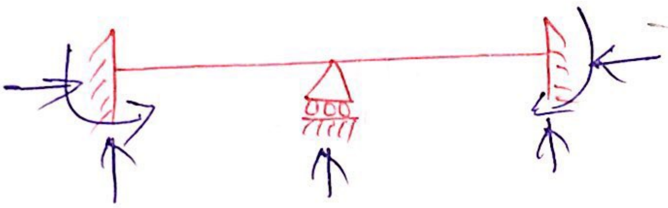


$R=5,\ E=3,\ A=0$

$D_{se}=5-3=2$

$\therefore D_s=D_{se}+D_{si}=2+0=2$

(5)



$R=7,\ E=3,\ A=0$

$D_{se}=7-3+0=4$

$\therefore D_s=D_{se}+D_{si}=4+0=4$

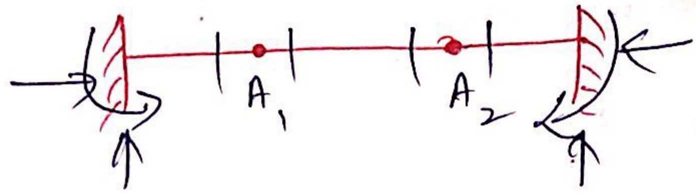
(6)

$R=6,\ E=3,\ A=0$

$D_{se}=6-3+0=3$

$\therefore D_s=D_{se}+D_{si}=3+0=3$

(7)



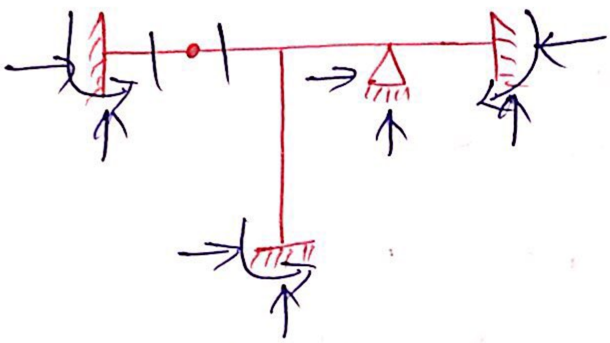
$R = 6,\ E = 3,\ A = A_1 + A_2;\quad A_1 = M - 1 = 2 - 1 = 1;\ A_2 = M - 1 = 2 - 1 = 1$

$A = 1 + 1 = 2$

$D_{se} = 6 - 3 - 2 = 1$

$\therefore D_s = D_{se} + D_{si} = 1 + 0 = 1$

(8)



$R = 11,\ E = 3,\ A = M - 1 = 2 - 1 = 1$

$D_{se} = 11 - 3 - 1 = 7$

$\therefore D_s = D_{se} + D_{si} = 7 + 0 = 7$

Degree of Kinematic Indeterminacy (D_K) :—

$$\boxed{D_K = 3j - R + A}$$

j = No. of joints

R = No. of Reactions

A = Eqll due to internal hinge. $(M-1)$.

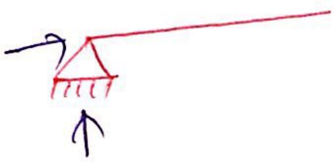
Problems :—

① 

$D_K = 3j - R + A$

$j = 1,\ R = 3,\ A = 0$

$D_K = 3(1) - 3 + 0 = 0.$

② 

$j = 1,\ R = 2,\ A = 0.$

$D_K = 3(1) - 2 + 0 = 1$

③ 

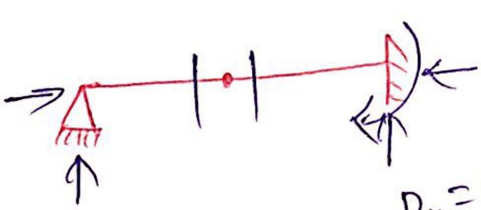
$j = 1,\ R = 1,\ A = 0$

$D_K = 3(1) - 1 + 0 = 2$

④ 

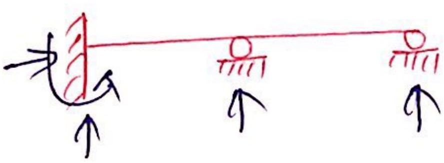
$j = 1,\ R = 0,\ A = (M-1) = (2-1) = 1$

$D_K = 3(1) - 0 + 1 = 4$

⑤ 

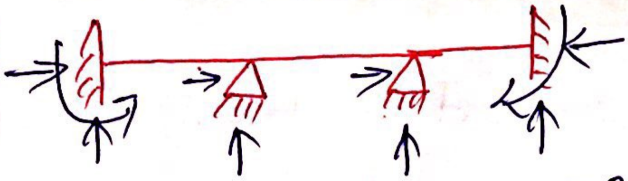
$j = 3,\ R = 5,\ A = (M-1) = (2-1) = 1$

$D_K = 3(3) - 5 + 1 = 5$

⑥ 

$j = 3,\ R = 5,\ A = 0$

$D_K = 3(3) - 5 + 0 = 4$

7 

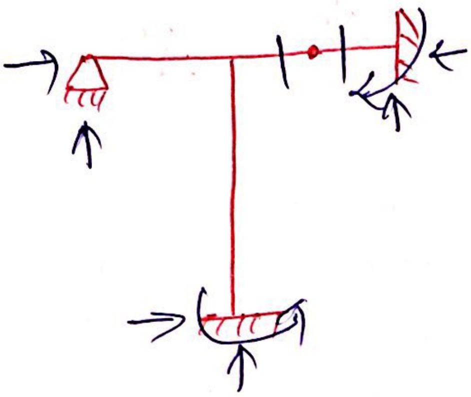
$j = 4, R = 10, A = 0$

$D_K = 3(4) - 10 + 0 = 2$

8 

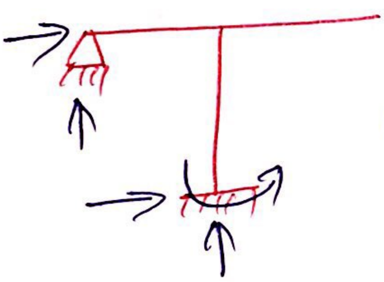
$j = 3, R = 7, A = 0$

$D_K = 3(3) - 7 + 0 = 2$

9 

$j = 5, R = 8, A = (m-1) = (2-1) = 1$

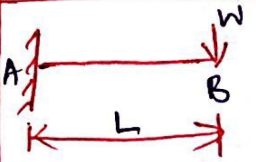
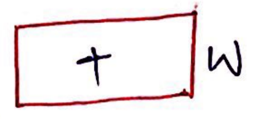
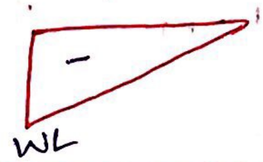
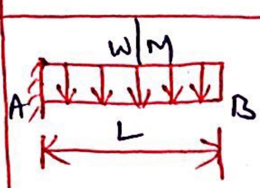
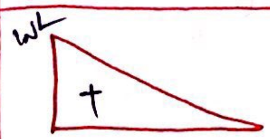
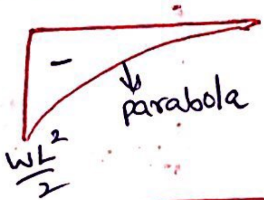
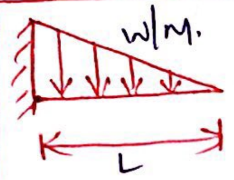
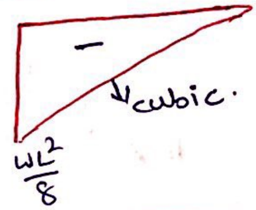
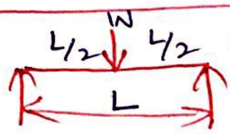
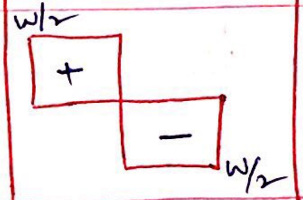
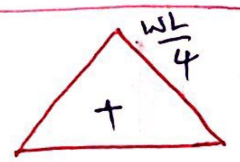
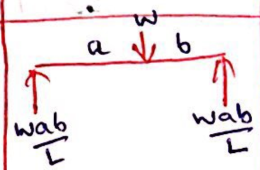
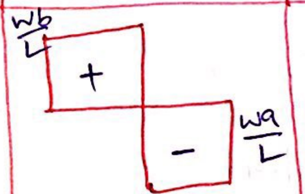
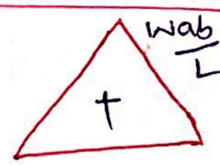
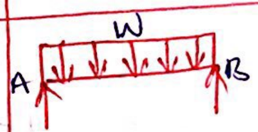
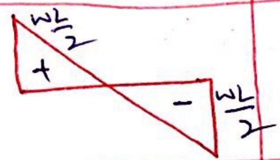
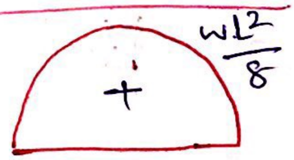
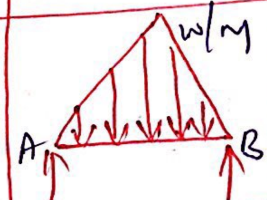
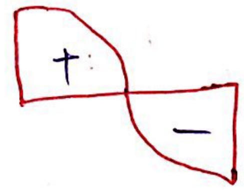
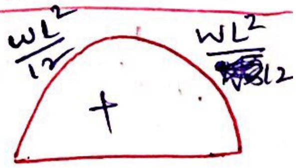
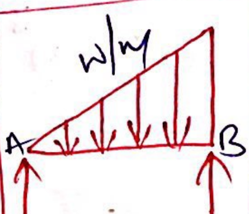
$D_K = 3(5) - 8 + 1 = 8$

10 

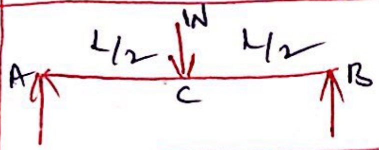
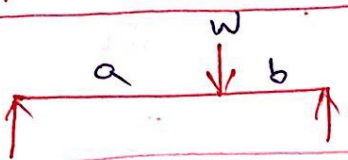
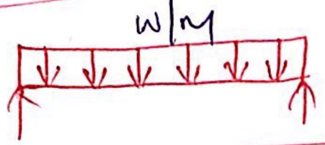
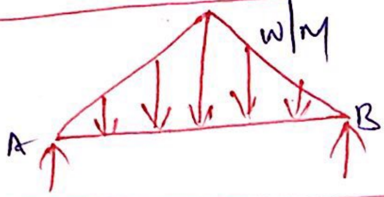
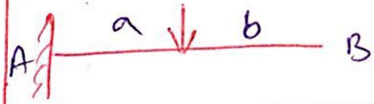
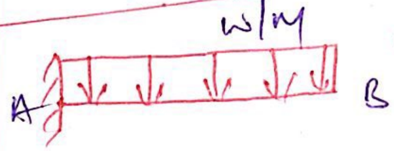
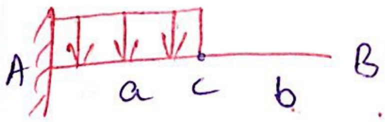
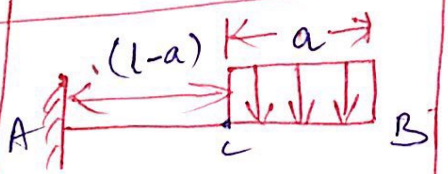
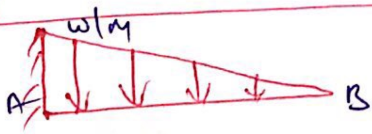
$j = 3, R = 5, A = 0$

$D_K = 3(3) - 5 + 0 = 4$

Expression for shear force and Bending moments for Some standard cases of Beams:

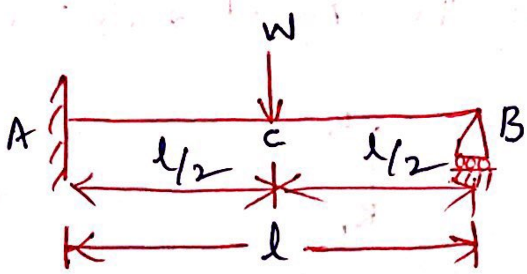
S.No.	Beam with external loading	Max. BM	Max. SF	Shape of SFD	Shape of BMD
1		WL	W		
2		$\frac{wL^2}{2}$	wL		
3		$\frac{wL^2}{8}$	$\frac{wL}{2}$		
4		$\frac{WL}{4}$	$\frac{W}{2}$		
5		$\frac{Wab}{L}$	$\frac{Wb}{L}$ @ A $\frac{Wa}{L}$ @ B		
6		$\frac{wL^2}{8}$	$\frac{wL}{2}$		
7		$\frac{wL^2}{12}$	$\frac{wL}{4}$		
8		$\frac{wL^2}{9\sqrt{3}}$	$\frac{wL}{6}$ @ A $\frac{wL}{3}$ @ B		

Expression for slope & deflection for some standard cases of beams :—

S.No	Beam with loading	Slope at support (θ)	Deflection (y)
1		$\frac{Wl^2}{16EI}$	$\frac{Wl^2}{48EI}$
2		$\frac{Wab}{6EIl}(a+2b)$	$\frac{Wa^2b^2}{3EIL}$
3		$\frac{wl^3}{24EI}$	$\frac{5wl^4}{384EI}$
4		$\frac{5wl^3}{192EI}$	$\frac{wl^4}{192EI}$
5		$\frac{Wl^2}{2EI}$	$\frac{Wl^3}{3EI}$
6		$\frac{Wa^2}{2EI}$	$\frac{Wa^3}{3EI} + \frac{Wa^2}{2EI}(l-a)$
7		$\frac{wl^3}{6EI}$	$\frac{wl^4}{8EI}$
8		$\frac{wa^3}{6EI}$	$\frac{wa^4}{8EI} + \frac{wa^3}{6EI}(l-a)$
9		$\frac{wl^3}{6EI} - w(l-a)^3$	$\frac{wl^4}{8EI} - \left[\frac{w(l-a)^4}{8EI} + \frac{w(l-a)^3}{6EI}\right]$
10			$\frac{wl^4}{30EI}$

Propped cantilever Beam subjected to point load at the center :—

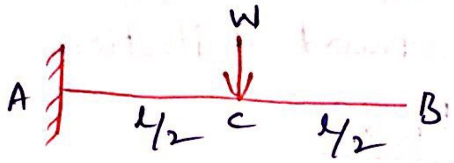
(A)



→ consider a cantilever beam AB, fixed at A, free at B and also prop at B.

→ point load acting at center of beam i.e. w(load) and A to B span is 'l'.

→ Ignore the effect of prop and calculate the downward deflection at the free end of the cantilever.



Here: From A to C ; $a = l/2$

From B to C ; $b = l/2$

The downward deflection at the free end is given by

$$y = \frac{wa^3}{3EI} + \frac{wa^2}{2EI}(b)$$

$$= \frac{w(l/2)^3}{3EI} + \frac{w(l/2)^2}{2EI}(l/2)$$

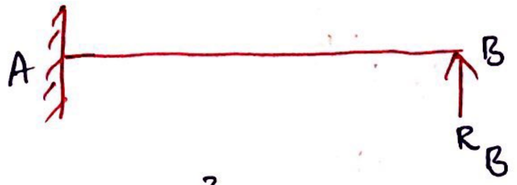
$$= \frac{wl^3}{24EI} + \frac{wl^2}{8EI}\cdot(l/2)$$

$$= \frac{wl^3}{24EI} + \frac{wl^3}{16EI}$$

$$= \frac{1}{8}\left(\frac{wl^3}{3EI} + \frac{wl^3}{2EI}\right)$$

$$\boxed{y = \frac{5wl^3}{48EI}} \rightarrow (1)$$

→ Remove the external load and apply vertical force of R_B at the free end and determine the upward deflection due to R_B:



$$y = \frac{Wl^3}{3EI}$$

where: $W = R_B$

$$\boxed{y = \frac{R_B \cdot l^3}{3EI}} \rightarrow (2)$$

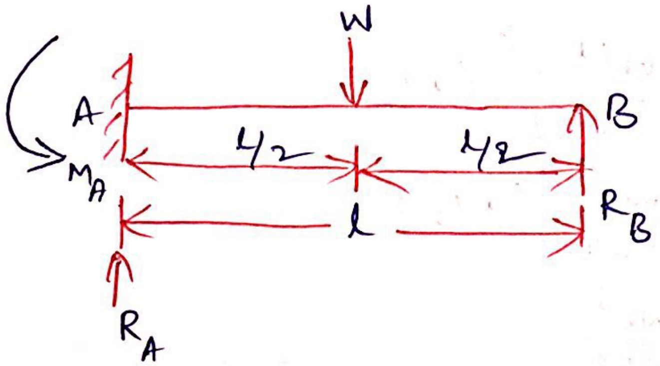
→ calculate the prop reaction 'R_B' by equating (1) & (2)

i.e. Downward deflection = upward deflection

$$\frac{5W\cancel{l^3}}{48\cancel{EI}} = \frac{R_B \cdot \cancel{l^3}}{3\cancel{EI}}$$

$$\boxed{R_B = \frac{5W}{16}}$$

Deflection of reactions at fixed support :-

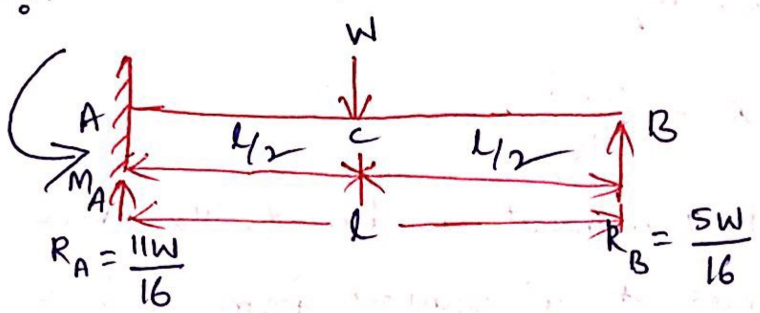


$\Sigma V = 0;$ $\quad R_A + R_B - W = 0$

$$R_A + \frac{5W}{16} = W$$

$$\boxed{R_A = \frac{11W}{16}}$$

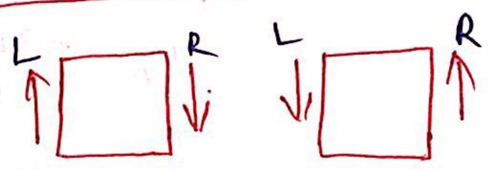
$M_A = ?$



$\Sigma M_A = 0$; $W \times \frac{l}{2} - R_B \times l - M_A = 0$

$\frac{Wl}{2} - \frac{5W}{16} \times l - M_A = 0$

$$\boxed{M_A = \frac{3Wl}{16}}$$



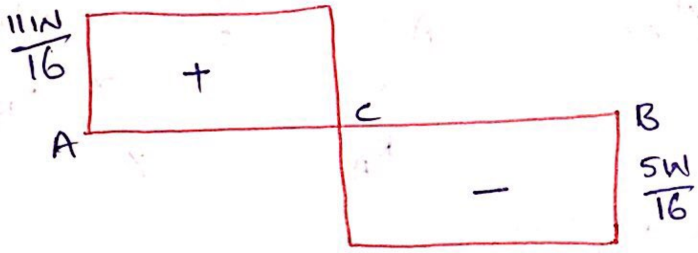
L to R:
upward (+ve)
Downward (-ve)

R to L:
upward (-ve)
Downward (+ve)

Shear force Diagram :—

$(SF)_{AC} = R_A$ (constant from A to C) $= \frac{11W}{16}$

$(SF)_{BC} = -R_B$ (constant from B to C) $= -\frac{5W}{16}$



$\frac{11W}{16} - W$

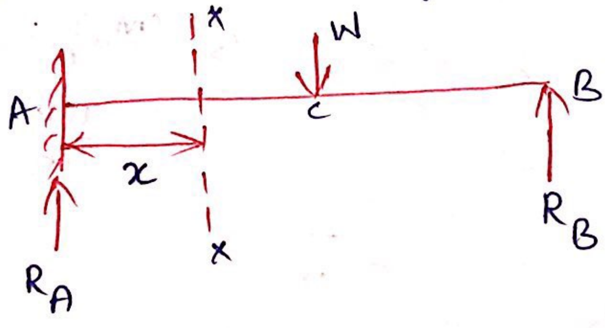
$\frac{11W - 16W}{16}$

$\frac{-5W}{16}$

Bending Moment Diagram :—

Consider any section at a distance 'x' from support 'A'.

The bending moment at section is given by ;

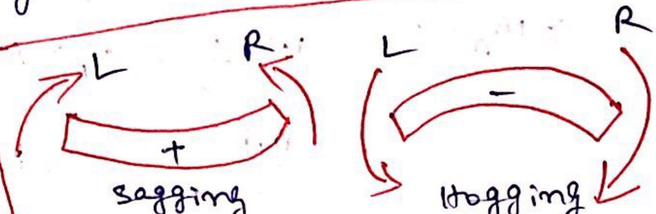


$$(M_x)_{AC} = R_A \times x - M_A$$

$$(M_x)_{AC} = \frac{11W}{16} x - \frac{3Wl}{16} \rightarrow (3)$$

variation of bending moment is linear. The above eqn is varried for all values of 'x' ranging from A to C.

i.e. 'x' ranges from 0 to $l/2$



L to R :
C.W. (+ve)
A.C.W. (−ve)

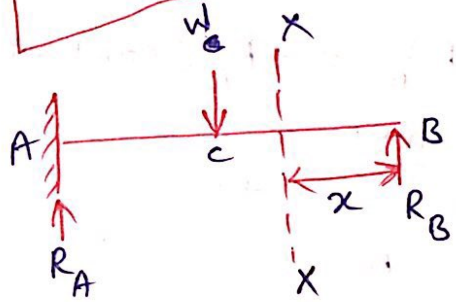
R to L :
C.W (−ve)
A.C.W (+ve)

At $x = 0$; $M_A = \frac{11W}{16}(0) - \frac{3Wl}{16}$

$$\boxed{M_A = -\frac{3Wl}{16}}$$

At $x = l/2$; $M_C = \frac{11W}{16}(l/2) - \frac{3Wl}{16}$

$$\boxed{M_C = \frac{5Wl}{32}}$$



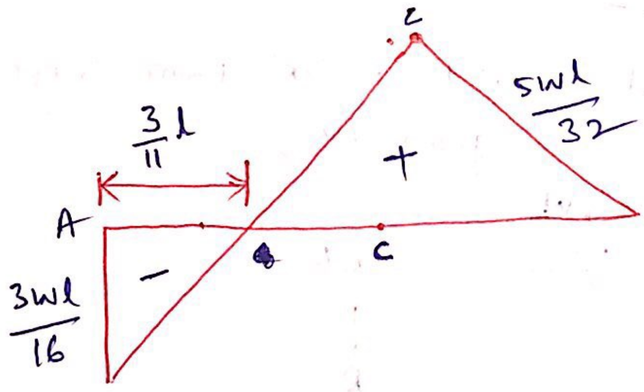
Consider any section from B to C.

$$(M_x)_{BC} = +R_B \times x = +\left(\frac{+5W}{16} \times x\right)$$

$$(M_x)_{BC} = \frac{5Wx}{16} \rightarrow (4)$$

At $x = 0$; $M_B = \frac{5W(0)}{16} \Rightarrow \boxed{M_B = 0}$

At $x = l/2$; $M_C = \frac{5W(l/2)}{16} \Rightarrow \boxed{M_C = \frac{5Wl}{32}}$



Location of point of contraflexure :—

from eqn (3)

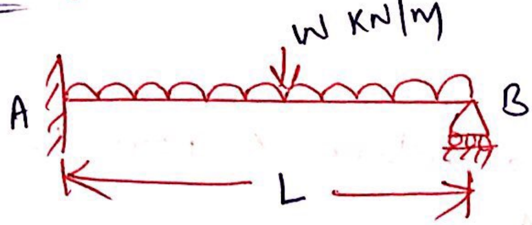
$$(M_x)_{AC} = \frac{11W}{16} x - \frac{3Wl}{16} = 0$$

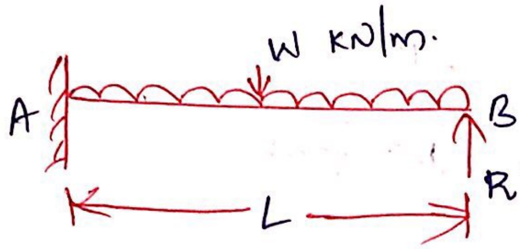
$$\frac{11\cancel{W}}{\cancel{16}} x = \frac{3\cancel{W}l}{\cancel{16}}$$

$$\boxed{x = \frac{3}{11} l}$$

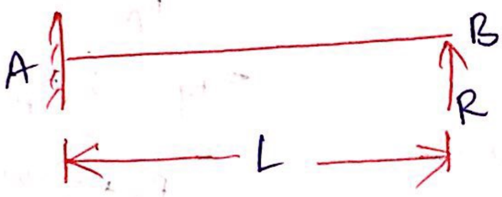
Propped cantilever beam subjected to uniformly distributed Load :- (8)

(A)





(1) ~~Down~~ Upward deflection at point 'B' due to 'R' (upward) without considering UDL.



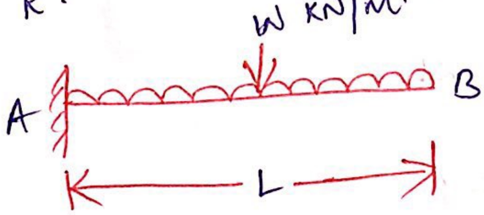
($\because R = W$)

($W = R$)

$$y = \frac{WL^3}{3EI}$$

$$y = \frac{RL^3}{3EI} \rightarrow (1)$$

(2) Downward deflection at point 'B' due to UDL without considering 'R'.



$$y = \frac{wL^4}{8EI} \rightarrow (2)$$

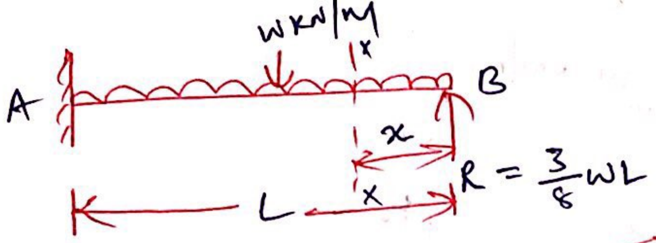
$$(1) = (2)$$

$$\frac{R\cancel{L^3}}{3\cancel{EI}} = \frac{wL^{\cancel{4}}}{8\cancel{EI}}$$

$$\boxed{R = \frac{3}{8} WL}$$

Shear force Diagram :—

Consider a section x-x at a distance 'x' from 'B'



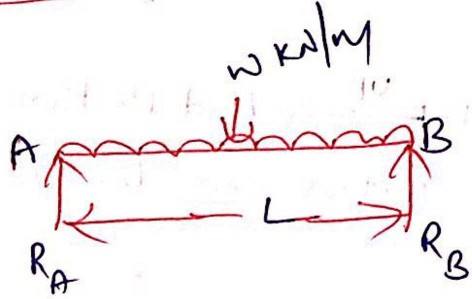
$(SF)_{x-x} = -R + W \cdot x$

$(SF)_{x-x} = \frac{-3}{8} WL + Wx \longrightarrow (3)$

At $x=0$; $\boxed{(SF)_B = \frac{-3}{8} WL}$

At $x = L$; $(SF)_A = \frac{-3}{8} WL + W \cdot L$

$$\boxed{(SF)_A = \frac{5}{8} WL}$$



$\Sigma f_y = 0$; $R_A + R_B - WL = 0$

$R_A + \frac{3}{8} WL - WL = 0.$

$R_A = WL - \frac{3}{8} WL$

$R_A = \frac{8WL - 3WL}{8}$

$$\boxed{R_A = \frac{5WL}{8}}$$

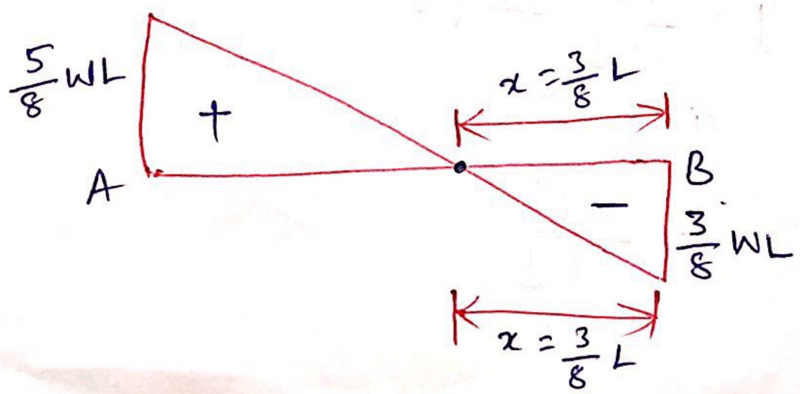
Zero shear force ~~at~~ from 'B'.

From eqn (3); $(SF)_0 = \frac{-3}{8} WL + W \cdot x$

$0 = \frac{-3}{8} WL + Wx$

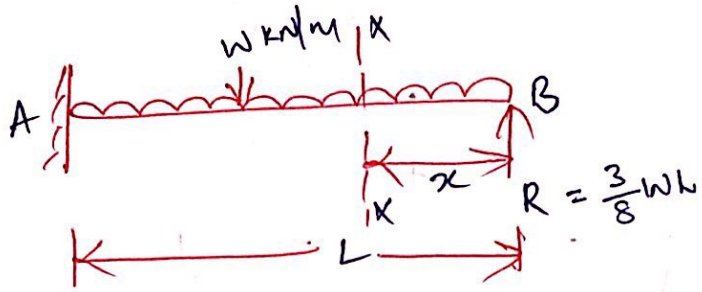
$\cancel{W}x = \frac{3}{8} \cancel{W}L$

$$\boxed{x = \frac{3}{8} L}$$



Bending moment Diagram :—

consider a section X-X at a distance 'x' from 'B'.



$$(BM)_{X-X} = R \cdot x - w \cdot x \cdot \frac{x}{2}$$

$$(BM)_{X-X} = Rx - \frac{wx^2}{2} \rightarrow (4)$$

At $x = 0$; $(BM)_B = R(0) - \frac{w(0)^2}{2}$

$$\boxed{(BM)_B = 0}$$

At $x = L$; $(BM)_A = R \cdot (L) - \frac{w(L)^2}{2}$

$$= \frac{3}{8}wL^2 - \frac{wL^2}{2}$$

$$\boxed{(BM)_A = \frac{-wL^2}{8}}$$

Maximum Bending Moment :—

sub. $x = \frac{3}{8}L$ in eqn (4)

$$(BM)_{max} = Rx - \frac{wx^2}{2}$$

$$= \frac{3}{8}wL(x) - \frac{wx^2}{2}$$

$$= \frac{3}{8}wL\left(\frac{3}{8}L\right) - \frac{w}{2}\left(\frac{3}{8} \cdot L\right)^2$$

$$\boxed{(BM)_{max} = \frac{9}{128}wL^2}$$

Point of contraflexure :—

From eqn (4)

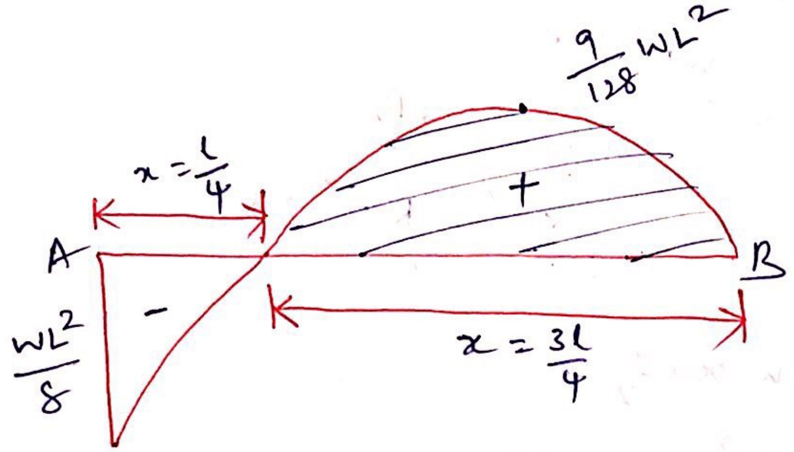
$$Rx - \frac{wx^2}{2} = 0$$

$$x \cdot \frac{3}{8}wl - \frac{wx^2}{2} = 0$$

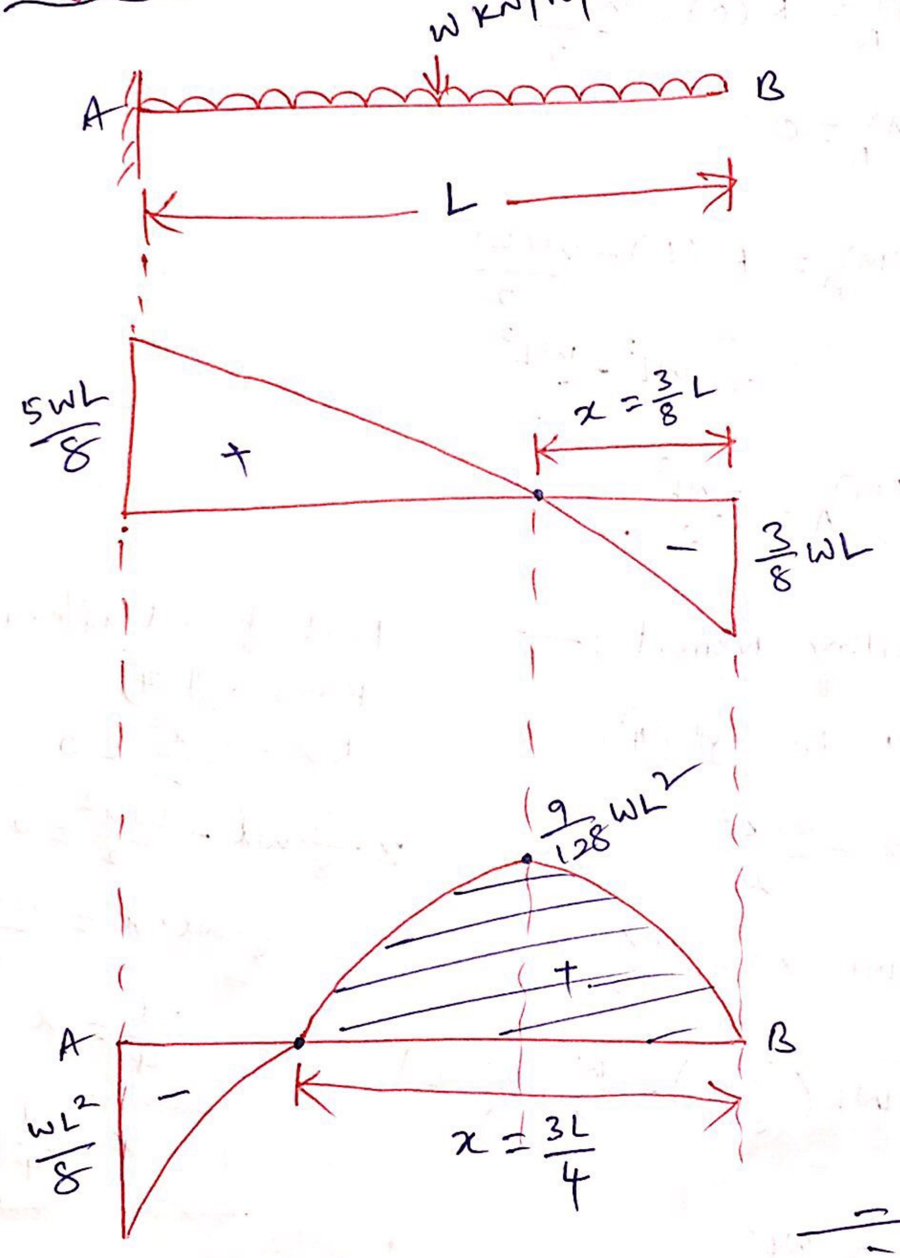
$$\frac{3}{\cancel{8}_4}\cancel{w}l \cdot \cancel{x} = \frac{\cancel{w}x^{\cancel{2}}}{\cancel{2}}$$

$$\frac{3l}{4} = x$$

$\boxed{\therefore x = \frac{3l}{4}}$ from 'B'.

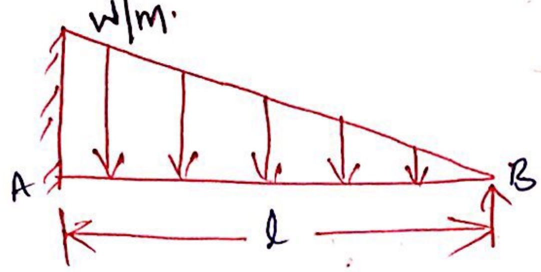


Final Diagram :-

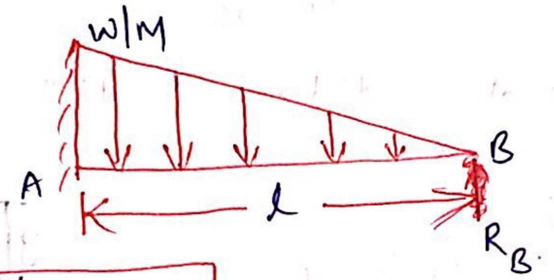


Analysis of propped cantilever beam carrying uniformly varying load zero at prop support and maximum at fixed support:

(A)

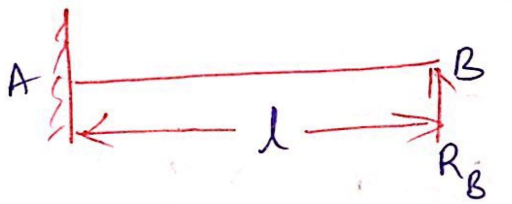


(1) Downward deflection i.e. Remove prop and apply external load.



$$\boxed{y = \frac{wl^4}{30EI}} \longrightarrow (1)$$

(2) upward deflection i.e. Remove external load and apply prop.



$\because R_B = W$

$W = R_B$

$$y = \frac{Wl^3}{3EI}$$

$$\boxed{y = \frac{R_B \cdot l^3}{3EI}} \longrightarrow (2)$$

(1) = (2)

Downward deflection = upward deflection.

$$\frac{wl^{4}}{\cancel{30}_{10}\cancel{EI}} = \frac{R_B \cdot \cancel{l^3}}{\cancel{3}\cancel{EI}}$$

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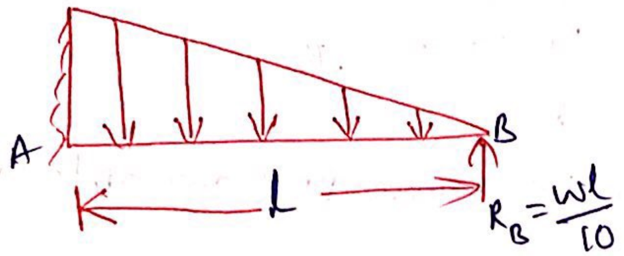
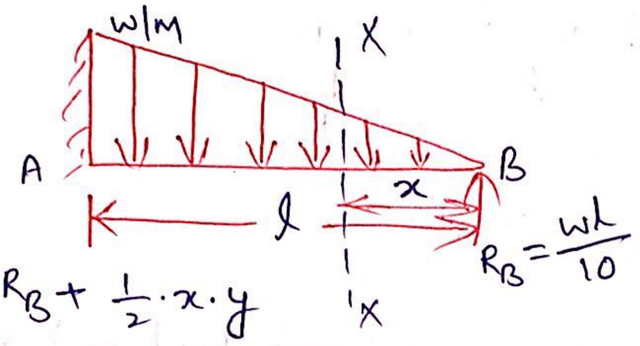
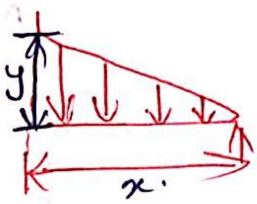
$$\boxed{\therefore R_B = \frac{wl}{10}}$$

shear force diagram :-



consider a section x-x at a distance 'x' from support 'B'.





$$\frac{w}{l} = \frac{y}{x}$$

$$y = \frac{wx}{l}.$$

$$(SF)_{x-x} = -R_B + \frac{1}{2} \cdot x \cdot y$$

$$(SF)_{x-x} = -R_B + \frac{1}{2} \cdot x \cdot \frac{wx}{l}.$$

$$(SF)_{x-x} = \frac{-wl}{10} + \frac{wx^2}{2l} \rightarrow (3)$$

Limits 0 to l (i.e. $x=0$ & $x=l$)

$$@\ x=0;\ (SF)_B = \frac{-wl}{10} + \frac{w(0)^2}{2l}$$

$$\boxed{(SF)_B = \frac{-wl}{10}}$$

$$@\ x=l;\ (SF)_A = \frac{-wl}{10} + \frac{w(l)^{\cancel{2}}}{2\cancel{l}} = \frac{-wl}{10} + \frac{wl}{2} = \frac{-wl+5wl}{10}$$

$$= \frac{\cancel{4}^{2}wl}{\cancel{10}_{5}} = \frac{2wl}{5}$$

$$\boxed{(SF)_A = \frac{2wl}{5}}$$

Zero shear force :— (from support B)

from equation (3) $(SF)_{x-x} = 0$

$$-\frac{wl}{10} + \frac{wx^2}{2l} = 0$$

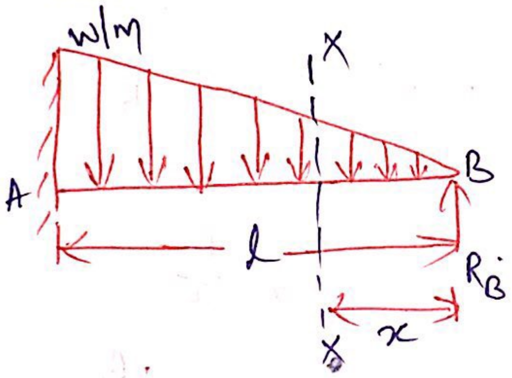
$$\frac{\cancel{w}x^2}{\cancel{2}l} = \frac{\cancel{w}l}{\cancel{10}\,5}$$

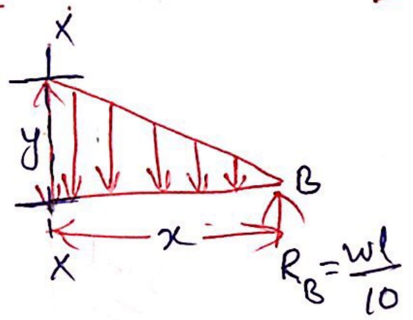
$$x^2 = \frac{l^2}{5}$$

$$\boxed{x = \frac{l}{\sqrt{5}}}$$ from 'B'.

Bending moment diagram :—

Consider a section X–X at a distance 'x' from 'B'.





$$\frac{w}{l} = \frac{y}{x}$$

$$y = \frac{wx}{l}$$

$$(BM)_{x-x} = R_B \times x - \frac{1}{2} \times x \times y \times \frac{x}{3}$$

$$= \frac{wl}{10} \times x - \frac{1}{2} \times x \times \frac{wx}{l} \times \frac{x}{3}$$

$$(BM)_{x-x} = \frac{wlx}{10} - \frac{wx^3}{6l} \rightarrow (4)$$

Limits 0 to l (i.e. $x = 0$ & $x = l$)

@ $x = 0$; $(BM)_B = \frac{wl(0)}{10} - \frac{w(0)^3}{6l}$

$$\boxed{(BM)_B = 0}$$

$$@\ x = l;\ (BM)_A = \frac{wl(l)}{10} - \frac{w(l)^3}{6l}$$

$$\boxed{(BM)_A = \frac{-wl^2}{15}}$$

Maximum Bending moment :—

$x = \frac{l}{\sqrt{5}}$ sub. in eqn (4)

$$(BM)_{max} = \frac{wlx}{10} - \frac{wx^3}{6l}$$

$$= \frac{wl\left(\frac{l}{\sqrt{5}}\right)}{10} - \frac{w\left(\frac{l}{\sqrt{5}}\right)^3}{6l}$$

$$\boxed{(BM)_{max} = \frac{wl^2}{15\sqrt{5}}}$$

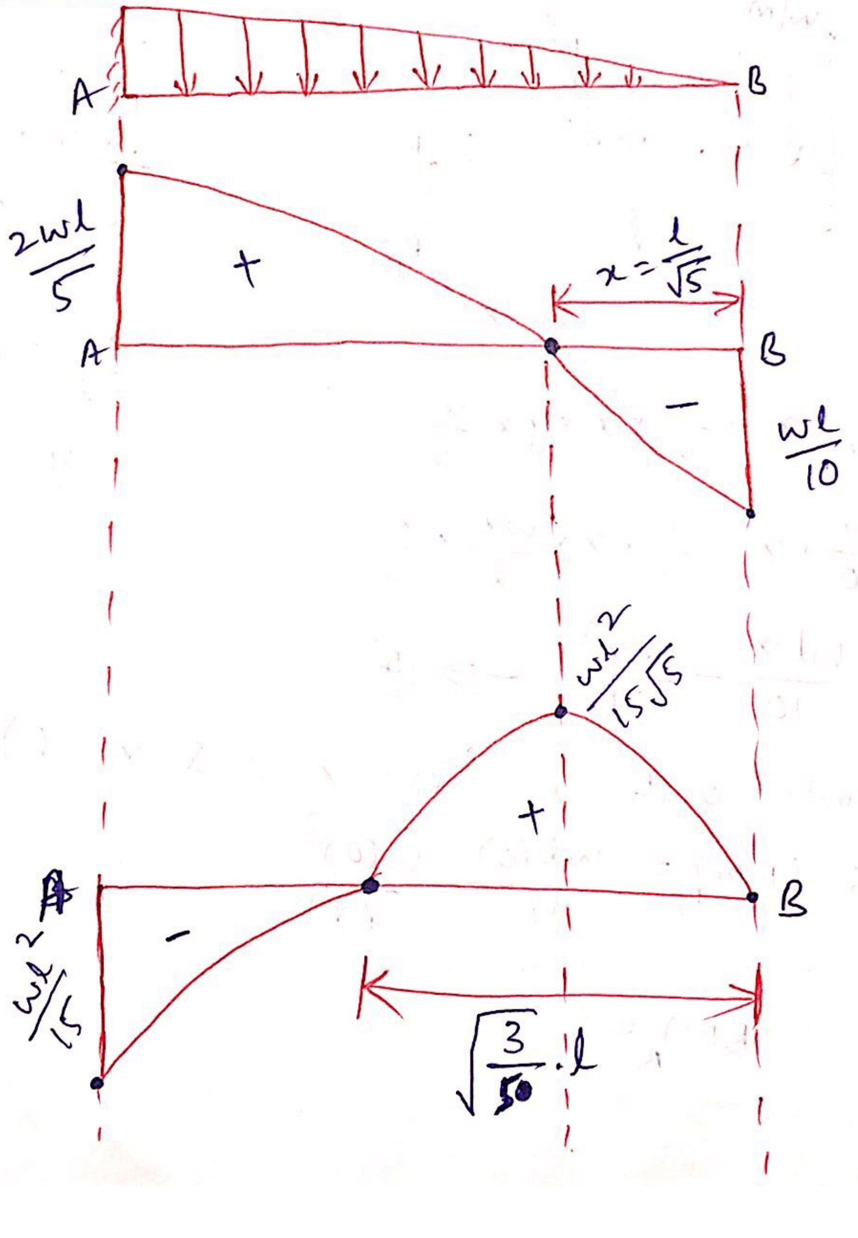
Location of point of contraflexure :—

From eqn (4) = 0.

$$\frac{wlx}{10} - \frac{wx^3}{6l} = 0.$$

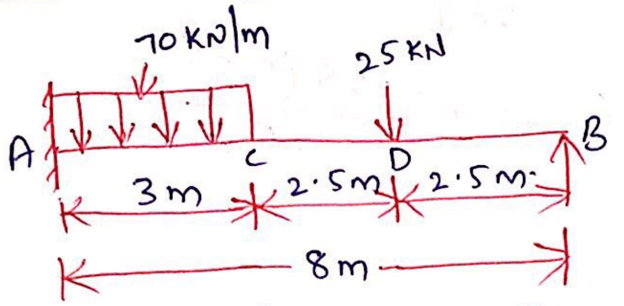
$$\frac{wlx}{10} = \frac{wx^3}{6l}$$

$$\boxed{x = \sqrt{\frac{3}{5}} \cdot l}$$



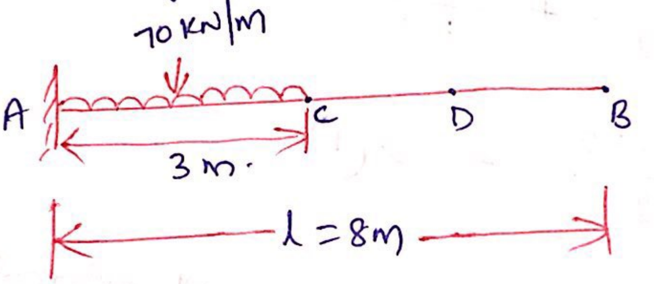
Problem on combination of UDL and point load :—

1



From above diagram, calculate prop reaction, shear force diagram, bending moment diagram and point of contraflexure.

(A) consider UDL acting on the cantilever beam. (Downward deflection)



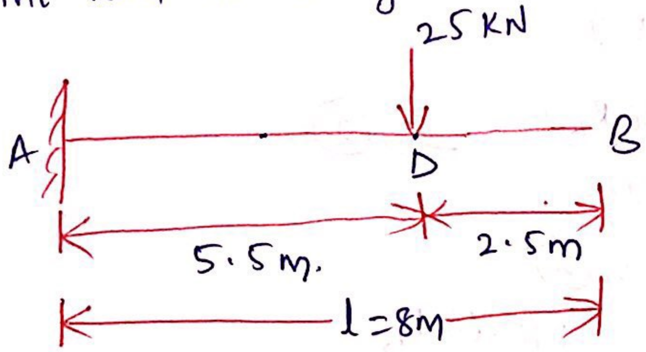
$$y_{B1} = \frac{wa^4}{8EI} + (l-a)\cdot\frac{wa^3}{6EI}$$

where: $a = 3m$, $l = 8m$, $(l-a) = (8-3) = 5m$, $w = 70\ KN/m$.

$$y_{B1} = \frac{70 \times (3)^4}{8EI} + (8-3)\cdot\frac{70 \times (3)^3}{6EI}$$

$$\boxed{y_{B1} = \frac{2283.75}{EI}}$$

consider point load is acting (Downward deflection)



where: $a = 5.5m$, $l = 8m$, $(l-a) = (8-5.5) = 2.5m$.

$W = 25\ KN$.

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$$y_{B2} = \frac{Wa^3}{3EI} + (l-a)\frac{Wa^2}{2EI}$$

$$= \frac{25(5.5)^3}{3EI} + (8-5.5)\cdot\frac{25(5.5)^2}{2EI}$$

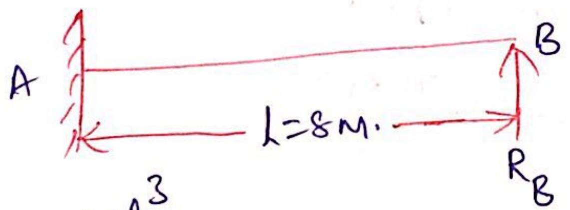
$$\boxed{y_{B2} = \frac{2331.77}{EI}}$$

where: $y_B = y_{B1} + y_{B2}$

$$y_B = \frac{2283.75}{EI} + \frac{2331.77}{EI}$$

$$\boxed{y_B = \frac{4615.52}{EI}} \longrightarrow (1)$$

calculate upward deflection i.e. remove all external loads and apply reaction at point B.



$(\because W = R_B)$

$$y = \frac{Wl^3}{3EI}$$

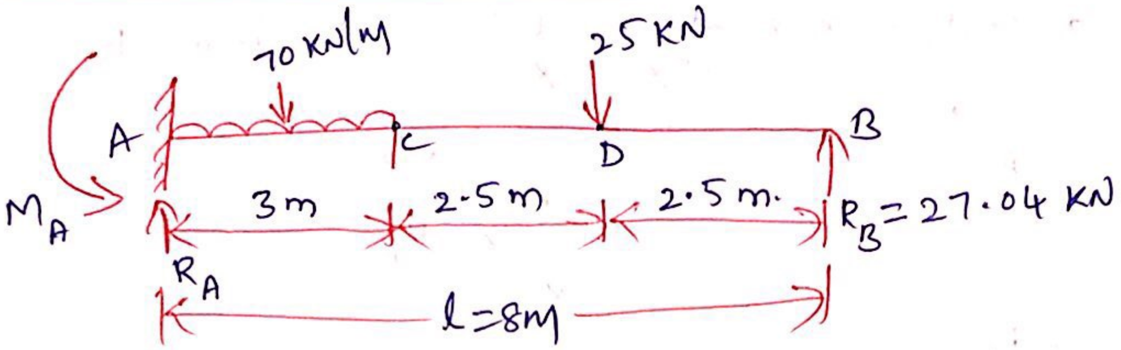
$$= \frac{R_B \times (8)^3}{3EI}$$

$$\boxed{y = \frac{170.66 R_B}{EI}} \longrightarrow (2)$$

$$(1) = (2)$$

$$\frac{4615.52}{\cancel{EI}} = \frac{170.66 R_B}{\cancel{EI}}$$

$$\boxed{R_B = 27.04 \text{ KN.}}$$



$\Sigma V = 0;\ R_A + R_B - 70 \times 3 - 25 = 0$

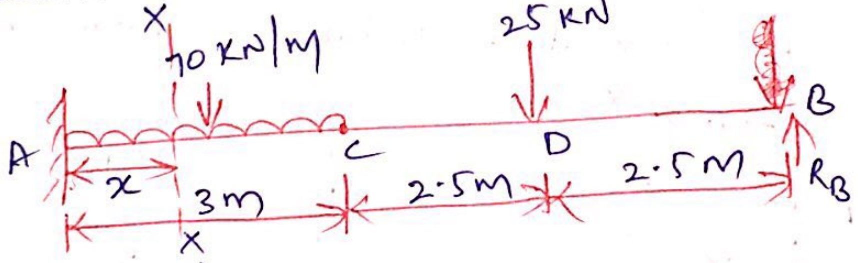
$$\boxed{R_A = 207.96 \text{ KN}}$$

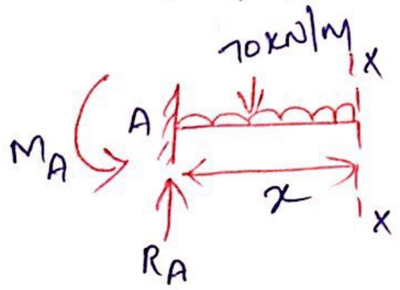
$\Sigma M_A = 0;\ -M_A + 70 \times 3 \times \frac{3}{2} + 25 \times 5.5 - (R_B \times 8) = 0$

$$\boxed{M_A = 236.18 \text{ KN.m.}}$$

Shear force diagram :— (AC) :-

consider a section X-X at a distance 'x' from support 'A'.





$(SF)_{X-X} = R_A - 70 \times x$

$(SF)_{X-X} = 207.96 - 70x \rightarrow$ (3)

@ $x = 0$; $\boxed{(SF)_A = 207.96 \text{KN.}}$

@ $x = 3M$; $(SF)_c = 207.96 - 70(3)$

$$\boxed{(SF)_c = -2.04 \text{ KN.}}$$

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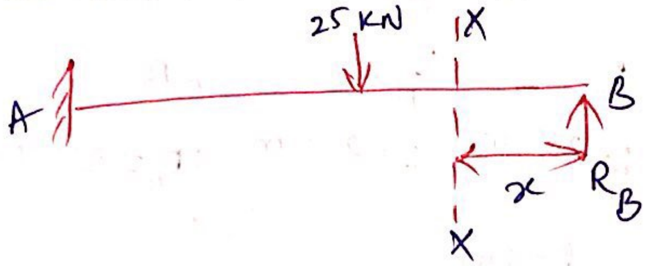
consider a section X-X at a distance 'x' from 'B'.



$(SF)_B = -R_B$

$$\boxed{(SF)_B = -27.04\ KN.}$$

Location of zero shear force :—

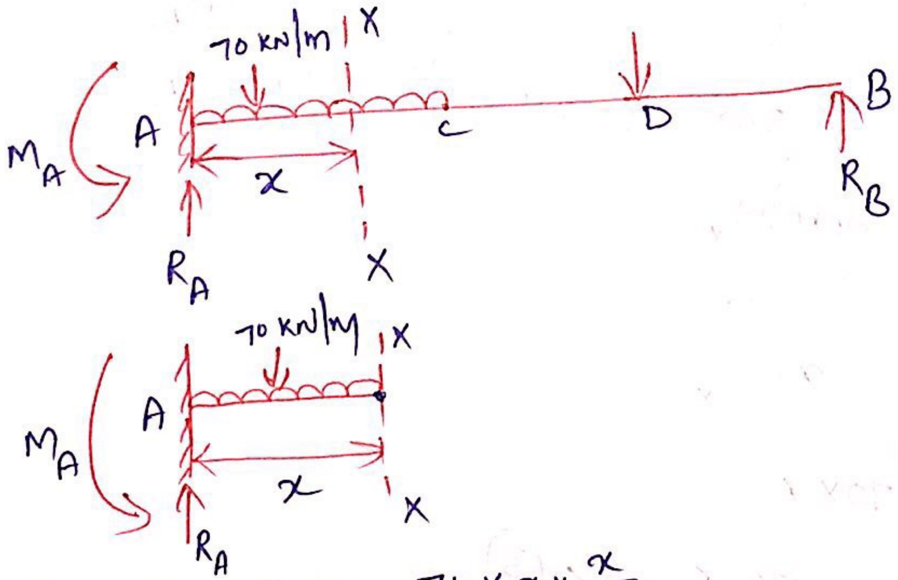
Eqn (3) equating to zero. i.e. $(SF)_{X-X} = 0$.

$207.96 - 70 \times x = 0$

$$\boxed{x = 2.97\ m.}$$

Bending moment diagram :—

consider a section X-X at a distance 'x' from 'A'



$(BM)_{X-X} = -M_A + R_A \times x - 70 \times x \times \frac{x}{2}$

$(BM)_{X-X} = -236.18 + 207.96\,x - \frac{70x^2}{2} \rightarrow (4)$

Apply limits 0 to 3

BM_A & BM_C.

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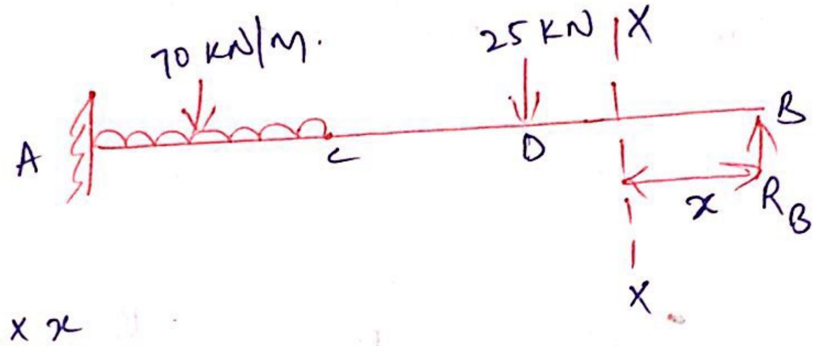
@ $x=0$; $(BM_A) = -236.18 + 207.96(0) - \frac{70(0)^2}{2}$

$$\boxed{(BM)_A = -236.18 \text{ KN-m.}}$$

@ $x=3$ m; $(BM)_C = -236.18 + 207.96(3) - \frac{70(3)^2}{2}$

$$\boxed{(BM)_C = 72.7 \text{ KN.m.}}$$

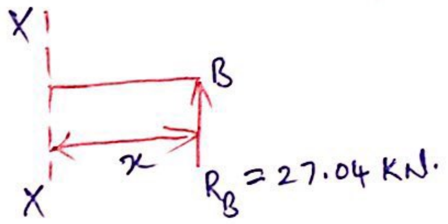
consider a section X-X at a distance 'x' from 'B'.



$(BM)_{X-X} = R_B \times x$

$(BM)_{X-X} = 27.04x \rightarrow$ (5)

Apply limits $x = 0$m to 2.5M.



@ $x=0$; $\boxed{(BM)_B = 0}$

@ $x=2.5$m; $(BM)_D = 27.04(2.5)$

$$\boxed{(BM)_D = 67.6 \text{ KN.m.}}$$

Maximum bending moment :— [From eqn (4)]

At $x = 2.97$ m. maximum bending moment occurs.

$(BM)_{max} = -236.18 + 207.96(2.97) - \frac{70}{2}(2.97)^2$.

$$\boxed{(BM)_{max} = 72.72 \text{ KN.m.}}$$

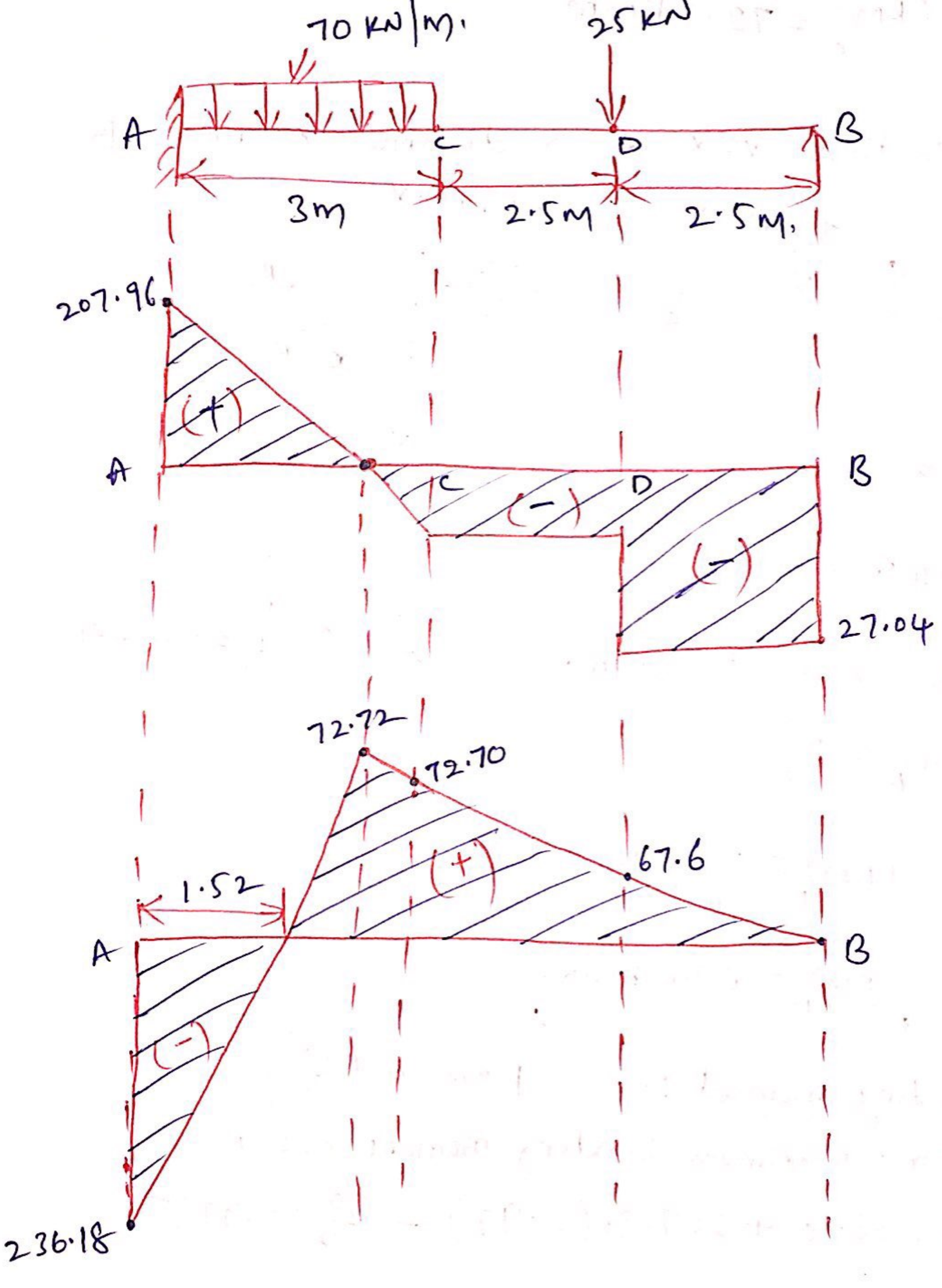
Location of point of contraflexure :-

Equating eqn (4) is zero.

$(BM)_{x-x} = 0.$

$$-236.18 + 207.96x - \frac{70x^2}{2} = 0$$

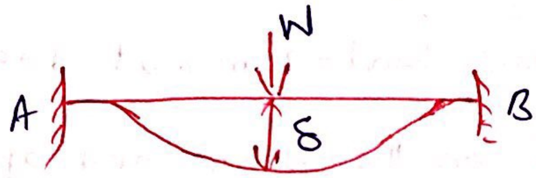
$$\boxed{x = 1.52\ m.}$$



Fixed Beams

Introduction :-

A beam is said to be fixed beam whose both ends are rigidly fixed, so that slope at the ends remains zero.



$\theta_A = 0$
$\theta_B = 0$

Advantages :-

(1) It is stiffer, stronger and more stable.

(2) The slope at both ends are zero.

(3) The fixing moment are developed at both ends which reduce the maximum bending moment at center of beam.

(4) The deflection at center is very much reduced.

Disadvantages :-

(1) Both ends of the beam must be at same level (or) else extra stress can be developed when they are not on same level.

(2) Stresses are produced in fixed end due to change in temp.

(3) The fixity of the beam is reduce due to vibration.

Analysis of Fixed Beams :—

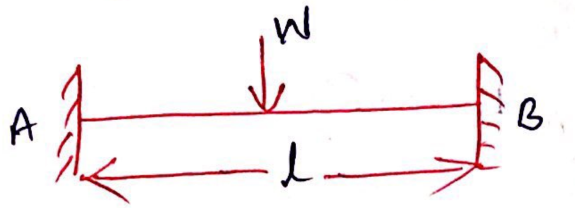
The analysis of a fixed beam carried out in the following stages.

(1) consider the beam to be a simply supported beam subjected to an external load and draw bending moment diagram.

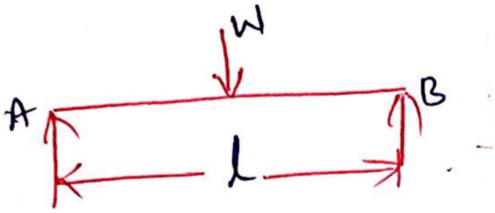
(2) Remove the external load on the beam and apply couples of M_A and M_B at the two ends of the beam and draw the bending moment diagram for this case. this bending moment diagram is known as fixed bending moment diagram.

(3) the resultant bending moment diagram can be obtained by super imposing the bending moment diagrams of the above cases.

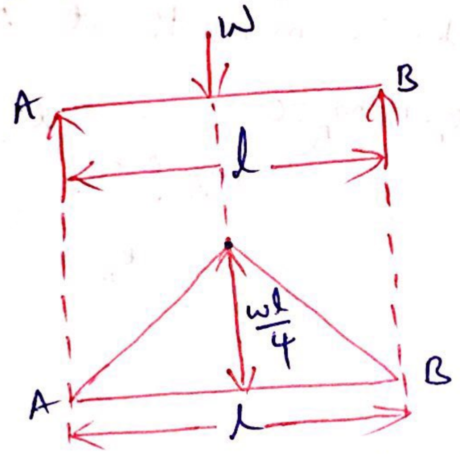
Fixed beam subjected to point load :—



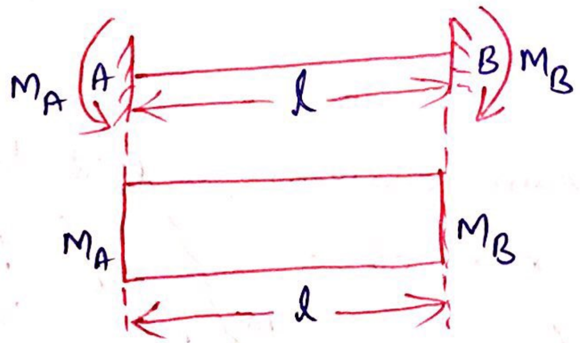
(A). (1) convert the fixed supports as simply supports.



(2) Draw the BMD for simply supported subjected to point load.



(3) Draw the BMD subjected to fixed ends.



(4) Equate the BMD areas

Area of triangle, $A_1 = \frac{1}{2} \times b \times h$

$$= \frac{1}{2} \times l \times \frac{wl}{4}$$

$$A_1 = \frac{wl^2}{8} \rightarrow (1)$$

Area of rectangle, $A_2 = ?$

Due to symmetry, $M_A = M_B$

$$A_2 = M_A \times l \rightarrow (2)$$

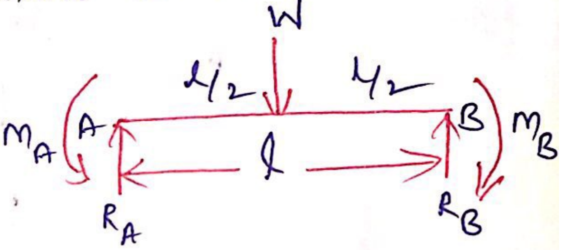
$$(1) = (2)$$

$$\frac{wl^{\cancel{2}}}{8} = M_A \times \cancel{l}$$

$$\boxed{M_A = \frac{wl}{8}}$$

$$\boxed{M_B = \frac{wl}{8}}$$

(5) Equate clockwise and anticlockwise moments at 'A'.



$$\Sigma M_A = 0; \; -M_A + (R_A \times 0) + W.\frac{l}{2} - R_B . l + M_B = 0$$

$$-\cancel{\frac{wl}{8}} + \frac{wl}{2} - R_B . l + \cancel{\frac{wl}{8}} = 0$$

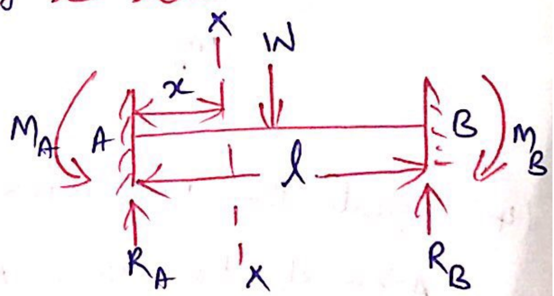
$$\frac{wl}{2} = R_B . \cancel{l}$$

$$\boxed{R_B = \frac{W}{2}}$$

$$\boxed{R_A = \frac{W}{2}}$$

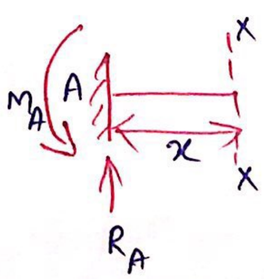
Deflection for a fixed beam carrying a point load :—

Let us consider a section $x - x$ at a distance 'x' from support 'A':



$$(BM)_{x-x} = -M_A + R_A \cdot x$$

$$(BM)_{x-x} = \frac{-wl}{8} + \frac{wx}{2}$$



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By using macaulay's method :

$$EI \cdot \frac{d^2y}{dx^2} = \frac{-wl}{8} + \frac{wx}{2} \rightarrow 3$$

By integrating above eqn (3)

$$EI \cdot \frac{dy}{dx} = -\frac{wlx}{8} + \frac{wx^2}{4} + c_1 \Rightarrow (4)$$

Apply boundary conditions i.e. $x = 0, \frac{dy}{dx} = 0$.

$$c_1 = 0$$

$c_1 = 0$ sub. in eqn (4)

$$EI \cdot \frac{dy}{dx} = \frac{-wlx}{8} + \frac{wx^2}{4} \Rightarrow (5)$$

Again integrating above eqn (5)

$$EI \cdot y = \frac{-wl}{8} \cdot \frac{x^2}{2} + \frac{w}{4} \cdot \frac{x^3}{3} + c_2$$

$$EI \cdot y = \frac{-wlx^2}{16} + \frac{wx^3}{12} + c_2 \Rightarrow (6)$$

Apply boundary conditions i.e. $x = 0, y = 0$

$$c_2 = 0$$

$c_2 = 0$ sub. in eqn (6)

$$EI \cdot y = \frac{-wlx^2}{16} + \frac{wx^3}{12} \Rightarrow (7)$$

Maximum deflection at mid span i.e. $x = \frac{l}{2}$

sub. $x = \frac{l}{2}$ in eqn (7)

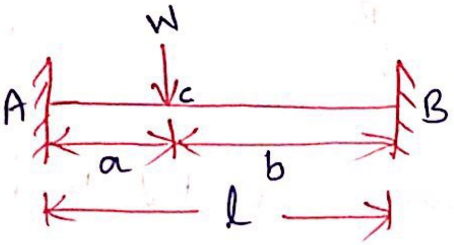
$$EI \cdot y = \frac{-wl}{16}\left(\frac{l}{2}\right)^2 + \frac{w}{12}\left(\frac{l}{2}\right)^3$$

$$EI \cdot y = \frac{-wl^3}{64} + \frac{wl^3}{96} \Rightarrow EI \cdot y = \frac{-wl^3}{192}$$

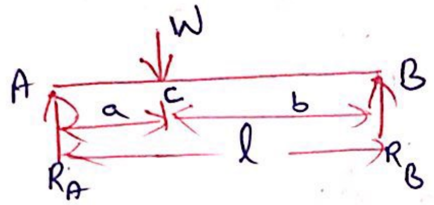
$$\boxed{y = \frac{-wl^3}{192 EI}} \qquad \boxed{\therefore y = \frac{-1}{4}\left[\frac{wl^3}{48EI}\right]}$$

Fixed beam with eccentric point load :—

Eccentric point load :- Load is non symmetric w.r.t. Centre of the axis.



convert fixed supports as simply supports.



$\Sigma V = 0;\ R_A + R_B - W = 0$

$R_A + R_B = W \rightarrow (1)$

$\Sigma M_B = 0;\ (R_A \times l) - (W \times b) = 0$

$$\boxed{R_A = \frac{Wb}{l}}$$

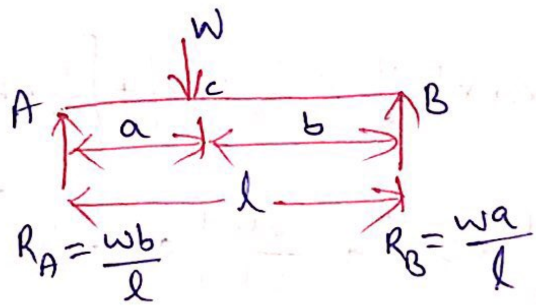
sub. 'R_A' in eq (1)

$\frac{Wb}{l} + R_B = W \Rightarrow R_B = W - \frac{Wb}{l} \Rightarrow R_B = \frac{Wl - Wb}{l} \Rightarrow R_B = \frac{W(l-b)}{l}$

$[\because l - b = a]$

$$\boxed{R_B = \frac{Wa}{l}}$$

Draw bending moment diagram for simply supported beam.

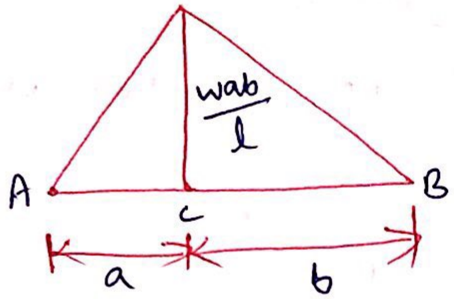


$M_A = 0,\ M_B = 0$

$M_c = ?$

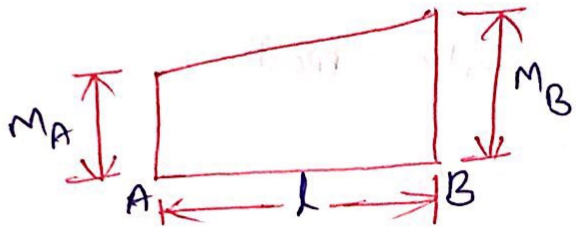
$(BM)_c = R_A (a)$

$$\boxed{(BM)_c = \frac{Wab}{l}}$$



$\rightarrow (1)$

Draw the bending moment diagram subjected to fixed end moments.



$\rightarrow (2)$ (Trapezoidal shape)

Since the beam is not symmetrical.

$$M_A \neq M_B$$

Calculate BMD for S.S.B and fixed ~~beam~~ end moments areas.

$$(1) = (2)$$

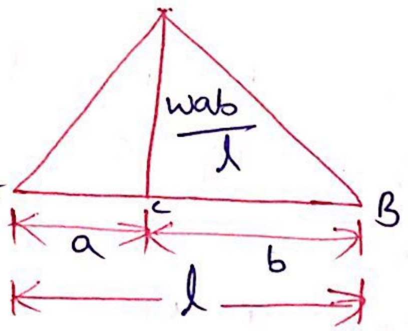
$$\frac{1}{2} \times b \times h = \frac{1}{2}(a+b)h.$$

$$\cancel{\frac{1}{2}} \times \cancel{l} \times \frac{wab}{\cancel{l}} = \cancel{\frac{1}{2}}(M_A + M_B)\cancel{l}$$

$$\boxed{M_A + M_B = \frac{wab}{l}} \rightarrow (3)$$

The bending moment diagram due to simply support beam

BMD about 'A' split into 2 triangles.

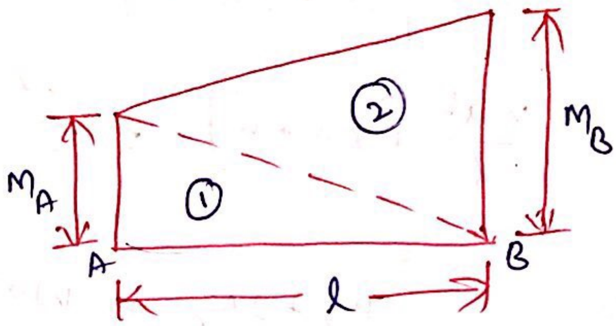


$$\left[\frac{1}{2} \times a \times \frac{wab}{l} \times \frac{2}{3}(a)\right] + \left[\frac{1}{2} \times b \times \frac{wab}{l} \times \left\{\frac{1}{3}(b) + a\right\}\right]$$

$$\frac{wab}{6l} \times 2a^2 + \frac{wab}{2l}\left[ab + \frac{b^2}{3}\right] \rightarrow (4)$$

Bending moment due to fixed moments about A:

$$\left[\frac{1}{2}\times l\times M_A\times\frac{l}{3}\right]+\left[\frac{1}{2}\times l\times M_B\times\frac{2}{3}l\right]$$



$$\frac{M_A\cdot l^2}{6}+\frac{2M_B\cdot l^2}{6}$$

$$\frac{l^2(M_A+2M_B)}{6} \rightarrow (5)$$

Now equating (4) & (5)

$$\frac{wab}{6l}\times 2a^2+\frac{wab}{2l}\left[ab+\frac{b^2}{3}\right]=\frac{l^2(M_A+2M_B)}{6}$$

$$\frac{wab}{6l}\times 2a^2+\frac{wab}{6l}(3ab+b^2)=\frac{l^2(M_A+2M_B)}{6}$$

$$\frac{wab}{6l}(2a^2+3ab+b^2)=\frac{l^2(M_A+2M_B)}{6}$$

$$\frac{wab}{6l}\left[2(l-b)^2+3(l-b)b+b^2\right]=\frac{l^2(M_A+2M_B)}{6}$$

$$\frac{wab}{6l}\left[2(l^2+b^2-2lb)+3lb-3b^2+b^2\right]=\frac{l^2(M_A+2M_B)}{6}$$

$$\frac{wab}{6l}\left[2l^2+\cancel{2b^2}-4lb+3lb-\cancel{3b^2}+\cancel{b^2}\right]=\frac{l^2(M_A+2M_B)}{6}$$

$$\frac{wab}{6l}\left[2l^2-bl\right]=\frac{l^2(M_A+2M_B)}{6}$$

$$\frac{wab\cancel{l}}{6\cancel{l}}\left[2l-b\right]=\frac{l^2(M_A+2M_B)}{6}$$

$$\frac{wab}{6}\left[2l-b\right]=\frac{l^2(M_A+2M_B)}{6}$$

$$\frac{wab}{6}\left[2(a+b)-b\right]=\frac{l^2(M_A+2M_B)}{6}$$

$$\frac{wab}{6}\left[2a+2b-b\right] = \frac{l^2(M_A+2M_B)}{6}$$

$$\frac{wab}{\cancel{6}}\left[2a+b\right] = \frac{l^2(M_A+2M_B)}{\cancel{6}}$$

$$\boxed{M_A+2M_B = \frac{wab(2a+b)}{l^2}} \rightarrow (6)$$

substract (3) − (6)

$$\cancel{M_A}+M_B-\cancel{M_A}-2M_B = \frac{wab}{l} - \frac{wab(2a+b)}{l^2}$$

$$-M_B = \frac{wab}{l}\left[1-\frac{(2a+b)}{l}\right]$$

$$-M_B = \frac{wab}{l}\left[\frac{l-2a-b}{l}\right] \qquad \left[b=(l-a)\right]$$

$$-M_B = \frac{wab}{l}\left[\frac{l-2a-(l-a)}{l}\right]$$

$$-M_B = \frac{wab}{l}\left[\frac{\cancel{l}-2a-\cancel{l}+a}{l}\right]$$

$$-M_B = \frac{wab}{l}\left[\frac{-a}{l}\right]$$

$$\cancel{-}M_B = \cancel{-}\frac{wa^2b}{l^2}$$

$$\boxed{M_B = \frac{wa^2b}{l^2}} \rightarrow (7)$$

sub. eqn (7) in (3)

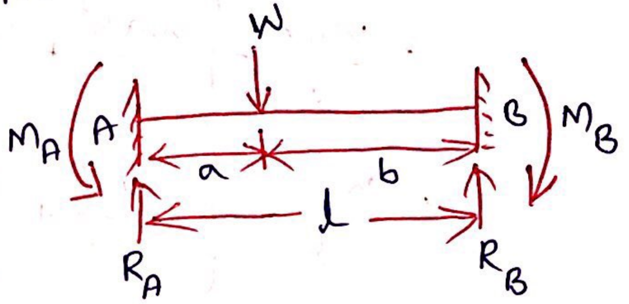
$$M_A+M_B = \frac{wab}{l}$$

$$M_A+\frac{wa^2b}{l^2} = \frac{wab}{l}$$

$$\boxed{M_A = \frac{wab^2}{l^2}} \rightarrow (8)$$

We know $M_A = \frac{wab^2}{l^2}$ and $M_B = \frac{wa^2b}{l^2}$

Find out reactions at fixed supports.



$\Sigma M_A = 0$

$-M_A + W(a) - R_B(l) + M_B = 0$

$M_B + Wa = M_A + R_B \cdot l$

$M_B - M_A + Wa = R_B \cdot l$

$$\boxed{R_B = \frac{(M_B - M_A) + Wa}{l}}$$

Similarly $\Sigma M_B = 0$

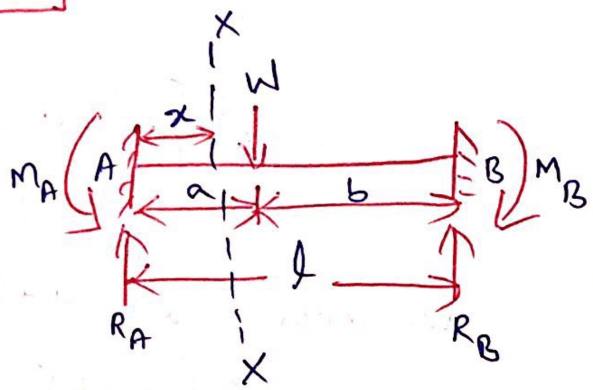
$-M_A + R_A(l) - W(b) + M_B = 0$

$R_A(l) + M_B = M_A + W(b)$

$R_A \cdot l = M_A - M_B + W \cdot b$

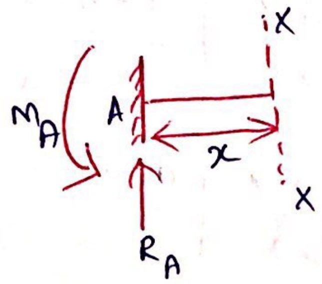
$$\boxed{R_A = \frac{(M_A - M_B) + Wb}{l}}$$

Bending moment $(BM)_{x-x}$:—



$(BM)_{x-x} = R_A(x) - M_A$

$(BM)_{x-x} = \left[\frac{(M_A - M_B) + Wb}{l}\right]x - M_A$



from macalay's method :

$$EI \cdot \frac{d^2y}{dx^2} = \left[\frac{(M_A - M_B) + Wb}{l}\right]x - M_A$$

$$= \frac{wbx}{l} - \left[M_A + (M_B - M_A) \cdot \frac{x}{l} \right]$$

$$= \frac{wbx}{l} - \left[\frac{wab^2}{l^2} + \left(\frac{wa^2b}{l^2} - \frac{wab^2}{l^2} \right) \cdot \frac{x}{l} \right]$$

$$= \frac{wbx}{l} - \left[\frac{wab^2}{l^2} + \frac{wab}{l^2}(a-b) \cdot \frac{x}{l} \right]$$

$$EI \cdot \frac{d^2y}{dx^2} = \frac{wbx}{l} - \frac{wab^2}{l^2} - \frac{wab(a-b)x}{l^3} \rightarrow (9)$$

Integrating above eqn. (9)

$$EI \cdot \frac{dy}{dx} = \frac{wb}{l} \cdot \frac{x^2}{2} - \frac{wab^2}{l^2} \cdot x - \frac{wab(a-b)}{l^3} \cdot \frac{x^2}{2} + c_1$$

Apply boundary conditions i.e. $x = 0$, $\frac{dy}{dx} = 0$

$$c_1 = 0$$

$$EI \cdot \frac{dy}{dx} = \frac{wbx^2}{2l} - \frac{wab^2x}{l^2} - \frac{wab(a-b) \cdot x^2}{2l^3} \rightarrow (10)$$

$$= \frac{wbx^2}{2l} \left[1 - \frac{a(a-b)}{l^2} \right] - \frac{wab^2x}{l^2}$$

$$= \frac{wbx^2}{2l} \left[\frac{l^2 - a^2 + ab}{l^2} \right] - \frac{wab^2x}{l^2}$$

we know $l = (a+b)$

$$EI \cdot \frac{dy}{dx} = \frac{wbx^2}{2l} \left[\frac{(a+b)^2 - a^2 + ab}{l^2} \right] - \frac{wab^2x}{l^2}$$

$$= \frac{wbx^2}{2l} \left[\frac{\cancel{a^2} + b^2 + 2ab - \cancel{a^2} + ab}{l^2} \right] - \frac{wab^2x}{l^2}$$

$$EI \cdot \frac{dy}{dx} = \frac{wbx^2}{2l}\left[\frac{3ab+b^2}{l^2}\right] - \frac{wab^2x}{l^2}$$ (7)

$$EI \cdot \frac{dy}{dx} = \frac{wb^2x^2(3a+b)}{2l^3} - \frac{wab^2x}{l^2} \rightarrow (11)$$

Integrating above eqn. (11)

$$EI \cdot y = \frac{wb^2(3a+b)}{2l^3} \cdot \frac{x^3}{3} - \frac{wab^2}{l^2} \cdot \frac{x^2}{2} + c_2$$

Apply boundary conditions i.e. $x=0, y=0$

$$c_2 = 0$$

$$EI \cdot y = \frac{wb^2(3a+b)x^3}{6l^3} - \frac{wab^2 \cdot x^2}{2l^2}$$

$$EI \cdot y = \frac{wb^2x^2}{6l^3}\left[x(3a+b) - 3al\right] \rightarrow (12)$$

we know for calculating max. deflection

$\frac{dy}{dx} = 0$ from eqn. (11)

$$\frac{wb^2x^2(3a+b)}{2l^3} - \frac{wab^2x}{l^2} = 0$$

$$\frac{wab^2\cancel{x}}{l^2} = \frac{wb^2}{2l^3}(3a+b)x^{\cancel{2}}$$

$$x = \frac{\cancel{w}a\cancel{b^2}}{\cancel{l^2}} \times \frac{2l^{\cancel{3}}}{\cancel{wb^2}(3a+b)}$$

$$\boxed{x = \frac{2al}{b+3a}}$$

sub. the 'x' value in eq^n

for maximum deflection

y_{max} = maximum deflection

$$EI \cdot y_{max} = \frac{wb^2x^2}{6l^3}\left[x(3a+b) - 3al\right] \rightarrow (13)$$

$$= \frac{wb^2}{6l^3}\left(\frac{2al}{3a+b}\right)^2\left[\frac{2al}{\cancel{(3a+b)}}\cancel{(3a+b)} - 3al\right]$$

$$= \frac{wb^2}{6l^3} \times \frac{4a^2l^2}{(3a+b)^2}\left[2al - 3al\right]$$

$$= \frac{wb^2}{\cancel{6}_3\cancel{l^3}} \times \frac{\cancel{4}^2a^2\cancel{l^2}}{(3a+b)^2}(-a\cancel{l})$$

$$= \frac{-wb^2}{3} \times \frac{2a^3}{(3a+b)^2}$$

$$EI \cdot y_{max} = \frac{-2}{3} \times \frac{wa^3b^2}{(3a+b)^2}$$

$$y_{max} = \frac{-2}{3EI} \times \frac{wa^3b^2}{(3a+b)^2} \rightarrow (14)$$

The deflection under the load when $x = a$ in eq^n (12)

$$EI \cdot y = \frac{wb^2x^2}{6l^3}\left[x(3a+b) - 3al\right]$$

$$= \frac{wb^2a^2}{6l^3}\left[a(3a+b) - 3al\right]$$

we know $l = (a+b)$

$$EI \cdot y = \frac{wb^2a^2}{6l^3}\left[a(3a+b) - 3a(a+b)\right]$$

$$= \frac{wb^2a^2}{6l^3}\left[\cancel{3a^2} + ab - \cancel{3a^2} - 3ab\right]$$

$$= \frac{wb^2a^2}{\cancel{6}_3 l^3}\left[-\cancel{2}ab\right]$$

$$EI \cdot y = \frac{-wa^3b^3}{3l^3}$$

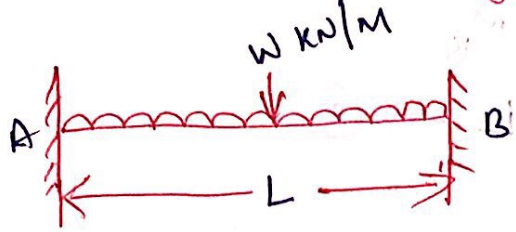
$$\boxed{y = -\left[\frac{wa^3b^3}{3EI \cdot l^3}\right]}$$ (Deflection for fixed beam)

$$\boxed{y = \frac{-ab}{l^2}\left[\frac{wa^2b^2}{3EIl}\right]}$$

Here $\frac{wa^2b^2}{3EIl}$ is the deflection for simply supported beam.

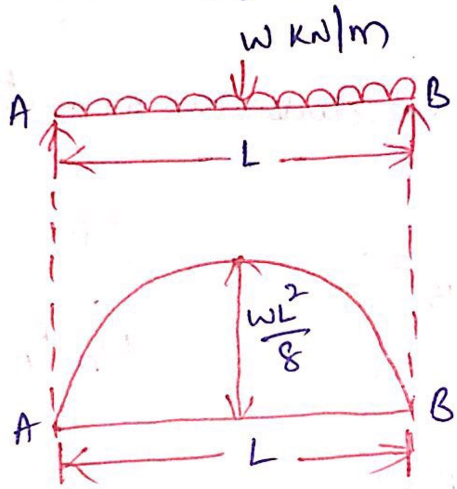
Fixed Beam subjected to uniformly Distributed Load :-

(A)

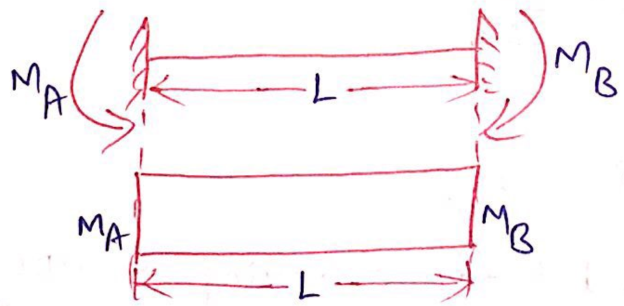


(1) convert the fixed supports as simply supports.

(2) Draw the BMD for simply supported subjected to UDL.



(3) Draw the BMD subjected to fixed ends



(4) Equate the BMD areas

Area of parabola, $A_1 = \frac{2}{3} \times b \times h$

$= \frac{2}{3} \times L \times \frac{WL^2}{8}$

$A_1 = \frac{WL^3}{12} \rightarrow (1)$

Area of rectangle, $A_2 = ?$

Due to symmetry $M_A = M_B$

$$A_2 = M_A \times L \rightarrow (2)$$

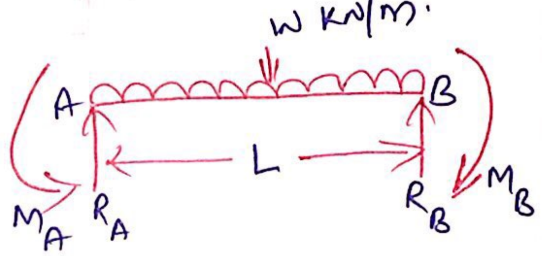
$$(1) = (2)$$

$$\frac{wL^{\cancel{3}\,2}}{12} = M_A \cdot \cancel{L}$$

$$\boxed{M_A = \frac{wL^2}{12}}$$

$$\boxed{M_B = \frac{wL^2}{12}}$$

(5) Equate clockwise and anticlockwise moments at A.



$\Sigma M_A = 0$

$$-M_A + w \times L \times \frac{L}{2} - R_B \times L + M_B = 0$$

$$-\frac{wL^2}{12} + \frac{wL^{\cancel{2}}}{2} - R_B \times L + \frac{wL^2}{12} = 0$$

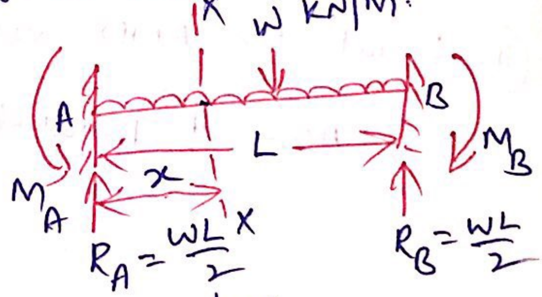
$$\frac{wL^{\cancel{2}}}{2} = R_B \times \cancel{L}$$

$$\boxed{R_B = \frac{wL}{2}}$$

$$\boxed{R_A = \frac{wL}{2}}$$

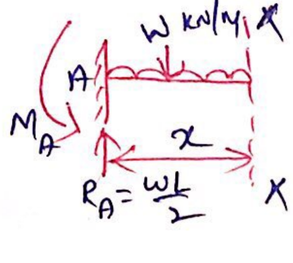
Deflection for a fixed beam carrying a UDL:—

Let us consider a section X–X at a distance 'x' from support 'A'.



$$M = R_A \times x - M_A - w \times x \times \frac{x}{2}$$

$$M = \frac{wL}{2} \times x - \frac{wL^2}{12} - \frac{wx^2}{2}$$



Scanned with CamScanner

By using macaulay's method :

$$EI \cdot \frac{d^2y}{dx^2} = \frac{WLx}{2} - \frac{WL^2}{12} - \frac{Wx^2}{2} \rightarrow (3)$$

Integrating eqn (3) w.r.t. 'x'

$$EI \cdot \frac{dy}{dx} = \frac{WL}{2} \cdot \frac{x^2}{2} - \frac{WL^2}{12} \times x - \frac{W}{2} \cdot \frac{x^3}{3} + c_1 \rightarrow (4)$$

Apply boundary conditions i.e. $x = 0$, $\frac{dy}{dx} = 0$ sub. in (4)

$c_1 = 0$.

'c_1' value sub. in eqn (4)

$$EI \cdot \frac{dy}{dx} = \frac{WL}{2} \cdot \frac{x^2}{2} - \frac{WL^2}{12} \cdot x - \frac{W}{2} \cdot \frac{x^3}{3} \rightarrow (5)$$

Again integrate eqn (5)

$$EI \cdot y = \frac{WL}{2} \times \frac{x^3}{2 \times 3} - \frac{WL^2}{12} \times \frac{x^2}{2} - \frac{W}{2} \times \frac{x^4}{3 \times 4} + c_2 \rightarrow (6)$$

Apply boundary conditions i.e. $x = 0, y = 0$

$c_2 = 0$

'c_2' value sub. in eqn (6)

$$EI \cdot y = \frac{WLx^3}{12} - \frac{WL^2x^2}{24} - \frac{Wx^4}{24} \rightarrow (7)$$

Maximum deflection at mid span i.e. $x = \frac{L}{2}$.

sub. $x = \frac{L}{2}$ in eqn (7)

$$EI \cdot y = \frac{WL}{12}\left(\frac{L}{2}\right)^3 - \frac{WL^2}{24}\left(\frac{L}{2}\right)^2 - \frac{W}{24}\left(\frac{L}{2}\right)^4$$

$$EI \cdot y = \cancel{\frac{WL^4}{96}} - \cancel{\frac{WL^4}{96}} - \frac{WL^4}{384}$$

$$EI \cdot y = \frac{-WL^4}{384}$$

$$\boxed{y = \frac{-WL^4}{384 EI}} \qquad \boxed{\therefore y = \frac{-1}{5}\left[\frac{5WL^4}{384 EI}\right]}$$

Problems :—

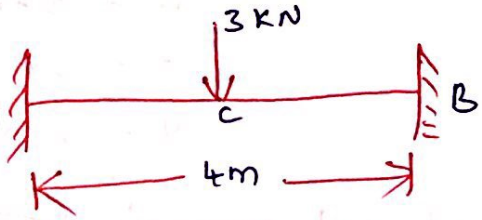
① A load of 3 KN is placed at the center of fixed beam of length 4 m. If young's modulus $E = 2.5 \times 10^6$ N/cm² and moment of inertia $I = 20,000$ cm⁴. Determine end moments and BM at center as simply supported beam and deflection under the load.

(A) Given : Length, $l = 4$ m.

load, $W = 3$ KN.

$E = 2.5 \times 10^6$ N/cm²

$I = 20,000$ cm⁴



Fixed end moments :—

$3\text{ KN} = 3000\text{ N}.$

$4\text{ m} = 400\text{ cm}.$

$$M_A = M_B = \frac{wl}{8}$$

$$M_A = \frac{3 \times 4}{8}$$

$$\boxed{M_A = 1.5\text{ KN.m.}}$$

$$\boxed{M_B = 1.5\text{ KN.m.}}$$

Bending moment :—

$$BM = \frac{wl}{4}$$

$$= \frac{3 \times \cancel{4}}{\cancel{4}}$$

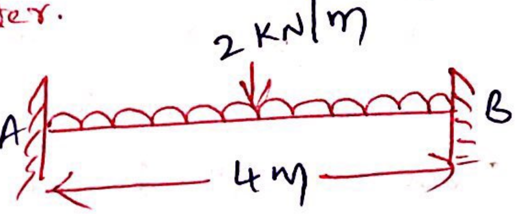
$$\boxed{BM = 3\text{ KN.m.}}$$

Deflection under the load :—

$$y_{max} = \frac{wl^3}{192EI} = \frac{3000\text{ N} \times (400\text{ cm})^3}{192 \times 2.5 \times 10^6 \frac{\text{N}}{\text{cm}^2} \times 20,000\text{ cm}^4}$$

$$\boxed{y_{max} = 0.02\text{ cm.}}$$

(2) A fixed beam of length 4m. carries a UDL of 2 kN/m. length over the whole span. If young's modulus, $E = 2\times10^6$ N/cm² and moment of inertia of the beam $I = 40{,}000$ cm⁴. Determine the end moments and deflection at the center.



(A) Given Data: $l = 4m$.

$w = 2 kN/m$.

$E = 2\times10^6 N/cm^2$

$I = 40{,}000 cm^4$.

Fixed end moments :—

$$M_A = M_B = \frac{wl^2}{12}$$

$$= \frac{2\times(4)^2}{12}$$

$$\boxed{M_A = 2.66\ kN.m}$$

$$\boxed{M_B = 2.66\ kN.m.}$$

Deflection at the center :—

$$y_{max} = \frac{wl^4}{384EI}$$

1 kN = 1000 N.

1m = 100cm.

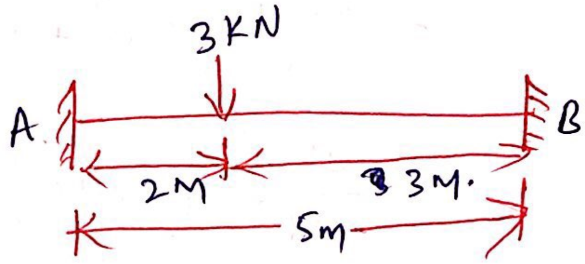
$$w = 2\frac{kN}{m} = 2\times1000\times\frac{1}{100} \Rightarrow w = 20\ N/cm.$$

$l = 4m = 400cm$.

$$y_{max} = \frac{20\times(400)^4}{384\times2\times10^6\times40{,}000}$$

$$\boxed{y_{max} = 0.0166\ cm.}$$

1) A load of 3 KN is placed at 2m. from left end 'A' of fixed beam AB of length 5m. If young's modulus $E = 2\times10^6$ N/cm² and M.O.I. of the beam $I = 10,000$ cm⁴. Determine the end moments and deflection under the load.



A) Given data: $W = 3$ KN

$l = 5$ m.

$a = 2$ m

$b = 3$ m

$E = 2\times10^6\ N/cm^2$

$I = 10,000\ cm^4$

Fixed end moments :-

$$M_A = \frac{wab^2}{l^2} = \frac{3\times2\times(3)^2}{(5)^2}$$

$$\boxed{M_A = 2.16\ KN.m.}$$

$$M_B = \frac{wa^2b}{l^2} = \frac{3\times(2)^2\times3}{(5)^2}$$

$$\boxed{M_B = 1.44\ KN.m.}$$

Deflection under the load :-

$W = 3$ KN $= 3\times1000$ N $= 3000$ N.

$l = 5$ m $= 5\times100$ cm $= 500$ cm.

$a = 2$ m $= 2\times100 = 200$ cm.

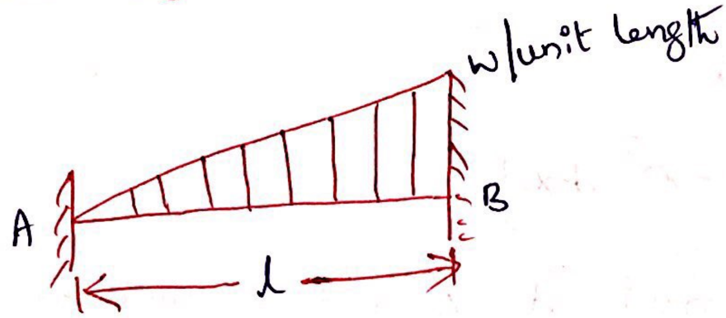
$b = 3$ m $= 3\times100 = 300$ cm.

$$y = \frac{wa^3b^3}{3EIl^3}$$

$$= \frac{3000\times(200)^3\times(300)^3}{3\times2\times10^6\times10,000\times(500)^3}$$

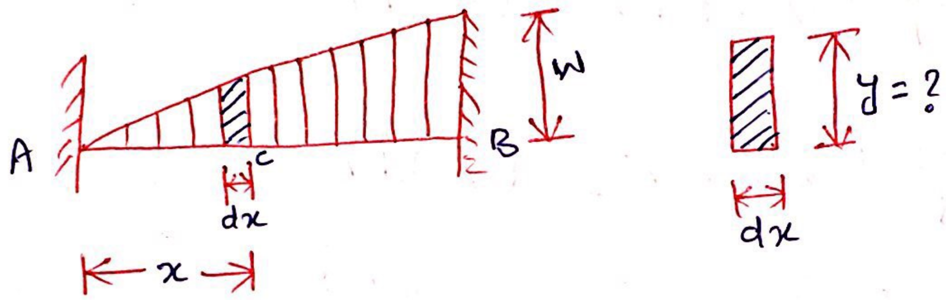
$$\boxed{y = 0.0864\ cm.}$$

Fixed beam subjected to UVL :—



Consider a fixed beam carrying gradually varying load 'O' at 'A' and w/unit length at 'B'.

Consider a strip of width "dx" at a distance "x" from 'A'.



From similar triangles

$$\frac{y}{x} = \frac{w}{l}$$

$$\boxed{y = \frac{wx}{l}}$$

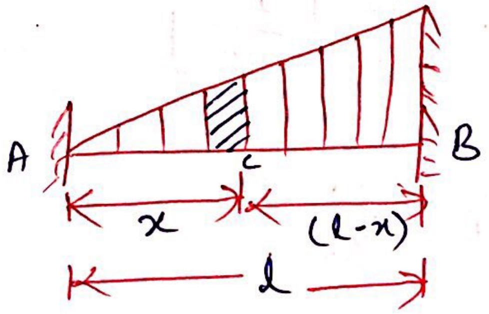
Load at the strip, [redacted] $= \frac{wx}{l}$

weight of the strip, $W = \frac{wx}{l} \times dx$ [Horizontal distance × vertical distance]

Assume the load is eccentric point load.

Fixing moment 'A' due to strip.

$$M_A = \frac{wab^2}{l^2}$$



$$M_A = \frac{\left(\frac{wx}{l}\right)dx \times x \times (l-x)^2}{l^2}$$

$$M_A = \frac{w}{l^3}\left[x^2 (l-x)^2 \cdot dx\right]$$

Total fixed end moment at 'A' will be given by integrating the equation from 0 to l.

$$M_A = \int_0^l \frac{w}{l^3}\left[x^2 (l-x)^2 \cdot dx\right].$$

$$= \frac{w}{l^3}\int_0^l x^2 (l^2 + x^2 - 2lx) \cdot dx$$

$$= \frac{w}{l^3}\int_0^l (x^2 l^2 + x^4 - 2lx^3) dx$$

$$= \frac{w}{l^3}\left[\frac{x^3}{3} \cdot l^2 + \frac{x^5}{5} - \cancel{2}l \cdot \frac{x^4}{\cancel{4}_2}\right]_0^l$$

$$= \frac{w}{l^3}\left[\frac{l^3}{3} \cdot l^2 + \frac{l^5}{5} - l \cdot \frac{l^4}{2}\right]$$

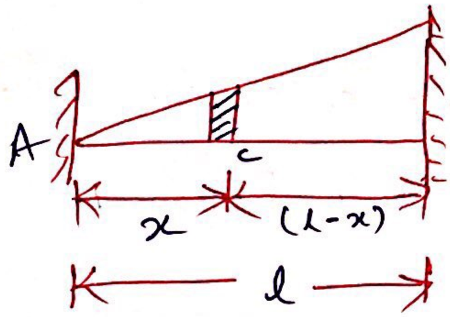
$$= \frac{w}{l^3}\left[\frac{l^5}{3} + \frac{l^5}{5} - \frac{l^5}{2}\right]$$

$$= \frac{w}{l^3}\left[\frac{10l^5 + 6l^5 - 15l^5}{30}\right]$$

$$= \frac{w}{\cancel{l^3}}\left[\frac{l^{\cancel{5}\,2}}{30}\right]$$

$$\boxed{M_A = \frac{wl^2}{30}}$$

fixed end moments at 'B'



$$M_B = \frac{wa^2b}{l^2}$$

$$= \frac{\left(\frac{wx}{l}\right)dx \times x^2 \times (l-x)}{l^2}$$

$$= \frac{w}{l^3}\left[x^3(l-x)\,dx\right]$$

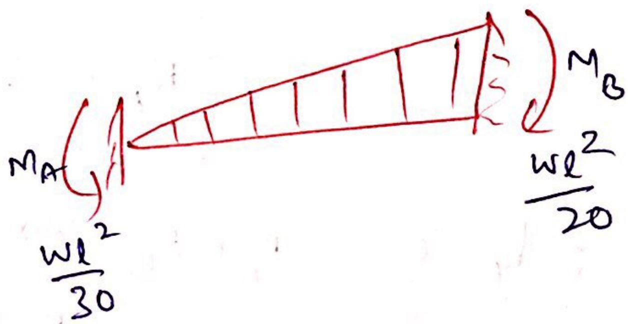
$$M_B = \frac{w}{l^3}\left[(lx^3 - x^4)\cdot dx\right]$$

for total fixed end moments at 'B', we are integrating

$$M_B = \int_0^l \frac{w}{l^3}\left[(lx^3 - x^4)\cdot dx\right]$$

$$= \frac{w}{l^3}\left[\int_0^l (lx^3 - x^4)\cdot dx\right]$$

$$= \frac{w}{l^3}\left[l\cdot\frac{x^4}{4} - \frac{x^5}{5}\right]_0^l$$

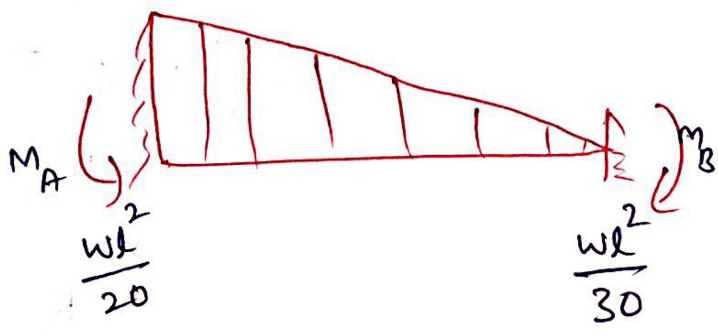


$$= \frac{w}{l^3}\left[l\cdot\frac{l^4}{4} - \frac{l^5}{5}\right]$$

$$= \frac{w}{l^3}\left[\frac{5l^5 - 4l^5}{20}\right]$$

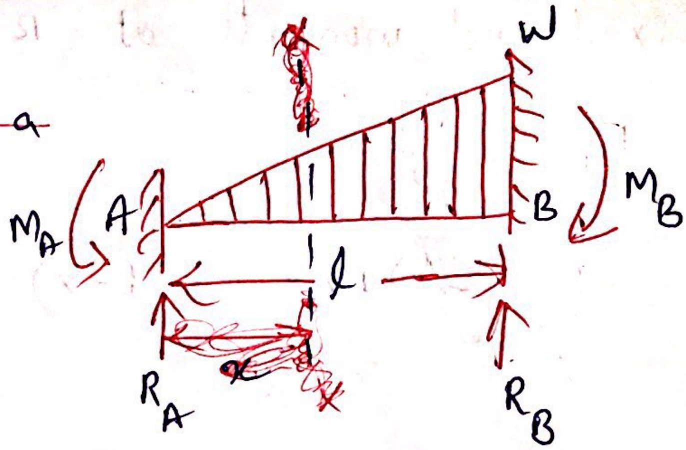
$$= \frac{w}{\cancel{l^3}}\left[\frac{l^{\cancel{5}\,2}}{20}\right]$$

$$\boxed{M_B = \frac{wl^2}{20}}$$



~~Consider a section x-x at a distance 'x' from A~~

~~$(BM)_{x-x} =$~~



$\Sigma M_B = 0;$

$$-M_A + R_A(l) + M_B - \frac{1}{2} \times b \times h \times \frac{l}{3} = 0$$

$$(R_A \times l) + \frac{wl^2}{20} = \frac{wl^2}{30} + \frac{1}{2} \times l \times w \times \frac{l}{3}$$

$$(R_A \times l) = \frac{wl^2}{30} - \frac{wl^2}{20} + \frac{wl^2}{6}$$

$$(R_A \times \cancel{l}) = \cancel{l}\left(\frac{wl}{30} - \frac{wl}{20} + \frac{wl}{6}\right)$$

$$R_A = \frac{2wl - 3wl + 10wl}{60}$$

$$\boxed{R_A = \frac{3wl}{20}}$$

$\Sigma f_y = 0;$ $\quad R_A + R_B - wl \cdot \frac{1}{2} = 0$

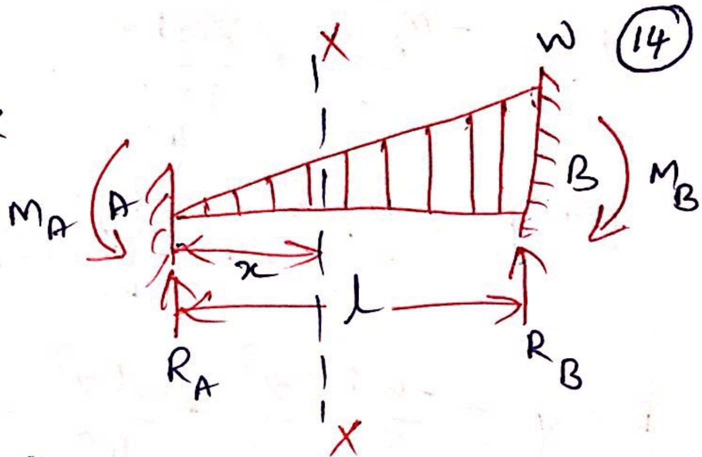
$$R_A + R_B = \frac{wl}{2}$$

$$\frac{3wl}{20} + R_B = \frac{wl}{2}$$

$$R_B = \frac{wl}{2} - \frac{3wl}{20}$$

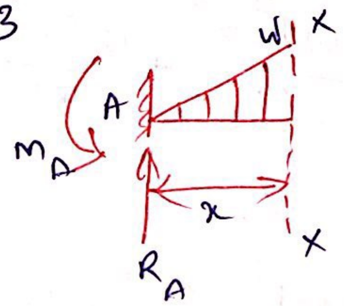
$$\boxed{R_B = \frac{7wl}{20}}$$

Let us consider a section X-X at a distance 'x' from 'A'.



$$(BM)_{x-x} = -M_A + R_A(x) - \frac{1}{2}\times b\times h\times\frac{x}{3}$$

$$= -M_A + \frac{3wl}{20}\times x - \frac{1}{2}\times x\times\frac{wx}{l}\times\frac{x}{3}$$



$$(BM)_{x-x} = EI.\frac{d^2y}{dx^2}$$

$$EI.\frac{d^2y}{dx^2} = -M_A + \frac{3wlx}{20} - \frac{wx^3}{6l} \rightarrow (1)$$

Integrating eqn (1)

$$EI.\frac{dy}{dx} = -M_A.x + \frac{3wl}{20}.\frac{x^2}{2} - \frac{w}{6l}.\frac{x^4}{4} + c_1$$

Apply boundary conditions i.e. $x=0$, $\frac{dy}{dx}=0$

$$c_1 = 0$$

$$EI.\frac{dy}{dx} = -M_A.x + \frac{3wlx^2}{40} - \frac{wx^4}{24l} \rightarrow (2)$$

Integrating eqn (2)

$$EI.y = -M_A.\frac{x^2}{2} + \frac{3wl}{40}.\frac{x^3}{3} - \frac{w}{24l}.\frac{x^5}{5} + c_2$$

Apply boundary conditions i.e. $x=0, y=0$

$$c_2 = 0$$

$$EI\,y = -\frac{M_A \cdot x^2}{2} + \frac{\cancel{3}wlx^3}{\cancel{120}\;40} - \frac{wx^5}{120l}$$

$$EI \cdot y = \frac{-M_A \cdot x^2}{2} + \frac{wlx^3}{40} - \frac{wx^5}{120l}$$

$$EI \cdot y = \frac{-wl^2}{30} \cdot \frac{x^2}{2} + \frac{wlx^3}{40} - \frac{wx^5}{120l}$$

$$EI \cdot y = \frac{-wl^2x^2}{60} + \frac{wlx^3}{40} - \frac{wx^5}{120l}$$

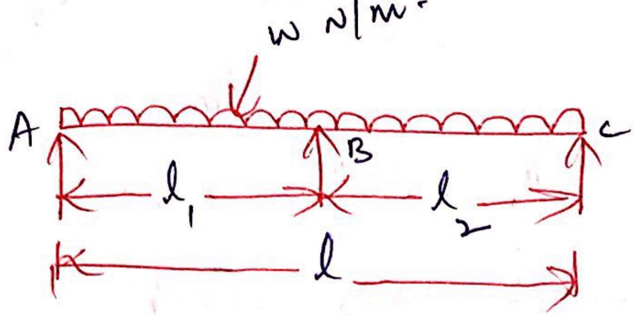
at $x = l/2$

$$EI \cdot y = \frac{-wl^2}{60}\left(\frac{l}{2}\right)^2 + \frac{wl}{40}\left(\frac{l}{2}\right)^3 - \frac{w}{120l}\left(\frac{l}{2}\right)^5$$

$$\boxed{y = \frac{-wl^4}{768\,EI}}$$

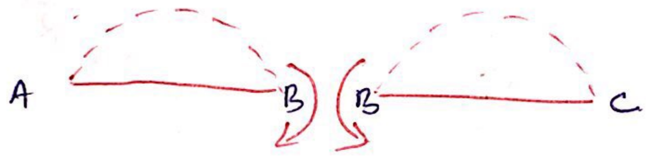
Continuous Beams

Introduction :–



(1) It's a beam having more than 2 supports.

(2) The intermediate supports are always subjected to bending moment (it is generally negative (Hogging case)).



concavity bending moment upwards.

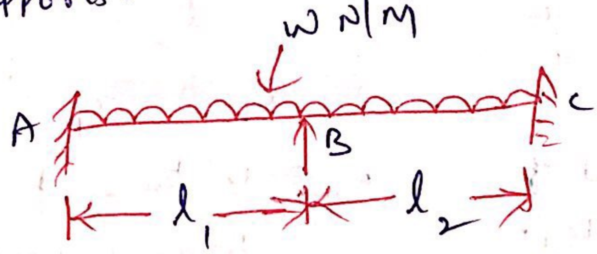
$M_B \Rightarrow$ (−ve) (Hogging)

(3) If end supports are simply supported, the BM at end supports will be zero.

i.e. $M_A = 0$

$M_C = 0$

(4) If end supports are fixed, there will be BM at fixed supports.

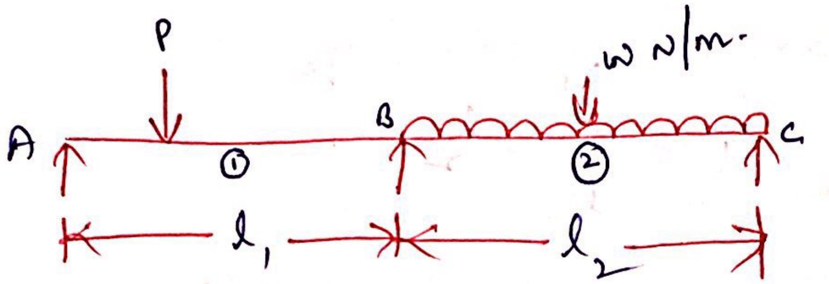


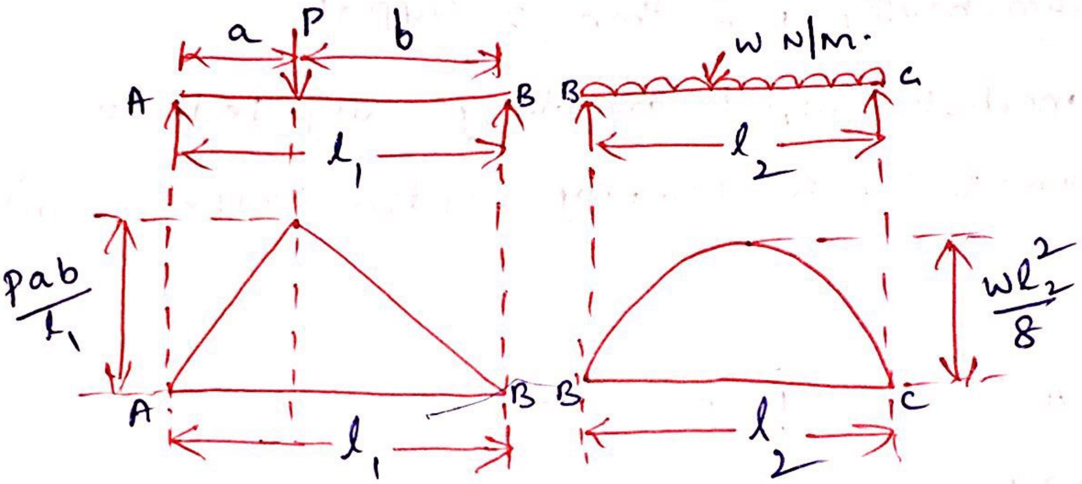
$M_A \neq 0$ & $M_C \neq 0$

Clapeyron's Theorem of Three Moments :—

It is having 3 distinct bending moment equations.

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 + \frac{6a_1 x_1}{l_1} + \frac{6a_2 x_2}{l_2} = 0$$





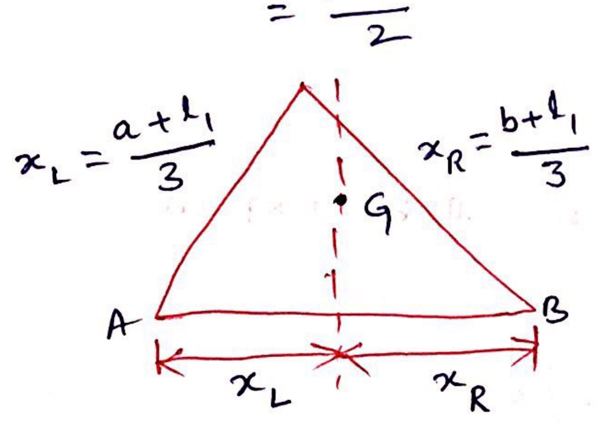
$a_1 \rightarrow$ Area of Beam 1's BMD $= \frac{1}{2} \times b \times h$

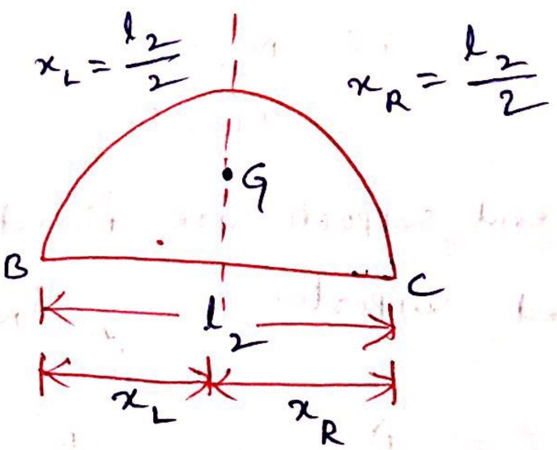
$= \frac{1}{2} \times \cancel{l_1} \times \frac{Pab}{\cancel{l_1}}$

$= \frac{Pab}{2}$

$a_2 \rightarrow$ Area of Beam 2's BMD $= \frac{2}{3} \times b \times h$

$= \frac{2}{3} \times l_2 \times \frac{wl_2^2}{8}$





$x_1 \rightarrow$ Distance of centroid of Beam 1 BMD from left

$x_2 \rightarrow$ Distance of centroid of Beam 2 BMD from right.

Scanned with CamScanner

Clapeyron's Theorem of Three Moments :—

(or)

Three Moment Theorem (TMT) :—

$$M_A L_1 + 2M_B(L_1 + L_2) + M_C L_2 + \frac{6A_1\bar{x}_L}{L_1} + \frac{6A_2\bar{x}_R}{L_2} = 0$$

Steps for TMT :—

(1) Draw the free bending moment diagram.

(2) calculate the area and ~~centroid~~ centroid of free bending moment diagram b/w two points.

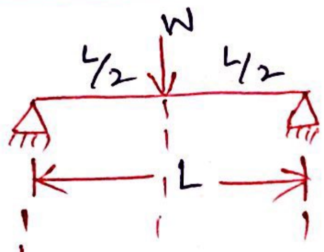
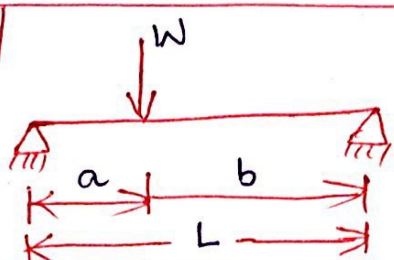
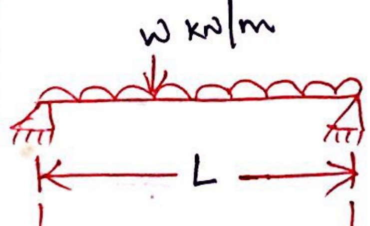
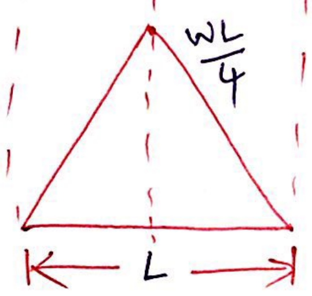
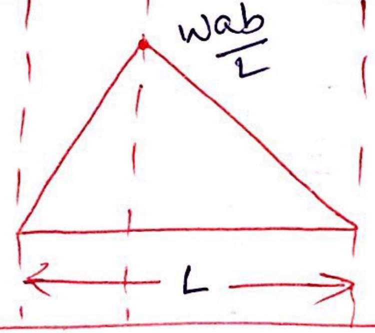
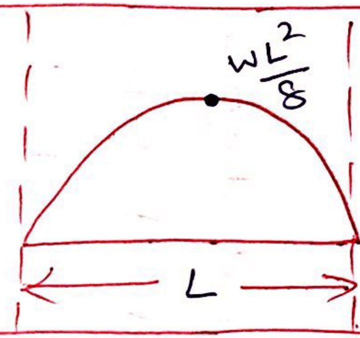
(3) Apply TMT (or) clapeyron's theorem.

(4) Find support reactions for two points AB, BC, CD separately.

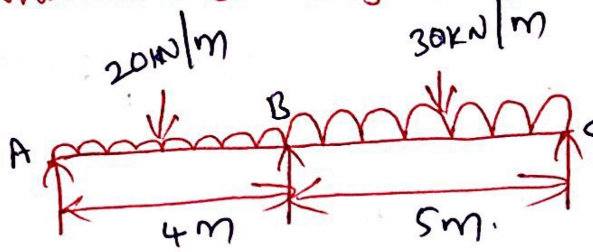
(5) Find final support reaction at support.

(6) Draw SFD & BMD & Net BMD.

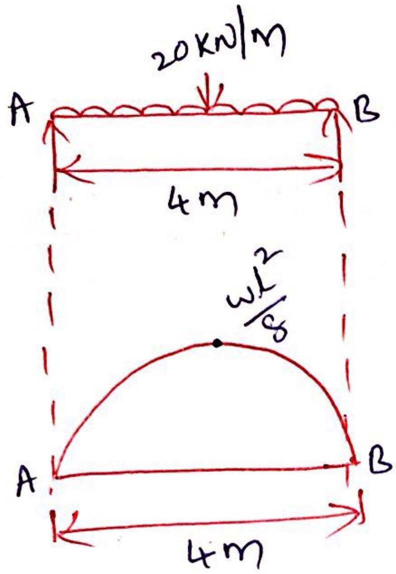
Table :-

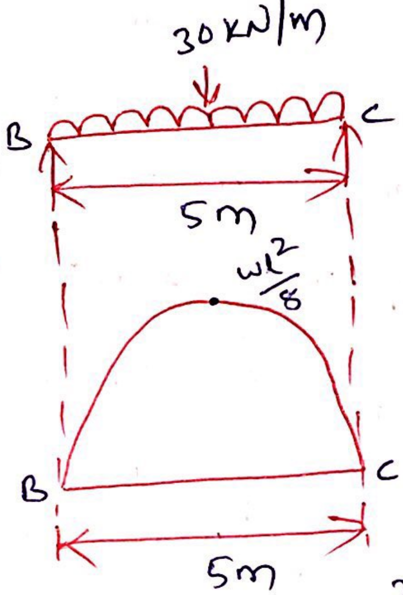
Diagram			
BMD			
Area	$\frac{1}{2} \times b \times h$ $\frac{1}{2} \times L \times \frac{WL}{4}$	$\frac{1}{2} \times b \times h$ $\frac{1}{2} \times L \times \frac{Wab}{L}$	$\frac{2}{3} \times b \times h$ $\frac{2}{3} \times L \times \frac{wL^2}{8}$
Centroid	x_L x_R $\frac{L}{2}$ $\frac{L}{2}$	x_L x_R $\frac{a+L}{3}$ $\frac{b+L}{3}$	x_L x_R $\frac{L}{2}$ $\frac{L}{2}$

(1) Analyse the continuous beam by using clapeyron's theorem. Find moment at 'B', Reactions, BMD, SFD. (3)



(A)





Maximum BM $= \frac{wl^2}{8}$

$= \frac{20 \times (4)^2}{8}$

$= 40$ KN.m.

Area, $A_1 = \frac{2}{3} \times b \times h$

$= \frac{2}{3} \times 4 \times 40$

a_1 (or) $A_1 = 106.67$

centroid, $x_L = \frac{l}{2}$

$= \frac{4}{2}$

$= 2$m.

$x_R = \frac{l}{2}$

$= \frac{4}{2}$

$= 2$m.

$\boxed{x_1 = x_L = 2m.}$

Maximum BM $= \frac{wl^2}{8}$

$= \frac{30 \times (5)^2}{8}$

$= 93.75$ KN.m.

$A_2 = \frac{2}{3} \times b \times h$

$= \frac{2}{3} \times 5 \times 93.75$

a_2 (or) $A_2 = 312.5$

centroid, $x_L = \frac{l}{2}$

$= \frac{5}{2}$

$= 2.5$m.

$x_R = \frac{l}{2}$

$= \frac{5}{2}$

$= 2.5$m.

$\boxed{x_2 = x_R = 2.5m}$

Apply clapeyron's three moment equation.

$$M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 + \frac{6a_1x_1}{l_1} + \frac{6a_2x_2}{l_2} = 0$$

where : $M_A = 0$, $M_C = 0$, $M_B = ?$.

$l_1 = 4m$, $l_2 = 5m$.

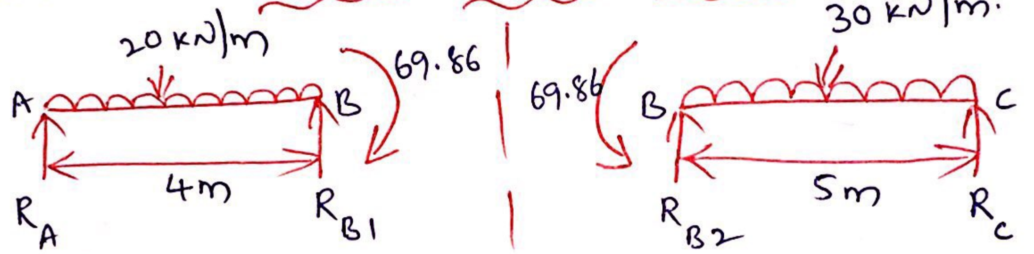
$a_1 = 106.67$, $a_2 = 312.5$

$x_1 = 2m$, $x_2 = 2.5m$.

$$0(4) + 2M_B(4+5) + 0(5) + \frac{6(106.67)(2)}{4} + \frac{6(312.5)(2.5)}{5} = 0.$$

$$\boxed{M_B = -69.86 \text{ KN.m.}}$$

Reactions :— ($R_A = ?$, $R_B = ?$, $R_C = ?$)



$\Sigma f_y = 0;$ $R_A + R_{B1} - (20\times4) = 0$

$R_A + R_{B1} = 80 \Rightarrow ①$

$\Sigma M_A = 0;$ $(-R_{B1}\times4) + (20\times4\times\frac{4}{2}) + 69.86 = 0$

$$\boxed{R_{B1} = 57.46 \text{ KN.}}$$

R_{B1} sub. in eq ①

$R_A + 57.46 = 80$

$$\boxed{R_A = 22.53 \text{ KN.}}$$

$\Sigma f_y = 0;$ $R_{B2} + R_C - (30\times5) = 0$

$R_{B2} + R_C = 150 \Rightarrow ②$

$\Sigma M_B = 0;$ $-69.86 + (30\times5\times\frac{5}{2}) - (R_C\times5) = 0$

$$\boxed{R_C = 61.03 \text{ KN.}}$$

R_C sub. in eq ②

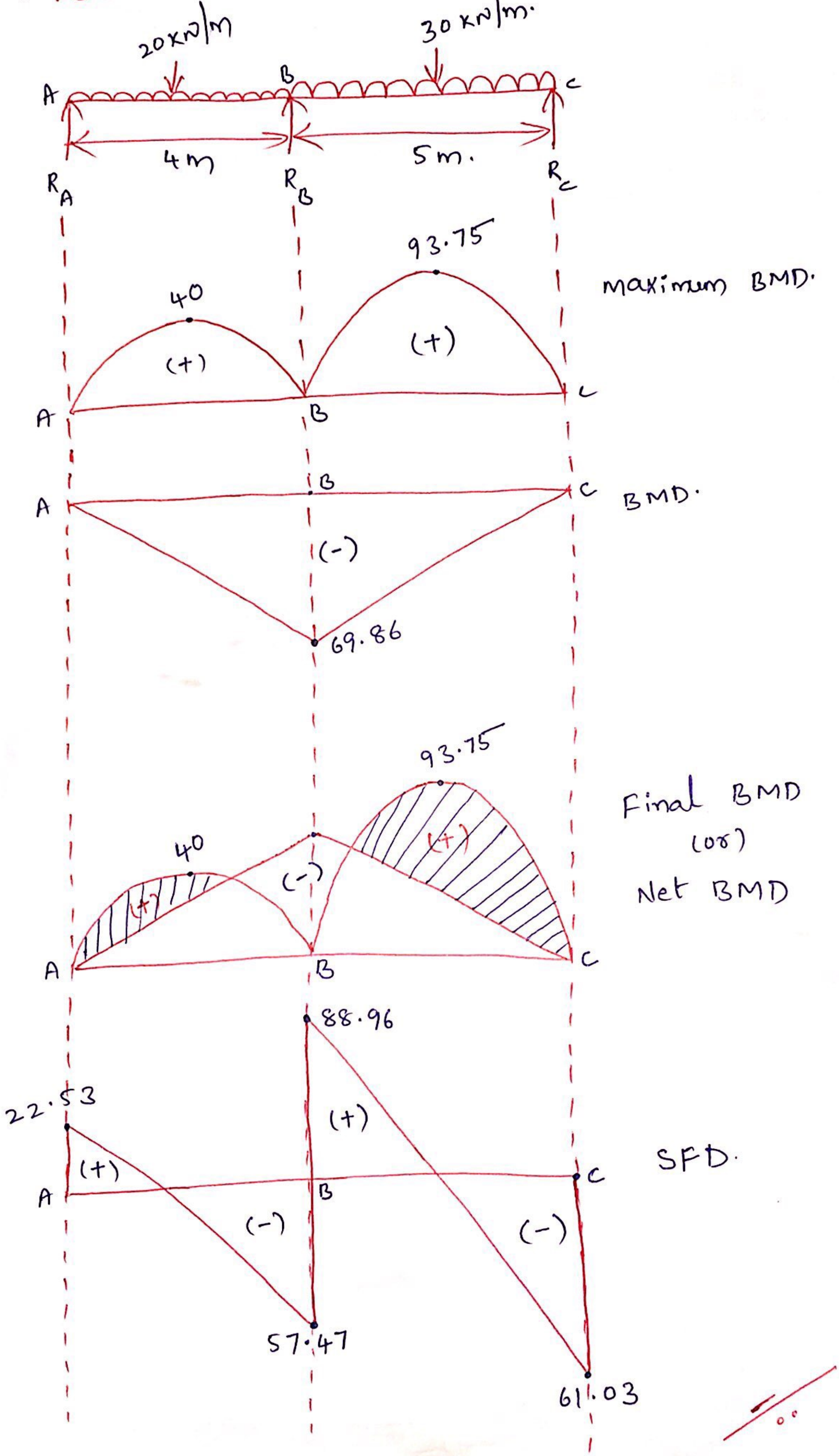
$R_{B2} + 61.03 = 150$

$$\boxed{R_{B2} = 88.79 \text{ KN.}}$$

$\therefore R_B = R_{B1} + R_{B2} = 57.46 + 88.79$

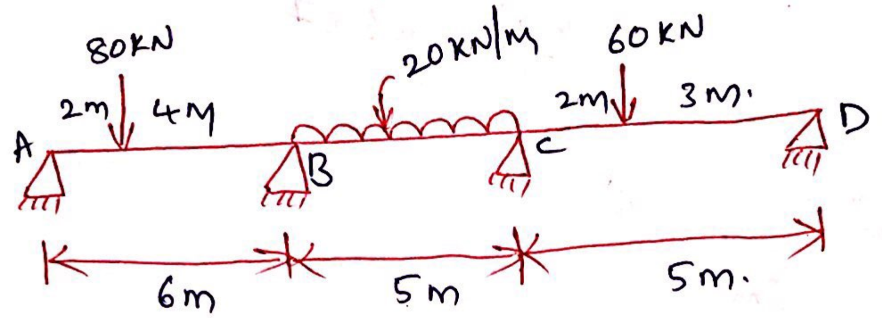
$$\boxed{R_B = 146.43 \text{ KN.}}$$

BMD & SFD :—

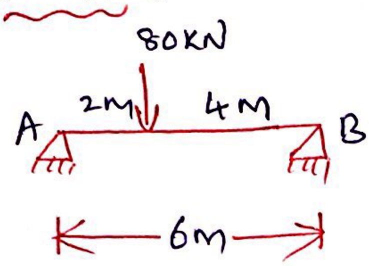


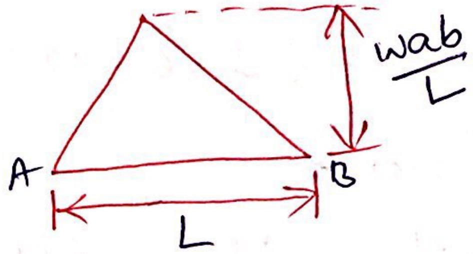
20 kN/m
30 kN/m.
A
B
C
4m
5m.
R_A
R_B
R_C
93.75
40
maximum BMD.
(+)
(+)
A
B
C
A
B
C
BMD.
(-)
69.86
93.75
Final BMD
(or)
Net BMD
40
(-)
(+)
(+)
A
B
C
88.96
22.53
(+)
(+)
SFD.
A
B
C
(-)
(-)
57.47
61.03

(i) Analyse the continuous beam by TMT & Draw SFD, BMD. (5)

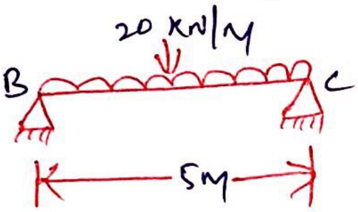


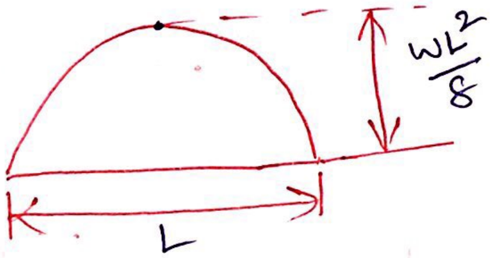
(A) Draw FBMD :—



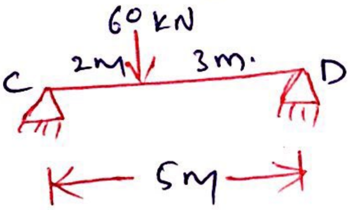


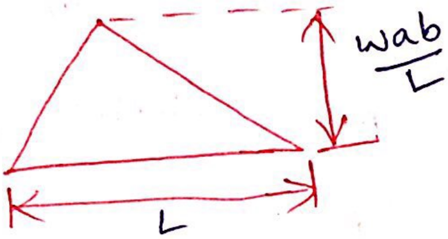
$$AB = \frac{wab}{L} = \frac{80 \times 2 \times 4}{6} = 106.67\ kN \cdot m$$





$$BC = \frac{WL^2}{8} = \frac{20 \times (5)^2}{8} = 62.5\ kN \cdot m.$$





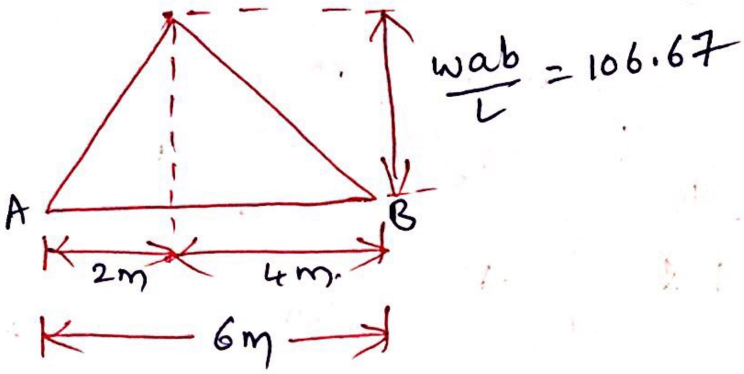
$$CD = \frac{wab}{l} = \frac{60 \times 2 \times 3}{5} = 72\ kN \cdot m.$$

Area and Centroid of FBMD :—

Area of AB, $A_1 = \frac{1}{2} \times b \times h$

Area of BC, $A_2 = \cancel{wL^2}\ \frac{2}{3} \times b \times h$

Area of CD, $A_3 = \frac{1}{2} \times b \times h.$

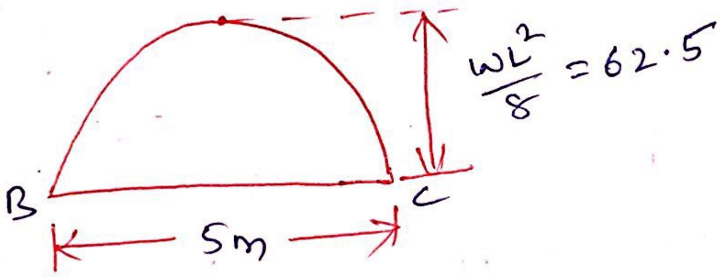


$A_1 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 6 \times 106.67$

$A_1 = 320.01$

$x_L = \frac{a+L}{3} = \frac{2+6}{3} = 2.67\,m.$

$x_R = \frac{b+L}{3} = \frac{4+6}{3} = 3.33\,m.$

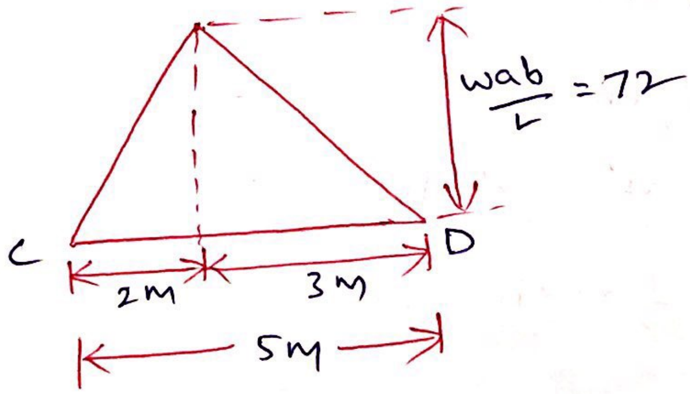


$A_2 = \frac{2}{3} \times b \times h = \frac{2}{3} \times 5 \times 62.5$

$A_2 = 208.33$

$x_L = \frac{L}{2} = \frac{5}{2} = 2.5\,m.$

$x_R = \frac{L}{2} = \frac{5}{2} = 2.5\,m.$



$A_3 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 5 \times 72$

$A_3 = 180$

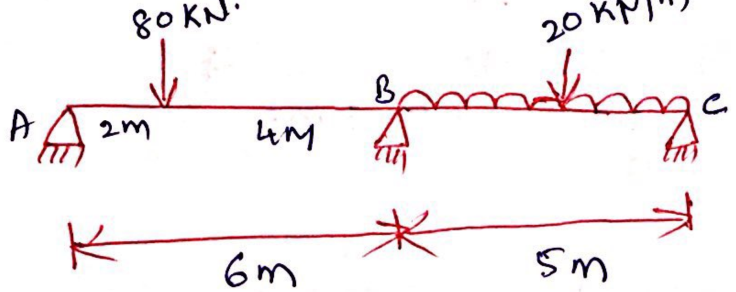
$x_L = \frac{a+L}{3} = \frac{2+5}{3} = 2.33\,m.$

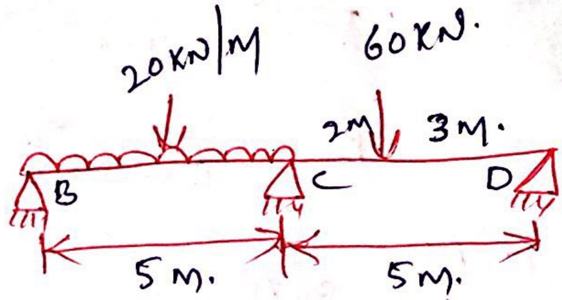
$x_R = \frac{b+L}{3} = \frac{3+5}{3} = 2.67\,m.$

Apply TMT :—

ABC

$M_A = 0, \quad M_D = 0$





$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 + \frac{6A_1\bar{x}_L}{l_1} + \frac{6A_2\bar{x}_R}{l_2} = 0$$

$$(0)(6) + 2M_B(6+5) + M_C(5) + \frac{6(320.01)(2.67)}{6} + \frac{6(208.33)(2.5)}{5} = 0$$

$$22M_B + 5M_C = -1479.41 \rightarrow (1)$$

ABCD

$$M_A l_1 + 2M_B(l_1 + l_2) + M_C l_2 + \frac{6A_1\bar{x}_L}{l_1} + \frac{6A_2\bar{x}_R}{l_2} = 0$$

BCD

$$M_B l_2 + 2M_C(l_2 + l_3) + M_D l_3 + \frac{6A_2\bar{x}_L}{l_2} + \frac{6A_3\bar{x}_R}{l_3} = 0$$

$$M_B(5) + 2M_C(5+5) + 0(5) + \frac{6(208.33)(2.5)}{5} + \frac{6(180)(2.67)}{5} = 0$$

$$5M_B + 20M_C = -1201.71 \rightarrow (2)$$

By solving eqns (1) & (2)

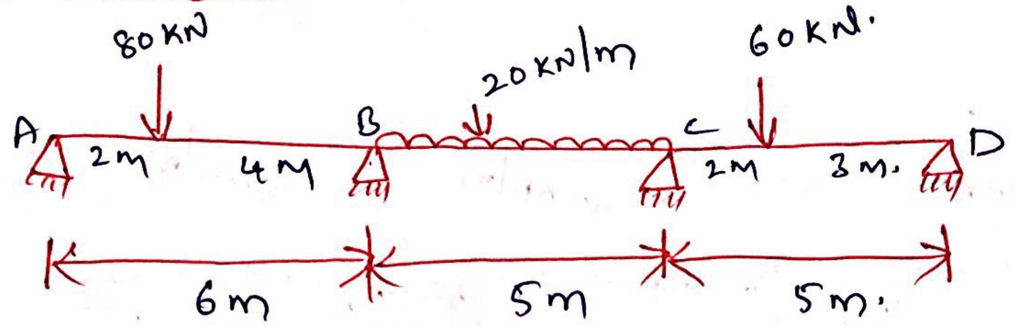
$$22M_B + 5M_C = -1479.41$$

$$5M_B + 20M_C = -1201.71$$

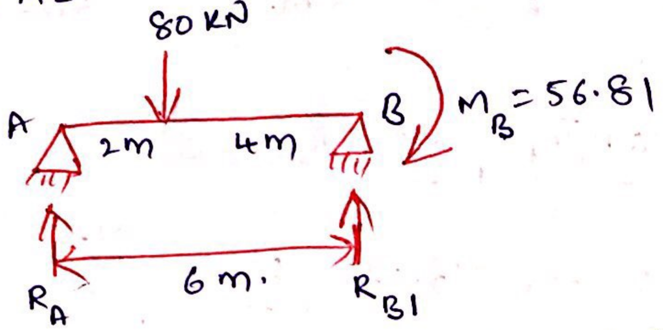
$$\boxed{M_B = -56.81 \text{ KN.m.}}$$

$$\boxed{M_C = -45.88 \text{ KN.m.}}$$

Support Reactions :—



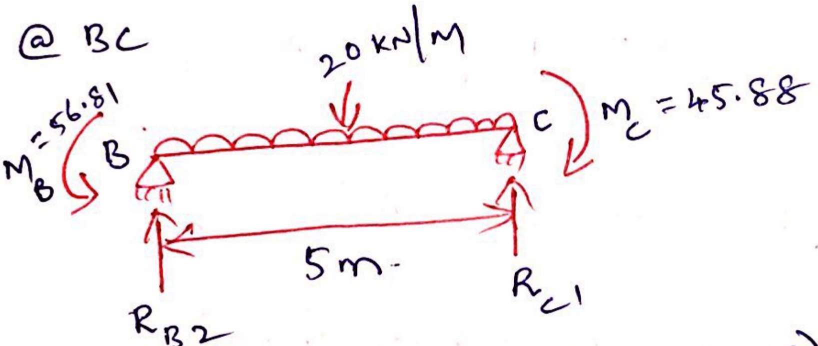
@ AB



$\Sigma M_A = 0$; $(80 \times 2) - (R_{B1} \times 6) + 56.81 = 0 \Rightarrow \boxed{R_{B1} = 36.135\ KN}$

$\Sigma f_y = 0$; $R_A + R_{B1} - 80 = 0 \Rightarrow R_A + 36.135 = 0 \Rightarrow \boxed{R_A = 43.865}$

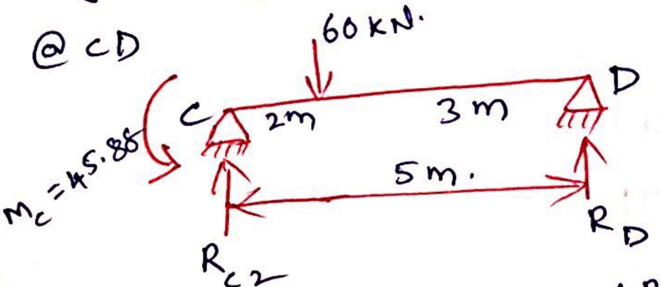
@ BC



$\Sigma M_B = 0$; $-56.81 + (20 \times 5 \times \frac{5}{2}) - (R_{C1} \times 5) + 45.88 = 0 \Rightarrow \boxed{R_{C1} = 47.814\ KN}$

$\Sigma f_y = 0$; $R_{B2} + R_{C1} - (20 \times 5) = 0 \Rightarrow R_{B2} + 47.814 - 100 = 0 \Rightarrow \boxed{R_{B2} = 52.186\ KN}$

@ CD



$\Sigma M_C = 0$; $-45.88 + (60 \times 2) - (R_D \times 5) = 0 \Rightarrow \boxed{R_D = 14.824\ KN.}$

$\Sigma f_y = 0$; $R_{C2} + R_D - 60 = 0 \Rightarrow R_{C2} + 14.824 - 60 = 0 \Rightarrow \boxed{R_{C2} = 45.176\ KN.}$

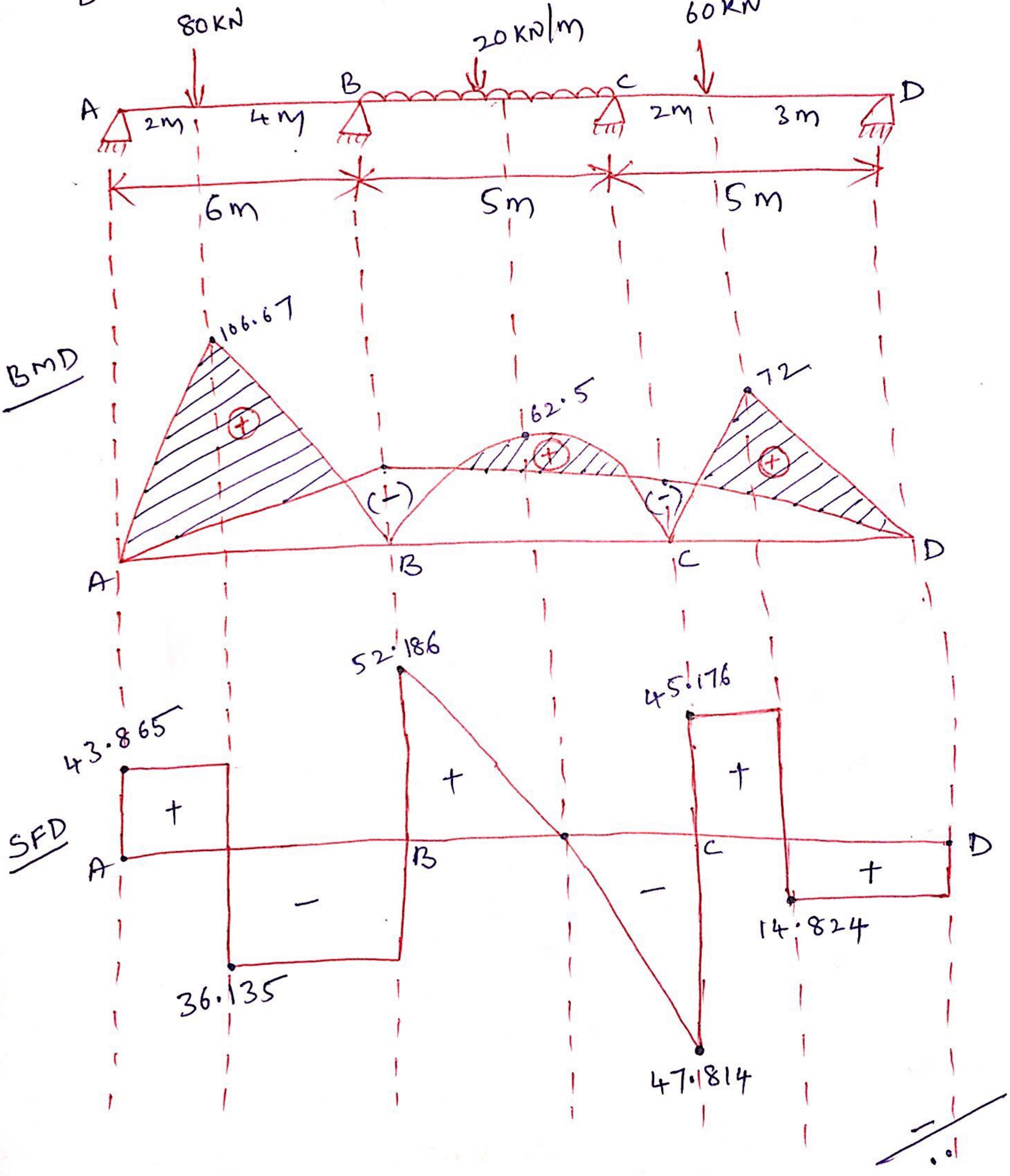
Final support Reactions :—

$R_A = 43.865$ KN.

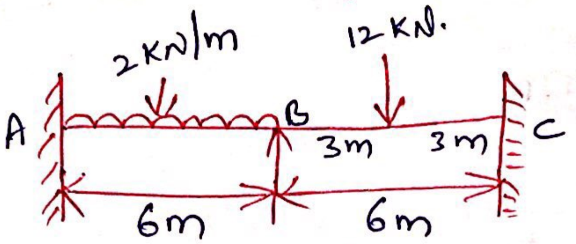
$R_B = R_{B1} + R_{B2} = 36.135 + 52.186 = 88.321$ KN.

$R_C = R_{C1} + R_{C2} = 47.814 + 45.176 = 92.99$ KN.

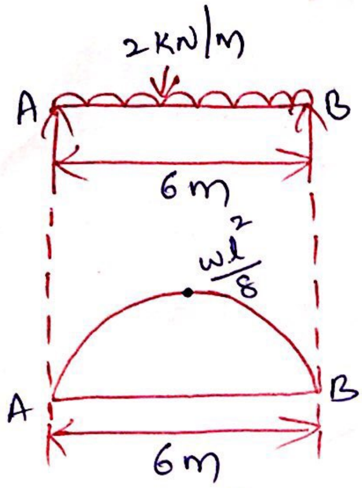
$R_D = 14.824$ KN.

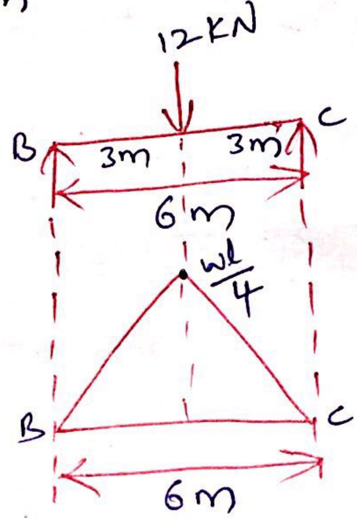


① Evaluate the BM & SF of beam shown in fig. ⑧



Ⓐ





Max. BM $= \frac{wl^2}{8}$

$= \frac{2 \times (6)^2}{8}$

$= 9$ KN.m.

Max. BM $= \frac{wl}{4}$

$= \frac{12 \times 6}{4}$

$= 18$ KN.m.

Area, $A_1 = \frac{2}{3} \times b \times h$

$= \frac{2}{\cancel{3}} \times \overset{2}{\cancel{6}} \times 9$

$= 36$

Area, $A_2 = \frac{1}{2} \times b \times h$

$= \frac{1}{\cancel{2}} \times \overset{3}{\cancel{6}} \times 18$

$= 54$

Centroid :

$x_L = \frac{l}{2} = \frac{6}{2} = 3$ m

$x_R = \frac{l}{2} = \frac{6}{2} = 3$ m.

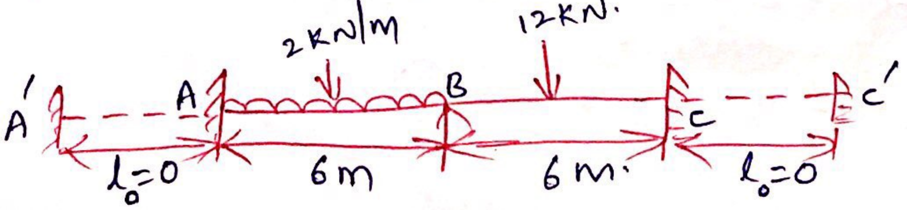
Centroid :

$x_L = \frac{l}{2} = \frac{6}{2} = 3$ m.

$x_R = \frac{l}{2} = \frac{6}{2} = 3$ m.

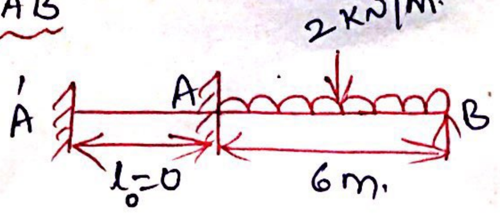
Apply TMT.

we will assume an imaginary span on both sides. (A' & c').



Scanned with CamScanner

span $A'AB$



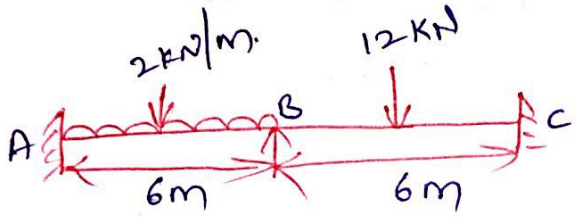
$$M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 + \frac{6A_1 x_L}{l_1} + \frac{6A_2 x_R}{l_2} = 0.$$

$$\cancel{M'_A l_0}^{0} + 2M_A(l_0+l_1) + M_B l_1 + \cancel{\frac{6A_0 x_L}{l_0}}^{0} + \frac{6A_1 x_R}{l_1} = 0$$

$$2M_A(6) + M_B(6) + \frac{6(36)(3)}{6} = 0$$

$$12M_A + 6M_B = -108 \rightarrow (1)$$

span ABC

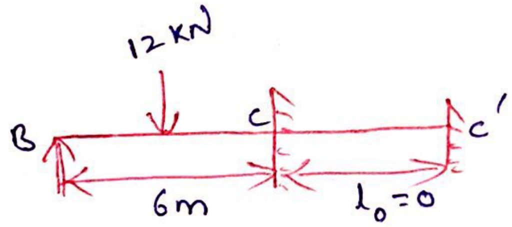


$$M_A l_1 + 2M_B(l_1+l_2) + M_C \cdot l_2 + \frac{6A_1 x_L}{l_1} + \frac{6A_2 x_R}{l_2} = 0.$$

$$M_A(6) + 2M_B(6+6) + M_C(6) + \frac{6(36)(3)}{6} + \frac{6(54)(3)}{6} = 0$$

$$6M_A + 24M_B + 6M_C = -270 \rightarrow (2)$$

Span BCC'



$$M_A \cdot l_1 + 2M_B(l_1+l_2) + M_C \cdot l_2 + \frac{6A_1 x_L}{l_1} + \frac{6A_2 x_R}{l_2} = 0.$$

$$M_B \cdot l_2 + 2M_C(l_2+l_0) + M'_C \cdot l_0 + \frac{6A_2 x_L}{l_2} + \frac{6A_0 \cdot x_R}{l_0} = 0$$

$$M_B(6) + 2M_C(6+0) + 0 + \frac{6(54)(3)}{6} + 0 = 0$$

$$6M_B + 12M_C = -162 \rightarrow (3)$$

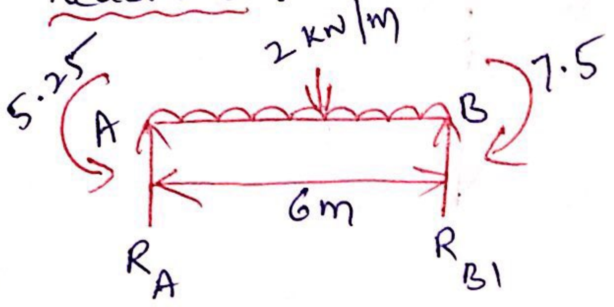
Solving eqns 1, 2, 3

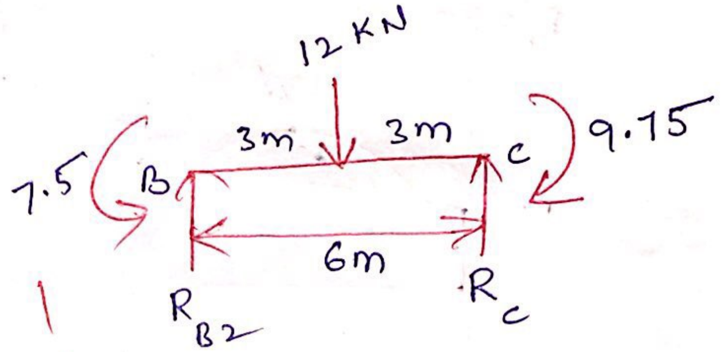
$M_A = -5.25$ KN.m.

$M_B = -7.5$ KN.m.

$M_C = -9.75$ KN.m.

Reactions :—





$\Sigma f_y = 0$;

$R_A + R_{B1} - (2\times 6) = 0$

$R_A + R_{B1} = 12 \rightarrow$ (4)

$\Sigma M_A = 0$;

$-5.25 + \left(2\times\frac{6}{2}\times\frac{6}{2}\right) + 7.5 - (R_{B1}\times 6) = 0$

$$\boxed{R_{B1} = 6.375 \text{ KN}}$$

R_{B1} sub. in (4)

$R_A + 6.375 = 12$

$$\boxed{R_A = 5.625 \text{ KN}}$$

$\Sigma f_y = 0$;

$R_{B2} + R_C - 12 = 0$

$R_{B2} + R_C = 12 \rightarrow$ (5)

$\Sigma M_B = 0$;

$-7.5 + (12\times 3) + 9.75 - (R_C\times 6) = 0$

$$\boxed{R_C = 6.375 \text{ KN.}}$$

R_C sub. in (5)

$R_{B2} + 6.375 = 12$

$$\boxed{R_{B2} = 5.625 \text{ KN.}}$$

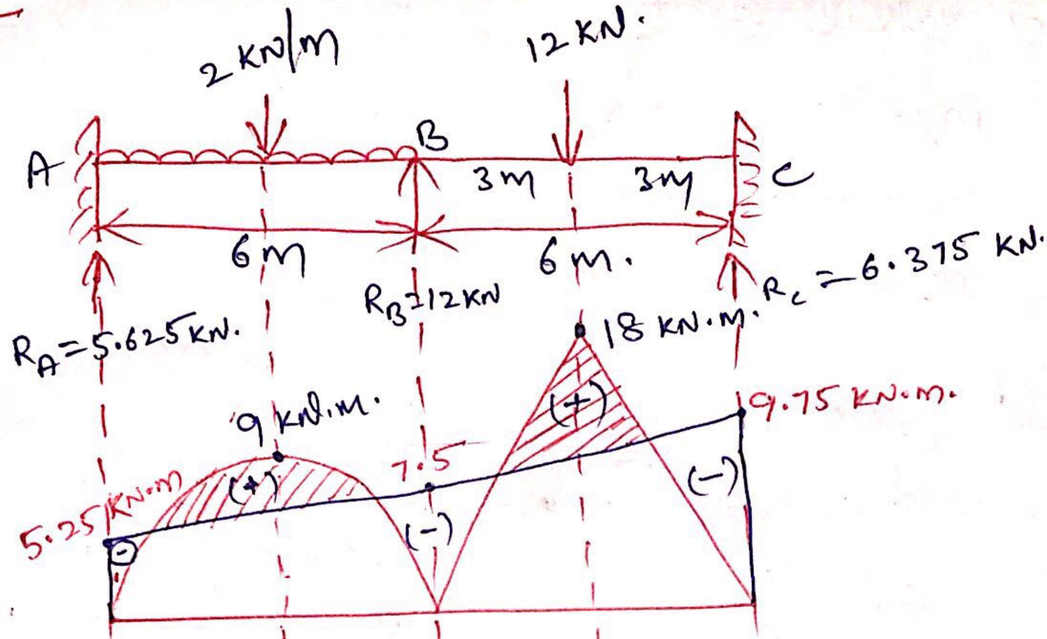
Final support Reactions :—

$R_A = 5.625$ KN.

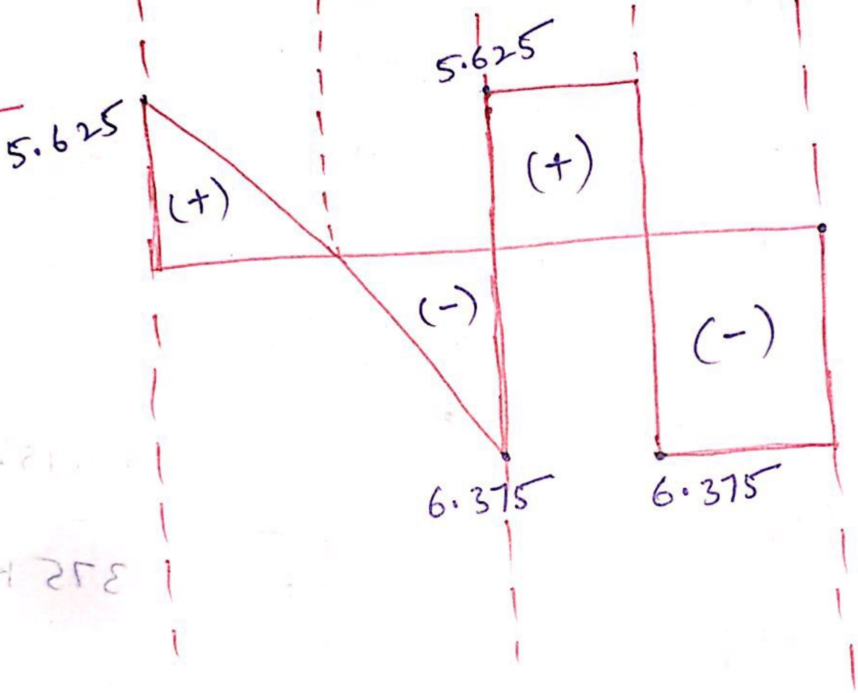
$R_B = R_{B1} + R_{B2} = 6.375 + 5.625 = 12$ KN.

$R_C = 6.375$ KN.

BMD :-



SFD :-



0 KN.

$+5.625$ KN.

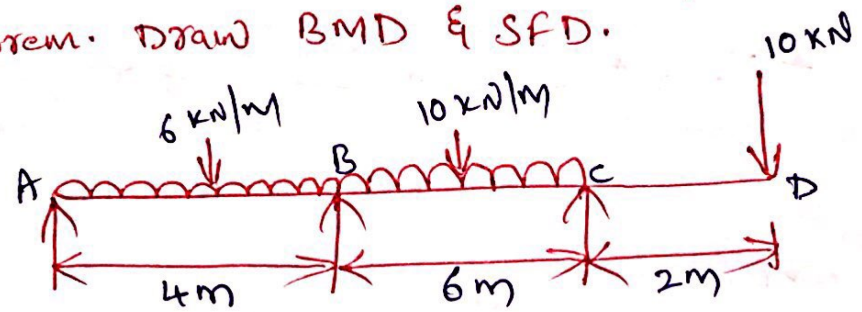
$5.625 - (2 \times 6) = -6.375$ KN.

$-6.375 + 12 = +5.625$ KN.

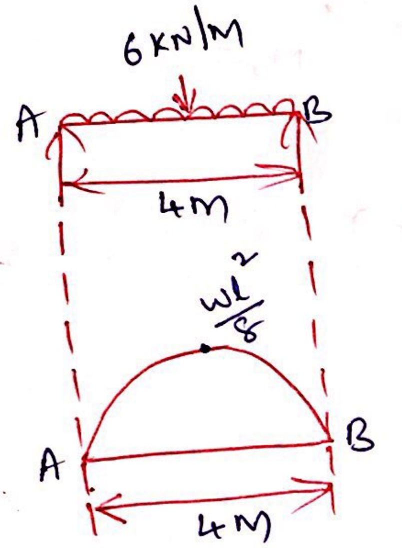
$5.625 - 12 = -6.375$ KN.

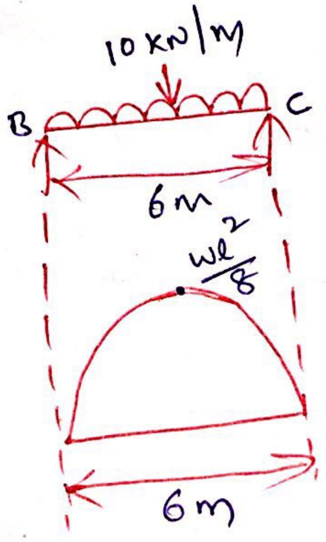
$-6.375 + 6.375 = 0$ KN.

① Analyse the continuous beam by TMT (or) clapeyron's theorem. Draw BMD & SFD. (10)



(A)





Max. BM, AB $= \frac{wl^2}{8}$

$= \frac{6 \times (4)^2}{8}$

$= 12$ KN.m.

Area, $A_1 = \frac{2}{3} \times b \times h$

$= \frac{2}{\cancel{3}} \times 4 \times \cancel{12}^{4}$

$= 32$

Centroid :

$x_L = \frac{l}{2} = \frac{4}{2} = 2m.$

$x_R = \frac{l}{2} = \frac{4}{2} = 2m.$

Max. BM, BC $= \frac{wl^2}{8}$

$= \frac{10 \times (6)^2}{8}$

$= 45$ KN.m.

Area, $A_2 = \frac{2}{3} \times b \times h$

$= \frac{2}{\cancel{3}} \times \cancel{6}^{2} \times 45$

$= 180$

Centroid :

$x_L = \frac{l}{2} = \frac{6}{2} = 3m.$

$x_R = \frac{l}{2} = \frac{6}{2} = 3m.$

Apply TMT (or) Clapeyron's Theorem :

$$M_A l_1 + 2M_B(l_1+l_2) + M_C l_2 + \frac{6A_1 x_L}{l_1} + \frac{6A_2 x_R}{l_2} = 0$$

$M_A = 0$

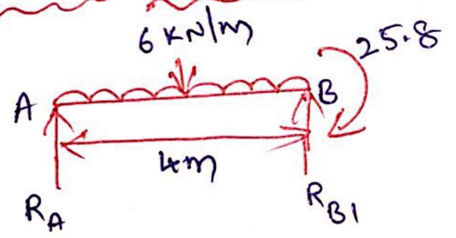
$M_C = -10 \times 2 = -20$ kN.m.

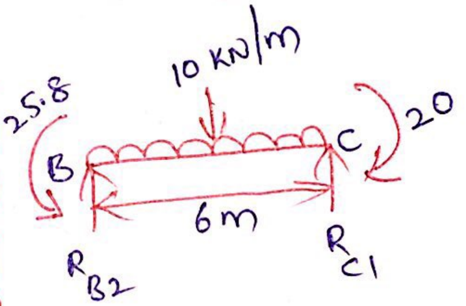
$l_1 = 4$ m	$A_1 = 32$	$x_L = 2$ m	$M_B = ?$
$l_2 = 6$ m	$A_2 = 180$	$x_R = 3$ m.	

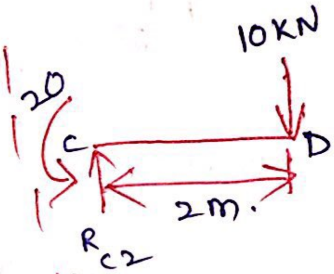
$$0(4) + 2M_B(4+6) + (-20)(6) + \frac{6(32)(2)}{4} + \frac{6(180)(3)}{6} = 0$$

$$\boxed{M_B = -25.8 \text{ kN.m.}}$$

Support Reactions :—







$\Sigma M_A = 0;$

$(6 \times 4 \times \frac{4}{2}) - (R_{B1} \times 4) + 25.8 = 0$

$\boxed{R_{B1} = 18.45 \text{ kN.}}$

$\Sigma f_y = 0;\ R_A + R_{B1} - (6 \times 4) = 0$

$R_A + 18.45 - 24 = 0$

$\boxed{R_A = 5.55 \text{ kN.}}$

$\Sigma M_A = 0;$

$-25.8 + (10 \times 6 \times \frac{6}{2}) - (R_{C1} \times 6) + 20 = 0$

$\boxed{R_{C1} = 29.03 \text{ kN.}}$

$\Sigma f_y = 0;\ R_{B2} + R_{C1} - (10 \times 6) = 0$

$R_{B2} + 29.03 - 60 = 0$

$\boxed{R_{B2} = 30.97 \text{ kN}}$

$\Sigma f_y = 0;$

$R_{C2} - 10 = 0$

$\boxed{R_{C2} = 10 \text{ kN.}}$

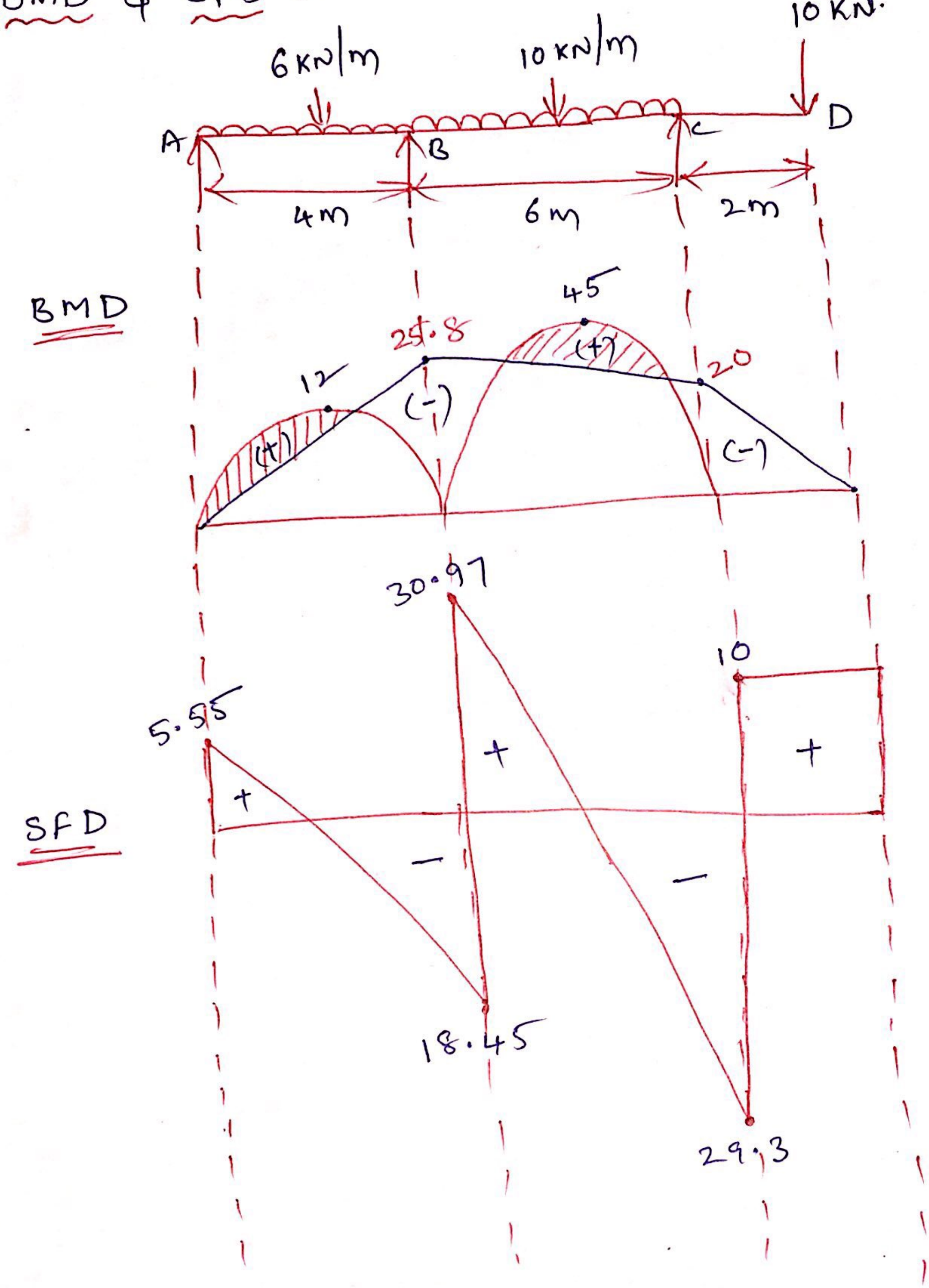
Final support Reactions :—

$R_A = 5.55$ kN.

$R_B = R_{B1} + R_{B2} = 18.45 + 30.97 = 49.42$ kN.

$R_C = R_{C1} + R_{C2} = 29.03 + 10 = 39.03$ kN.

BMD & SFD :-


6 KN/m
10 KN/m
10 KN.
A
B
C
D
4m
6m
2m
BMD
12
25.8
45
20
(+)
(-)
(+)
(-)
30.97
10
5.55
+
+
+
SFD
-
-
18.45
29.3

Slope Deflection method :-

Static Indeterminacy for frames :-

Degree of static indeterminacy (D_s)

$$\boxed{D_s = (3M + R) - (3j + A)}$$

where :

M = Members

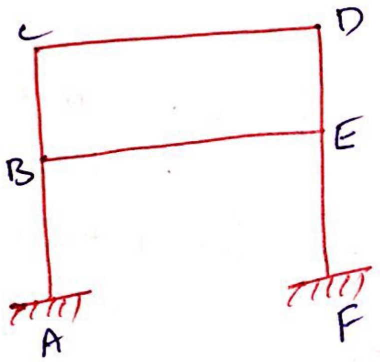
R = Reactions

j = joints

A = No. of additional eqlls due to internal hinge

$= (M - 1)$

(1)

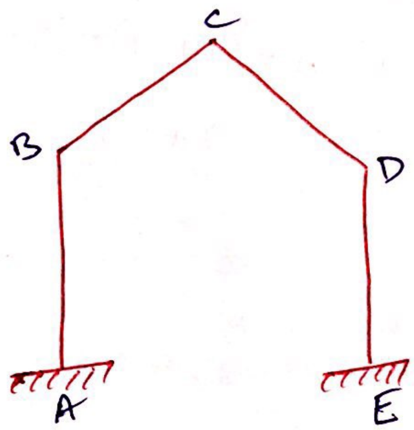


$M = 6,\ R = 6,\ j = 6,\ A = 0$

$D_s = (3 \times 6 + 6) - 3(6) + 0$

$$\boxed{D_s = 6}$$

(2)

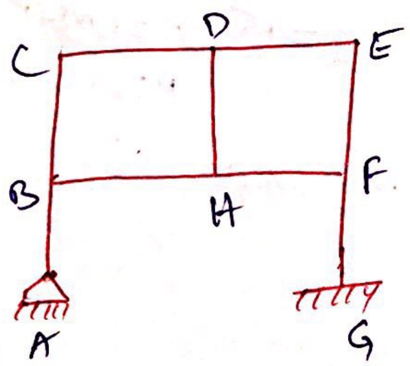


$R = 6,\ M = 4,\ j = 5,\ A = 0$

$D_s = (3 \times 4 + 6) - (3 \times 5 + 0)$

$$\boxed{D_s = 3}$$

(3)

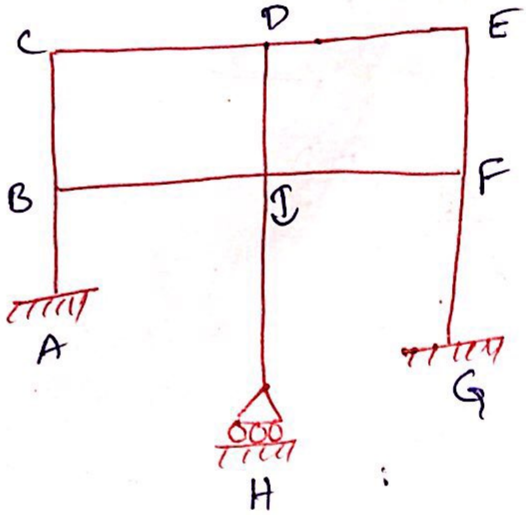


$R = 5,\ M = 9,\ j = 8$

$D_s = (3 \times 9 + 5) - (3 \times 8 + 0)$

$$\boxed{D_s = 8}$$

(4)

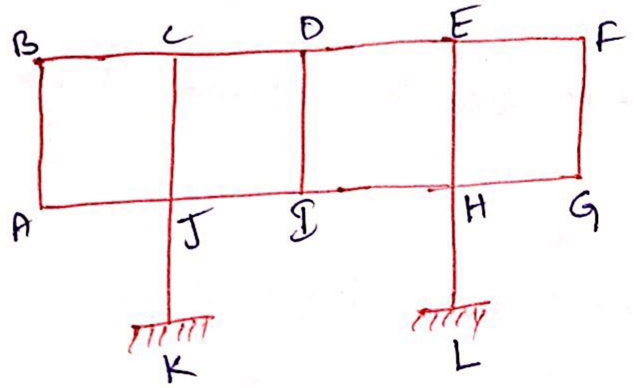


$R = 7, M = 10, j = 9, A = 0$

$D_S = (3 \times 10 + 7) - (3 \times 9 + 0) = 0$

$\boxed{D_S = 10}$

(5)

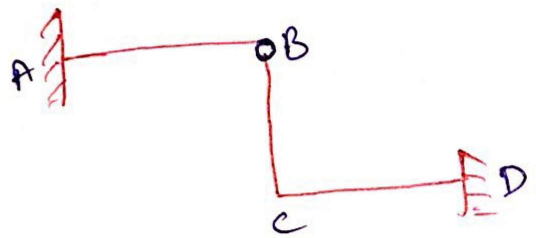


$R = 6, M = 15, 12 = j; A = 0$

$D_S = (3 \times 15 + 6) - (3 \times 12 + 0)$

$\boxed{D_S = 15}$

(6)



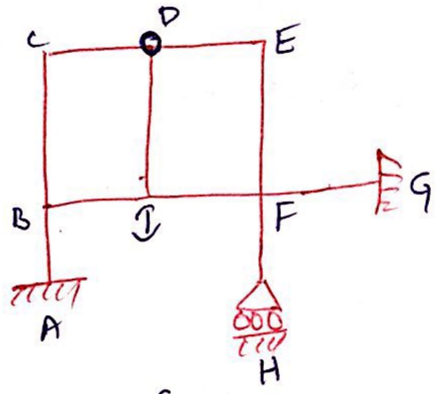
$R = 6, M = 3, j = 4, A = (m - 1)$

$A = (2 - 1) = 1$

$D_S = (3 \times 3 + 6) - (3 \times 4 + 1)$

$\boxed{D_S = 2}$

(7)

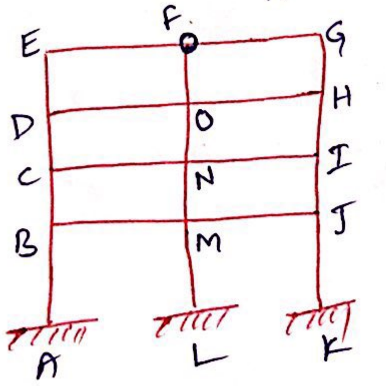


$R = 7, M = 10, j = 9, \quad A = (m - 1) = (3 - 1) = 2$

$D_S = (3 \times 10 + 7) - (3 \times 9 + 2)$

$\boxed{D_S = 8}$

(8)



$R = 9, M = 20, j = 15, A = (M - 1)$

$A = (3 - 1) = 2$

$D_S = (3 \times 20 + 9) - \{3 \times 15 + 2)$

$\boxed{D_S = 22}$

Kinematic Indeterminacy for frames :—

Degree of Kinematic indeterminacy (D_K)

$$\boxed{D_K = 3j - R + A}$$

where :

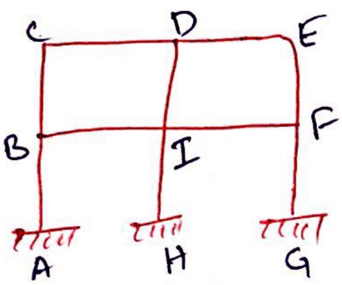
j = joints

R = Reactions

A = No. of additional eqns due to internal hinge

$= (M-1)$.

(1)

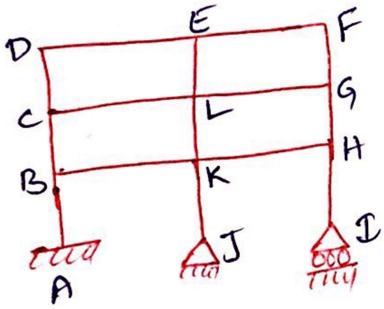


$R = 9,\ j = 9,\ A = 0$

$D_K = 3(9) - 9 + 0$

$\boxed{D_K = 18}$

(2)

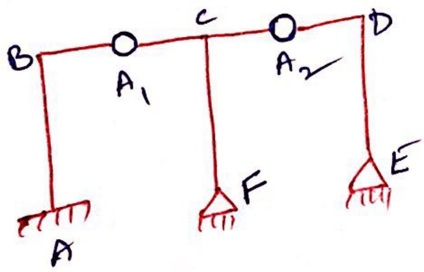


$R = 6,\ j = 12,\ A = 0$

$D_K = 3(12) - 6 + 0$

$\boxed{D_K = 30}$

(3)



$R = 7,\ j = 8,\ A_1 = (M-1) = (2-1) = 1$

$A_2 = (M-1) = (2-1) = 1$

$\therefore A = A_1 + A_2$

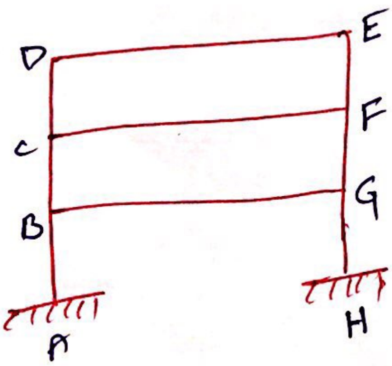
$A = 1 + 1$

$A = 2$

$D_K = 3(8) - 7 + 2$

$\boxed{D_K = 19}$

(4)



$R = 6,\ j = 8,\ A = 0$

$D_K = 3(8) - 6 + 0$

$\boxed{D_K = 18}$

Slope Deflection method :—

It was derived by G. A. Maney. Here the joint displacements (i.e. θ, Δ) are taken as unknown and the joint moments are derived by the force displacement relation which is called as slope deflection equation.

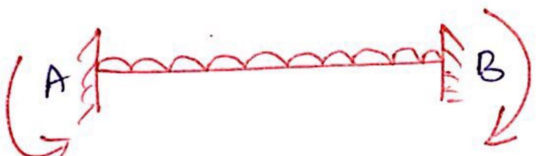
Note :— In this method axial force and shear force effects are neglected and only bending moment effects are only considered.

Sign convention :— Fixed end moments :—

clockwise +ve

Anticlockwise −ve

Ex :-

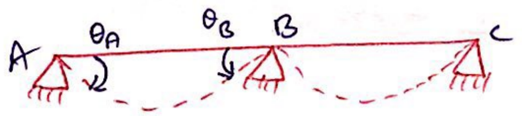


$$M_{F_{AB}} = \frac{-wl^2}{12} \qquad M_{F_{BA}} = \frac{+wl^2}{12}$$

Rotations :—

clockwise → +ve

Anticlockwise → −ve

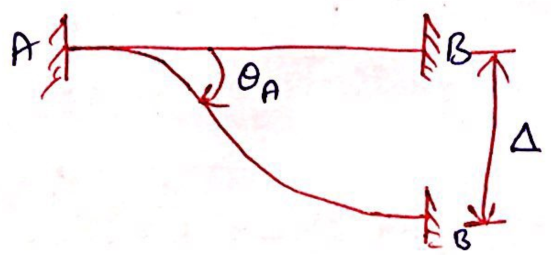


θ_A → +ve

θ_B → −ve

Settlement :—

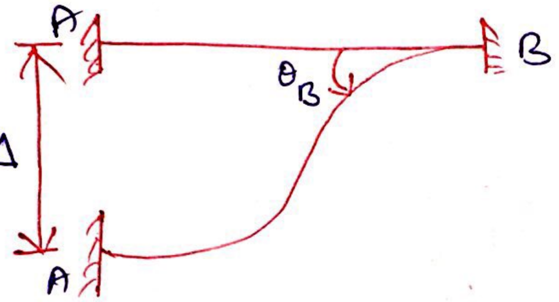
$\Delta \rightarrow$ +ve when it leads to clockwise rotation.



$\theta_A \rightarrow$ clockwise

$\Delta \Rightarrow$ +ve.

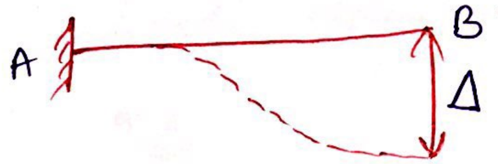
$\Delta \rightarrow$ -ve when it leads to anticlockwise rotation.



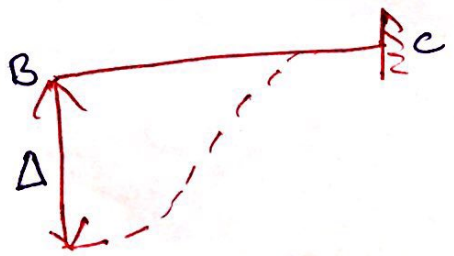
$\theta_B \rightarrow$ Anticlock wise

$\Delta \Rightarrow$ -ve.

(OR)



Right side down then Δ +ve for AB & BA.

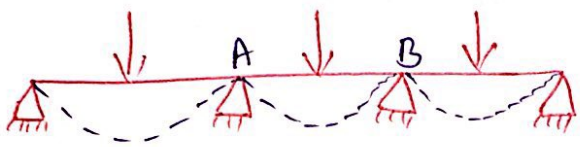


Left side down then Δ -ve. for BC, & CB.

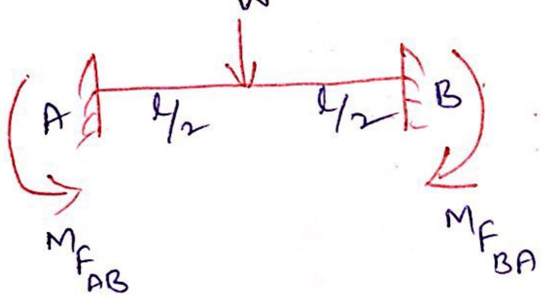
Slope Deflection Equation :—

Considering a continuous beam where A, B are the intermediate supports, the final end moments developed at A & B (M_{AB}, M_{BA}) will be due to the effects of :

(i) Effect of loading
(ii) Rotation of joint A (θ_A)
(iii) Rotation of joint B (θ_B)
(iv) Effect of settlement of support.



(i) Effect of loading :—



(ii) Rotation of joint A :—

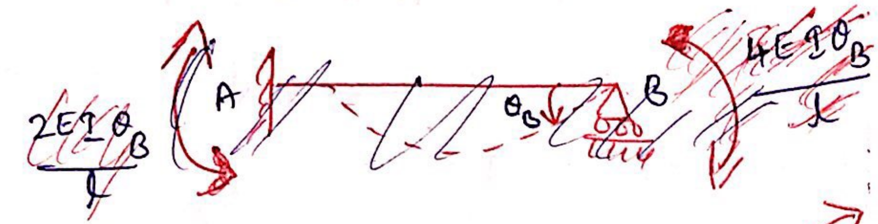
$$\theta_A = \frac{Ml}{4EI}$$

$$M = \frac{4EI\theta_A}{l}$$

$$\overline{M}_{AB1} = \frac{4EI\theta_A}{l}$$

$$\overline{M}_{BA1} = \frac{2EI\theta_A}{l}$$

(iii) Rotation of joint B :—

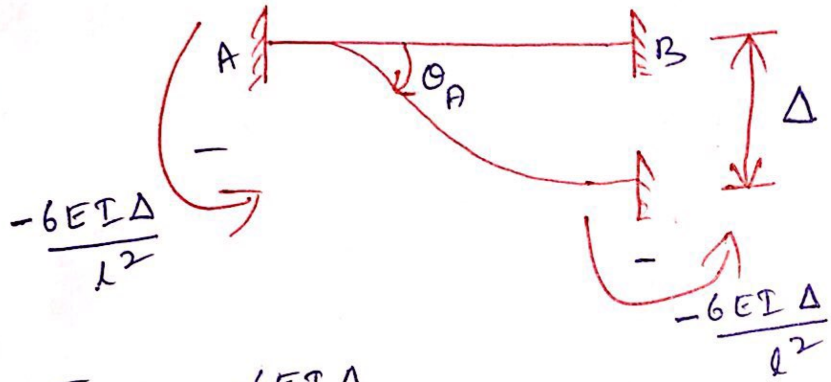


$$\bar{M}_{AB2} = \frac{2EI\theta_B}{l}$$

$$\bar{M}_{BA2} = \frac{4EI\theta_B}{l}$$

(iv) Effect of Settlement of support :—

Let support 'B' sinks by 'Δ'.



$$\bar{M}_{AB3} = \frac{-6EI\Delta}{l^2}$$

$$\bar{M}_{BA3} = \frac{-6EI\Delta}{l^2}$$

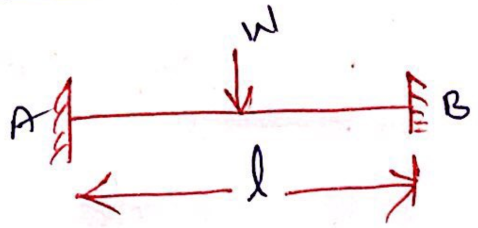
Final end moments :—

$$M_{AB} = M_{F_{AB}} + \bar{M}_{AB1} + \bar{M}_{AB2} + \bar{M}_{AB3}$$

$$M_{AB} = M_{F_{AB}} + \frac{4EI\theta_A}{l} + \frac{2EI\theta_B}{l} - \frac{6EI\Delta}{l^2}$$

$$\boxed{M_{AB} = M_{F_{AB}} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)}$$

Slope Deflection Method :—



$$M_{AB} = M_{F_{AB}} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$

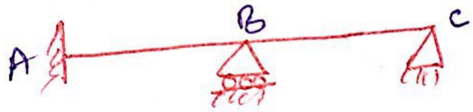
$$M_{BA} = M_{F_{BA}} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\Delta}{l}\right)$$

Δ = If deflection is given in question take 'Δ' (or)
Take $\Delta = 0$ if nothing is mentioned in question.

Same formula for all types of beam.

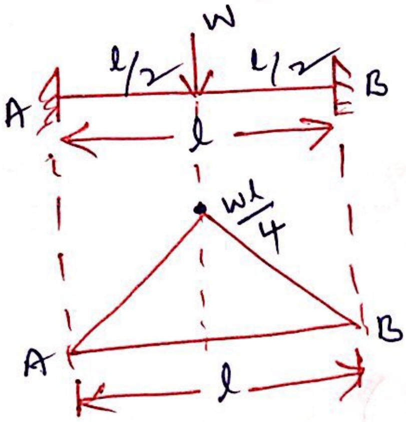
* At fixed end support then slope is zero (i.e. $\theta = 0$)
* while calculating fixed end moments consider all the points are fixed.

Ex:



Fixed end moments :—

(1)

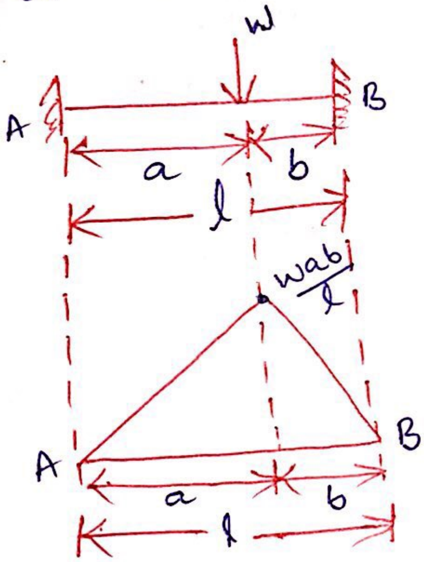


$$\text{Max. BM} = \frac{wl}{4}$$

$$M_{F_{AB}} = \frac{-wl}{8}$$

$$M_{F_{BA}} = \frac{wl}{8}$$

(2)

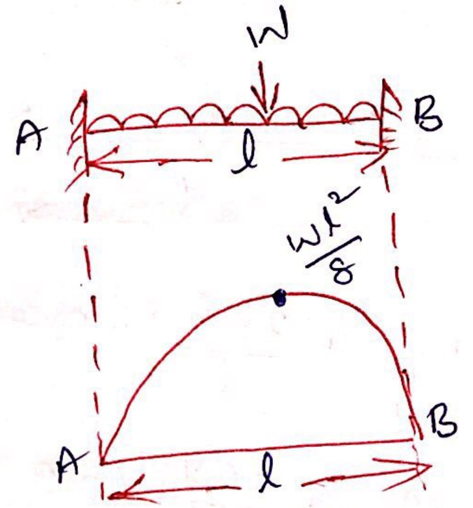


$$\text{Max. BM} = \frac{wab}{l}$$

$$M_{F_{AB}} = \frac{-wab^2}{l^2}$$

$$M_{F_{BA}} = \frac{wa^2b}{l^2}$$

(3)

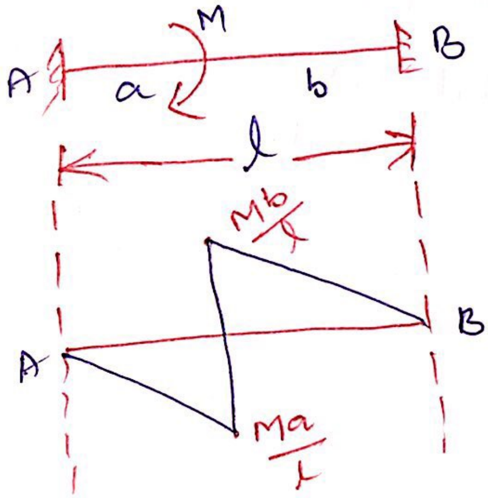


$$\text{Max. BM} = \frac{wl^2}{8}$$

$$M_{F_{AB}} = \frac{-wl^2}{12}$$

$$M_{F_{BA}} = \frac{wl^2}{12}$$

(4)



$$M_{F_{AB}} = \frac{Mb(2a-b)}{l^2}$$

$$M_{F_{BA}} = \frac{Ma(2b-a)}{l^2}$$

Steps for slope deflection method :—

(1) Find fixed end moments i.e. AB, BC, CD
BA, CB, DC.

(2) Apply slope deflection eqn for each span

(3) Apply joint equilibrium equations.

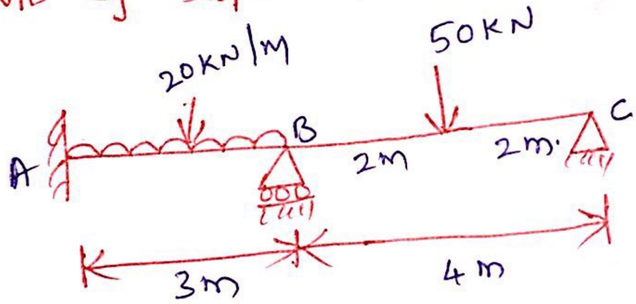
(4) Find final moments.

(5) Find support reactions.

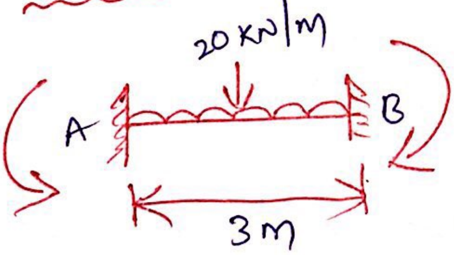
(6) Final support reactions.

(7) Draw BMD & SFD.

(1) Draw BMD by slope deflection method.



(A) Fixed end moments :—

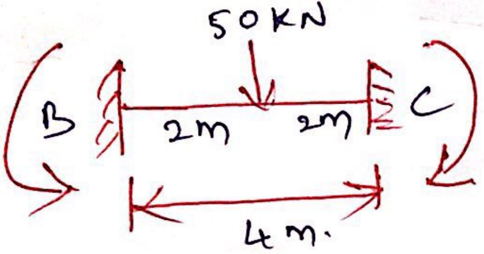


$$M_{F_{AB}} = \frac{-wl^2}{12} = -\frac{20\times(3)^2}{12}$$

$$\boxed{M_{F_{AB}} = -15\ KN\cdot m.}$$

$$M_{F_{BA}} = \frac{wl^2}{12} = \frac{20\times(3)^2}{12}$$

$$\boxed{M_{F_{BA}} = 15\ KN\cdot m.}$$



$$M_{F_{BC}} = \frac{-wl}{8} = \frac{-50 \times 4}{8}$$

$$\boxed{M_{F_{BC}} = -25 \text{ KN.m.}}$$

$$M_{F_{CB}} = \frac{wl}{8} = \frac{50 \times 4}{8}$$

$$\boxed{M_{F_{CB}} = 25 \text{ KN.m.}}$$

Apply slope deflection equation :-

$$\left(\theta_A = 0, \text{ fixed} \quad \Delta = 0 \right)$$

$$M_{AB} = M_{F_{AB}} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$

$$= -15 + \frac{2EI}{3}\left(2 \times 0 + \theta_B - \frac{3 \times 0}{3}\right)$$

$$M_{AB} = -15 + \frac{2EI\theta_B}{3} \rightarrow (1)$$

$$M_{BA} = M_{F_{BA}} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\Delta}{l}\right)$$

$$= 15 + \frac{2EI}{l}\left(2\theta_B + 0 - \frac{3 \times 0}{l}\right)$$

$$M_{BA} = 15 + \frac{4EI\theta_B}{3} \rightarrow (2)$$

$$M_{BC} = M_{F_{BC}} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\Delta}{l}\right)$$

$$M_{BC} = -25 + EI\theta_B + \frac{EI\theta_C}{2} \rightarrow (3)$$

$$M_{CB} = M_{F_{BC}} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\Delta}{l}\right)$$

$$M_{CB} = 25 + EI\theta_C + \frac{EI\theta_B}{2} \rightarrow (4)$$

Apply joint equilibrium eqns :—

Joint 'B' (BA, BC)

$$M_{BA} + M_{BC} = 0$$

$$15 + \frac{4EI\theta_B}{3} + \left(-25 + EI\theta_B + \frac{1}{2}EI\theta_C\right) = 0$$

$$EI\theta_B\left(\frac{4}{3}+1\right) + \frac{EI\theta_C}{2} = -15 + 25 \rightarrow (5)$$

Joint 'C' (CB)

$$M_{CB} = 0$$

$$25 + \frac{EI\theta_B}{2} + EI\theta_C = 0$$

$$\frac{EI\theta_B}{2} + EI\theta_C = -25 \rightarrow (6)$$

By solving eqns (5) & (6)

$$\boxed{\begin{aligned} EI\theta_B &= 10.8 \\ EI\theta_C &= -30.4 \end{aligned}}$$

Final moments :—

$EI\theta_B$ & $EI\theta_C$ values sub. in eqns (1), (2), (3), (4)

$$M_{AB} = -15 + \frac{2EI\theta_B}{3} = -15 + \frac{2(10.8)}{3}$$

$$\boxed{M_{AB} = -7.8 \text{ KN.m.}}$$

$$M_{BA} = 15 + \frac{4EI\theta_B}{3} = 15 + \frac{4(10.8)}{3}$$

$$\boxed{M_{BA} = 29.4 \text{ KN.m.}}$$

$$M_{BC} = -25 + EI\theta_B + \frac{EI\theta_C}{2}$$

$$= -25 + 10.8 + \frac{(-30.4)}{2}$$

$$\boxed{M_{BC} = -29.4 \text{ KN.m.}}$$

$$M_{CB} = 25 + EI\theta_C + \frac{EI\theta_B}{2}$$

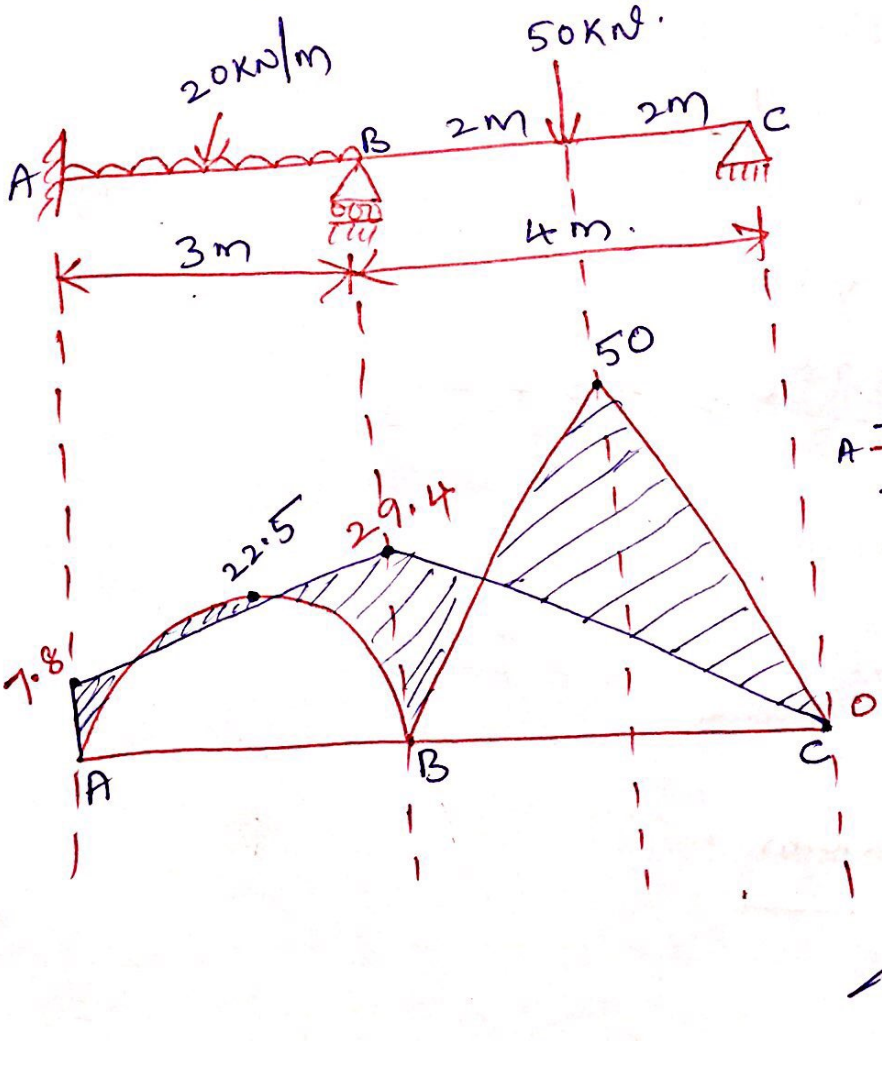
$$= 25 + (-30.4) + \frac{10.8}{2}$$

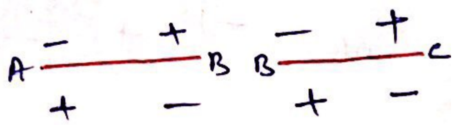
$$\boxed{M_{CB} = 0}$$

Final BMD :-

$$\text{Max. BM, AB} = \frac{wl^2}{8} = \frac{20 \times (3)^2}{8} = \boxed{22.5 \text{ KN.m.}}$$

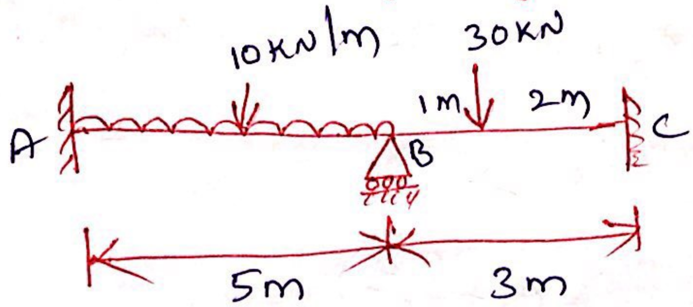
$$\text{Max. BM, BC} = \frac{wl}{4} = \frac{50 \times 4}{4} = \boxed{50 \text{ KN.m.}}$$





(2) Draw BMD & SFD by using slope deflection method.

(8)



(A) Fixed end moments :-

$$M_{F_{AB}} = \frac{-wl^2}{12} = \frac{-10 \times (5)^2}{12}$$

$$\boxed{M_{F_{AB}} = -20.833 \text{ KN.m.}}$$

$$M_{F_{BA}} = \frac{wl^2}{12} = \frac{10 \times (5)^2}{12}$$

$$\boxed{M_{F_{BA}} = 20.833 \text{ KN.m.}}$$

$$M_{F_{BC}} = \frac{-wab^2}{l^2} = \frac{-30 \times (1)(2)^2}{(3)^2}$$

$$\boxed{M_{F_{BC}} = -13.33 \text{ KN.m.}}$$

$$M_{F_{CB}} = \frac{wa^2b}{l^2} = \frac{30 \times (1)^2 (2)}{(3)^2}$$

$$\boxed{M_{F_{CB}} = 6.667 \text{ KN.m.}}$$

Apply slope deflection equation :—

$$M_{AB} = M_{F_{AB}} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right) \qquad (\theta_A = 0, \text{fixed}) \; (\Delta = 0)$$

$$M_{AB} = -20.833 + \frac{2EI\theta_B}{5} \rightarrow (1)$$

$$M_{BA} = M_{F_{BA}} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\Delta}{l}\right)$$

$$M_{BA} = 20.833 + \frac{4EI\theta_B}{5} \rightarrow (2)$$

$$M_{BC} = M_{F_{BC}} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\Delta}{l}\right) \qquad (\theta_C = 0, \text{fixed}) \; (\Delta = 0)$$

$$M_{BC} = -13.33 + \frac{4EI\theta_B}{3} \rightarrow (3)$$

$$M_{CB} = M_{F_{CB}} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\Delta}{l}\right)$$

$$M_{CB} = 6.667 + \frac{2EI\theta_B}{3} \rightarrow (4)$$

Apply joint equilibrium equations :—

Joint B (BA, BC)

$$M_{BA} + M_{BC} = 0$$

$$20.833 + \frac{4EI\theta_B}{5} + \left(-13.33 + \frac{4EI\theta_B}{3}\right) = 0$$

$$\boxed{EI\theta_B = -3.51}$$

$EI\theta_B$ value sub. in eqns (1), (2), (3), (4)

Scanned with CamScanner

Final moments :—

$EI\theta_B$ Sub. in eqns (1), (2), (3), (4)

$$M_{AB} = -20.833 + \frac{2(-3.51)}{5}$$

$$\boxed{M_{AB} = -22.23 \text{ KN.m.}}$$

$$M_{BA} = 20.833 + \frac{4(-3.51)}{5}$$

$$\boxed{M_{BA} = 18.01 \text{ KN.m.}}$$

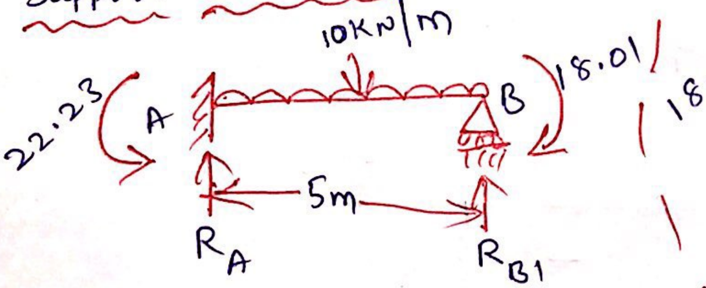
$$M_{BC} = -13.33 + \frac{4(-3.51)}{3}$$

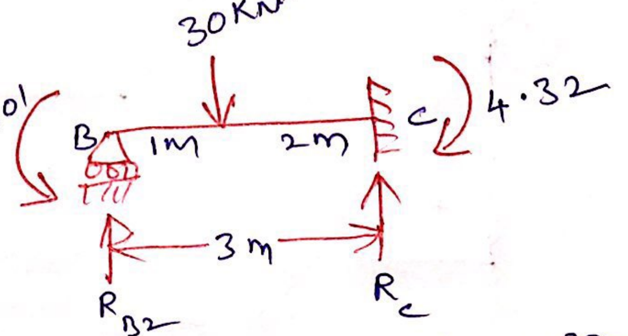
$$\boxed{M_{BC} = -18.01 \text{ KN.m.}}$$

$$M_{CB} = 6.667 + \frac{2(-3.51)}{3}$$

$$\boxed{M_{CB} = 4.32 \text{ KN.m.}}$$

Support Reactions :—





$\Sigma M_A = 0;\ -22.23 + (10 \times 5 \times \frac{5}{2}) + 18.01 - (R_B \times 5) = 0$

$$\boxed{R_B = 24.15 \text{ KN}}$$

$\Sigma f_y = 0;\ R_A + R_{B1} - (10 \times 5) = 0$

$$\boxed{R_A = 25.85 \text{ KN}}$$

$\Sigma M_B = 0;\ -18.01 + (30 \times 1) + 4.32 - (R_c \times 3) = 0$

$$\boxed{R_c = 5.43 \text{ KN}}$$

$\Sigma f_y = 0;\ R_{B2} + R_c - 30 = 0$

$$\boxed{R_{B2} = 24.57 \text{ KN.}}$$

Final support Reactions :—

$R_A = 25.85$ KN.

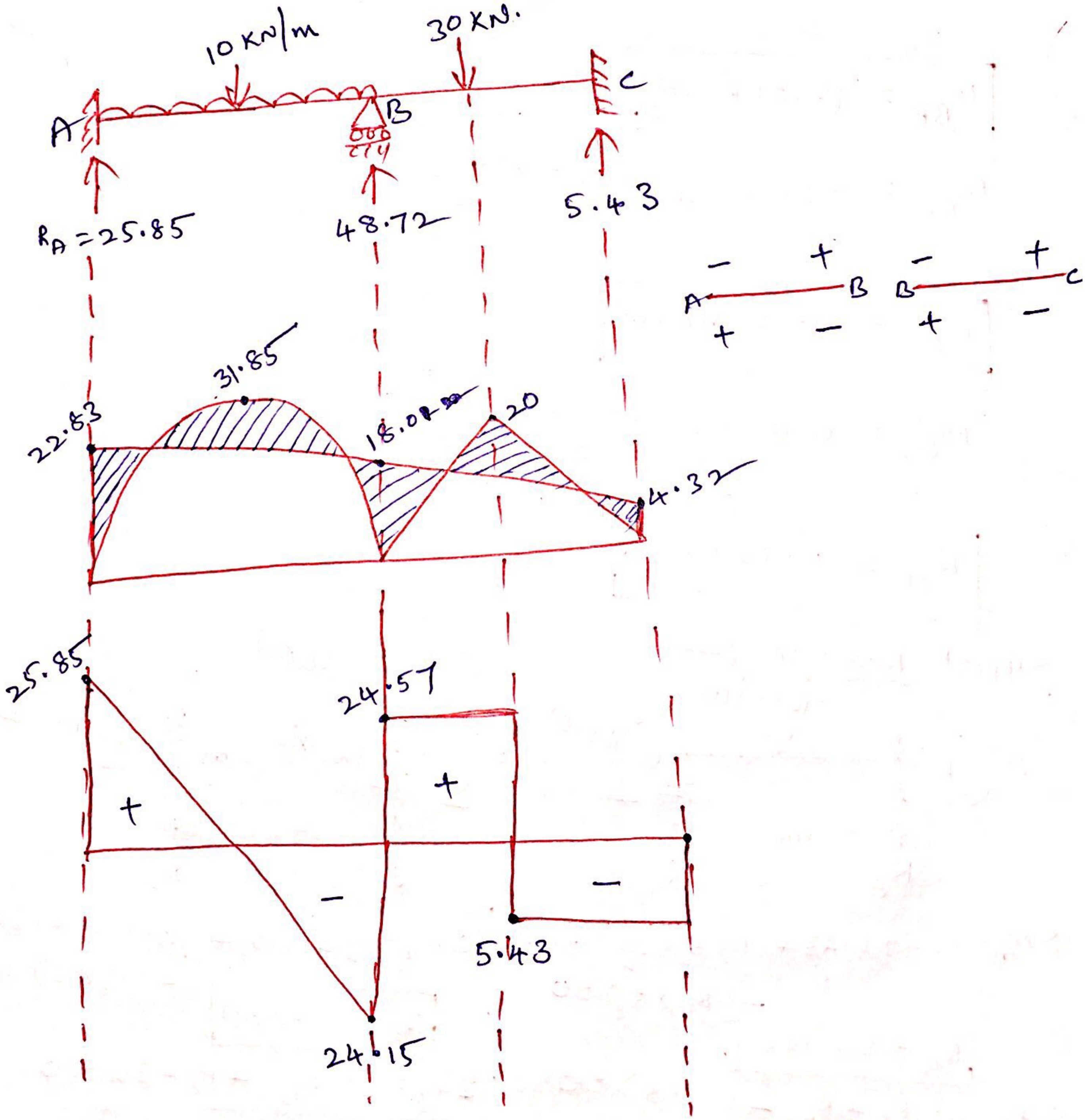
$R_B = R_{B1} + R_{B2} = 24.15 + 24.57 = 48.72$ KN.

$R_C = 5.43$ KN.

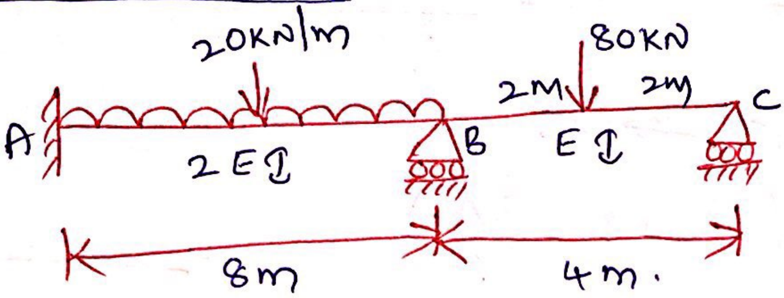
Final BMD & SFD :—

$$\text{Max. BM, } AB = \frac{wl^2}{8} = \frac{10 \times (5)^2}{8} = 31.85 \text{ KN.m.}$$

$$\text{Max. BM, } BC = \frac{wab}{l} = \frac{30 \times 1 \times 2}{3} = 20 \text{ KN.m.}$$



(3) Analyse the continuous beam shown in below by slope deflection method, if joint 'B' sinks by 10 mm. Given $EI = 4000\,KN{\cdot}m^2$. Draw BMD.

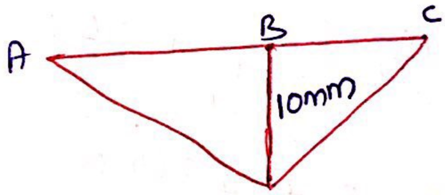


Fixed end moments :—

$$M_{F_{AB}} = \frac{-wl^2}{12} = -\frac{20\times(8)^2}{12} = -106.67\ KN{\cdot}m.$$

$$M_{F_{BA}} = \frac{wl^2}{12} = \frac{20\times(8)^2}{12} = 106.67\ KN{\cdot}m.$$

$$M_{F_{BC}} = \frac{-wl}{8} = -\frac{80\times4}{8} = -40\ KN{\cdot}m.$$



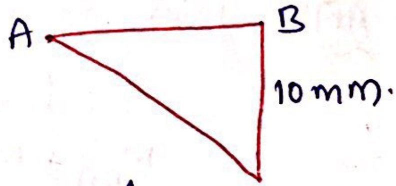
$$M_{F_{CB}} = \frac{wl}{8} = \frac{80\times4}{8} = 40\ KN{\cdot}m.$$

Apply slope deflection equation :—

$$M_{AB} = M_{F_{AB}} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$

$(\theta_A = 0,\ \text{fixed})$

$$= -106.67 + \frac{2(2EI)}{8}\left(2\times0 + \theta_B - \frac{3\times0.01}{8}\right)$$



$$= -106.67 + \frac{2(2\times4000)}{8}\left(\theta_B - 0.00375\right)$$

$\left(\begin{array}{l}\Delta = 10\,mm.\\ \Delta = 0.01\,m.\end{array}\right)$

$$= -106.67 + 2000(\theta_B - 0.00375)$$

$$= -106.67 + 2000\theta_B - 7.5$$

$$M_{AB} = 2000\theta_B - 114.17 \longrightarrow (1)$$

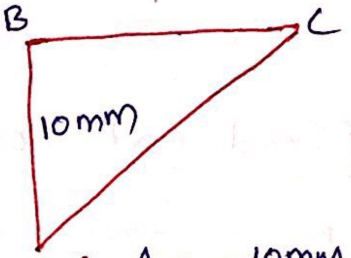
$$M_{BA} = M_{f_{BA}} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\Delta}{l}\right)$$

$$\left\{\begin{array}{l} \theta_A = 0, \text{ fixed} \\ \Delta = 10\text{ mm} \\ \Delta = 0.01\text{m.} \end{array}\right\}$$

$$= 106.67 + \frac{2(2\times4000)}{8}\left(2\theta_B + 0 - \frac{3\times0.01}{8}\right)$$

$$M_{BA} = 4000\,\theta_B + 99.17 \rightarrow (2)$$

$$M_{BC} = M_{f_{BC}} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\Delta}{l}\right)$$



$$= -40 + \frac{2(4000)}{4}\left(2\theta_B + \theta_C - \frac{3(-0.01)}{4}\right)$$

$$= -40 + 2000(2\theta_B + \theta_C + 0.0075)$$

$$\left(\begin{array}{l} \Delta = -10\text{mm} \\ \Delta = -0.01\text{m.} \end{array}\right)$$

$$M_{BC} = 4000\,\theta_B + 2000\,\theta_C - 25 \rightarrow (3)$$

$$M_{CB} = M_{f_{CB}} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\Delta}{l}\right)$$

$$\left(\begin{array}{l} \Delta = -10\text{mm} \\ \Delta = -0.01\text{m.} \end{array}\right)$$

$$= 40 + \frac{2(4000)}{4}\left(2\theta_C + \theta_B - \frac{3(-0.01)}{4}\right)$$

$$= 40 + 2000(2\theta_C + \theta_B + 0.0075)$$

$$M_{CB} = 2000\,\theta_B + 4000\,\theta_C + 55 \rightarrow (4)$$

Apply joint equilibrium ~~equil~~ equation :-

Joint 'B'

$$M_{BA} + M_{BC} = 0$$

$$4000\theta_B + 99.17 + (4000\,\theta_B + 2000\,\theta_C - 25) = 0$$

$$8000\,\theta_B + 2000\,\theta_C + 74.17 = 0 \rightarrow (5)$$

Joint 'C'

$$M_{CB} = 0$$

$$2000\theta_B + 4000\,\theta_C + 55 = 0 \rightarrow (6)$$

Solving eqns (5) & (6)

$$8000\,\theta_B + 2000\,\theta_c + 74.17 = 0 \quad \times 2$$

$$2000\,\theta_B + 4000\,\theta_c + 55 = 0$$

$$16000\,\theta_B + \cancel{4000\,\theta_c} + 148.34 = 0$$

$$\underset{(-)}{2000\,\theta_B} + \underset{(-)}{\cancel{4000\,\theta_c}} + \underset{(-)}{55} = 0$$

$$14000\,\theta_B + 93.34 = 0$$

$$\boxed{\theta_B = -0.0067}$$

'θ_B' sub. in eqn (6)

$$2000(-0.0067) + 4000\,\theta_c + 55 = 0$$

$$\boxed{\theta_c = -0.0104}$$

Final Moments :— sub. $EI\theta_B$ & $EI\theta_c$ in eqns (1), (2), (3), (4).

$$M_{AB} = 2000\,\theta_B - 114.17 = 2000(-0.0067) - 114.17$$

$$\boxed{M_{AB} = -127.57 \text{ KN.m.}}$$

$$M_{BA} = 4000\,\theta_B + 99.17 = 4000(-0.0067) + 99.17$$

$$\boxed{M_{BA} = 72.6 \text{ KN.m.}}$$

$$M_{BC} = 4000(-0.0067) + 2000(-0.0104) - 25$$

$$\boxed{M_{BC} = -72.6 \text{ KN.m.}}$$

$$M_{CB} = 2000(-0.0067) + 4000(-0.0104) + 55$$

$$\boxed{M_{CB} = 0}$$

Final BMD :—

$$\text{Maximum BM, } AB = \frac{wl^2}{8}$$

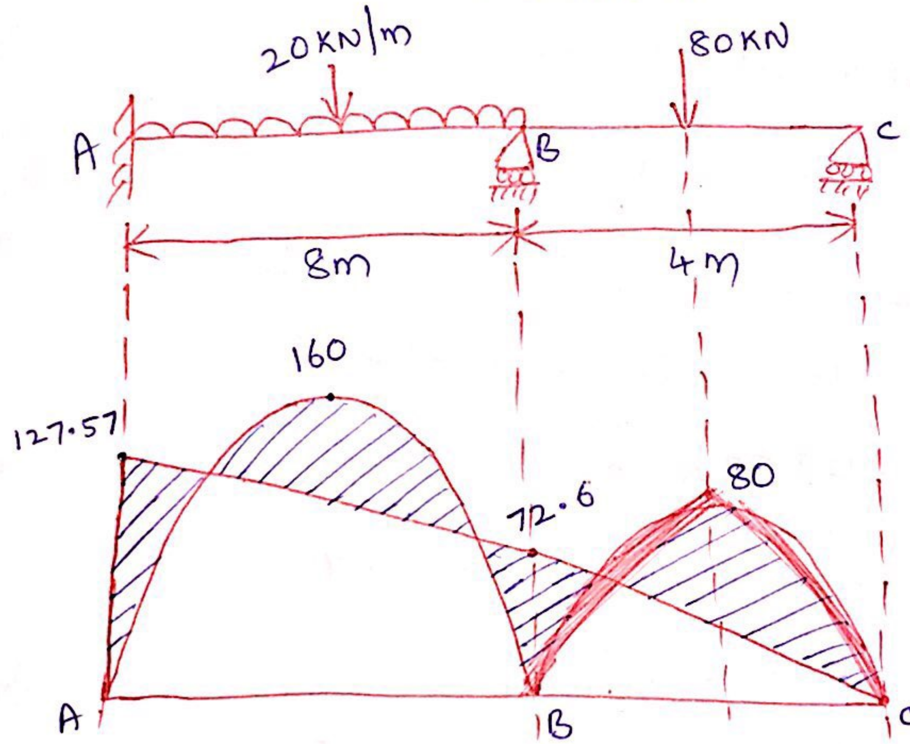
$$= \frac{20 \times (8)^2}{8}$$

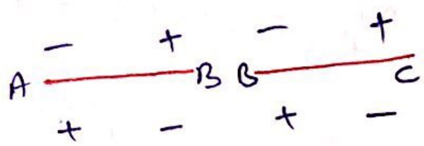
$$= 160 \text{ KN.m.}$$

$$\text{Maximum BM, } BC = \frac{wl}{4}$$

$$= \frac{80 \times 4}{4}$$

$$= 80 \text{ KN.m.}$$

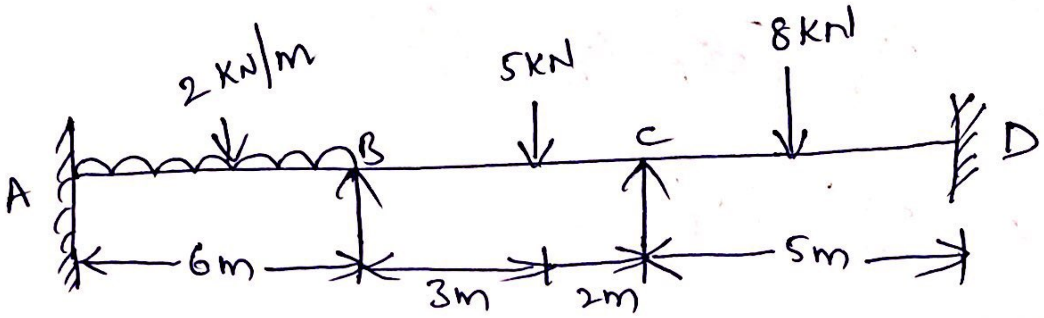




Max. BM for AB is parabola shape.

Max. BM for BC is Triangle shape.

(4) Analyse a continuous beam ABCD using SDM.



(A) Step (1) :- calculation of Bending moment :-

For span AB $= \frac{wl^2}{8} = \frac{2 \times 6^2}{8} = 9$ KN.m.

For span BC $= \frac{wab}{l} = \frac{5 \times 3 \times 2}{5} = 6$ KN.m.

For span CD $= \frac{wl}{4} = \frac{8 \times 5}{4} = 10$ KN.m.

Step (2) :- calculation of FEM :-

$$M_{F_{AB}} = \frac{-wl^2}{12} = \frac{-2 \times 6^2}{12} = -6 \text{ KN.m.}$$

$$M_{F_{BA}} = \frac{wl^2}{12} = \frac{2 \times 6^2}{12} = 6 \text{ KN.m.}$$

$$M_{F_{BC}} = \frac{-wab^2}{l^2} = \frac{-5 \times 3 \times 2^2}{5^2} = -2.4 \text{ KN.m.}$$

$$M_{F_{CB}} = \frac{wa^2b}{l^2} = \frac{5 \times 3^2 \times 2}{5^2} = 3.6 \text{ KN.m.}$$

$$M_{F_{CD}} = \frac{-wl}{8} = \frac{-8 \times 5}{8} = -5 \text{ KN.m.}$$

$$M_{F_{DC}} = \frac{wl}{8} = \frac{8 \times 5}{8} = 5 \text{ KN.m.}$$

Scanned with CamScanner

step (3):- Apply slope Deflection Equation :-

$$M_{AB} = M_{F_{AB}} + \frac{2EI}{l}\left[2\theta_A + \theta_B - \frac{3\Delta}{l}\right]$$

$$= -6 + \frac{2EI}{6}\left[\theta_B\right]$$

$$= \frac{EI\theta_B}{3} - 6 \rightarrow (1)$$

$$M_{BA} = M_{F_{BA}} + \frac{2EI}{l}\left[2\theta_B + \theta_A - \frac{3\Delta}{l}\right]$$

$$= \frac{2EI\theta_B}{3} + 6 \rightarrow (2)$$

$$M_{BC} = M_{F_{BC}} + \frac{2EI}{l}\left[2\theta_B + \theta_C - \frac{3\Delta}{l}\right]$$

$$= \frac{8EI\theta_B}{5} + \frac{4EI\theta_C}{5} - 2.4 \rightarrow (3)$$

$$M_{CB} = M_{F_{CB}} + \frac{2EI}{l}\left[2\theta_C + \theta_B - \frac{3\Delta}{l}\right]$$

$$= \frac{8EI\theta_C}{5} + \frac{4EI\theta_B}{5} + \overset{3.6}{\cancel{2.4}} \rightarrow (4)$$

$$M_{CD} = M_{F_{CD}} + \frac{2EI}{l}\left[2\theta_C + \theta_D - \frac{3\Delta}{l}\right]$$

$$= \frac{4EI\theta_C}{5} - 5 \rightarrow (5)$$

$$M_{DC} = M_{F_{DC}} + \frac{2EI}{l}\left[2\theta_D + \theta_C - \frac{3\Delta}{l}\right]$$

$$= \frac{2EI\theta_C}{5} + 5 \rightarrow (6)$$

step (4) :- Apply Equilibrium Equations :-

At joint B

$$M_{BA} + M_{BC} = 0$$

$$\frac{34 EI\theta_B}{15} + \frac{4EI\theta_c}{5} + 3.6 = 0 \rightarrow (7)$$

At joint C

$$M_{CB} + M_{CD} = 0$$

$$\frac{12}{5} EI\theta_c + \frac{4EI\theta_B}{5} - 1.4 = 0 \rightarrow (8)$$

From eqns (7) & (8)

$$EI\theta_B = -2.03 \text{ KN-m.}$$

$$EI\theta_c = 1.26 \text{ KN-m}$$

step (5) :- Final moments :-

$M_{AB} = -6.68$ KN.m.

$M_{BA} = 4.65$ KN.m.

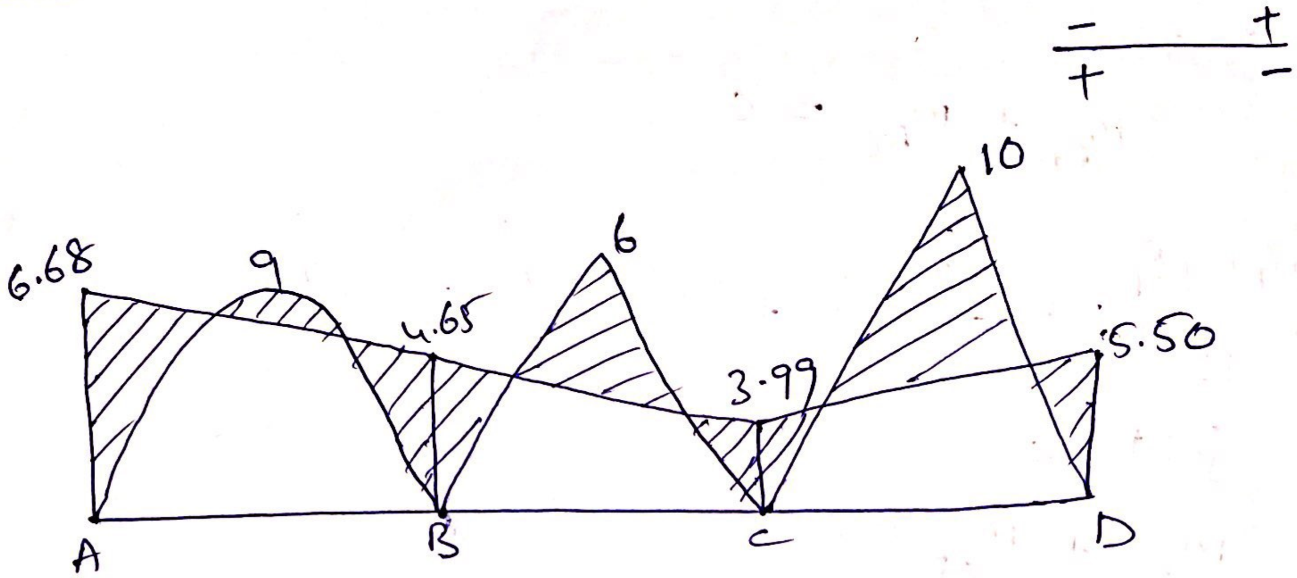
$M_{BC} = -4.65$ KN.m.

$M_{CB} = 3.99$ KN.m.

$M_{CD} = -3.99$ KN.m.

$M_{DC} = 5.50$ KN.M.

step (6) : Draw BMD :-



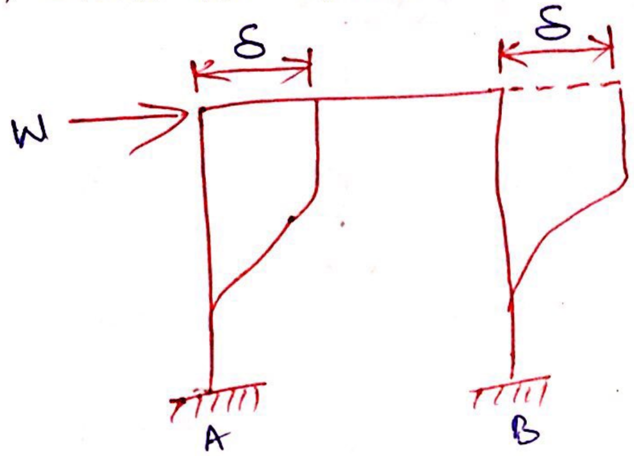
Sway frame and Non sway frame :—

(1) check for symmetrical loading

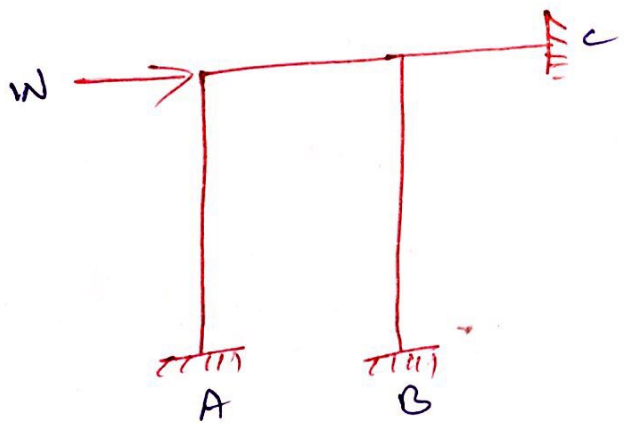
(2) check for symmetrical EI is given.

(3) check for vertical member length.

(4) check for symmetrical supports.

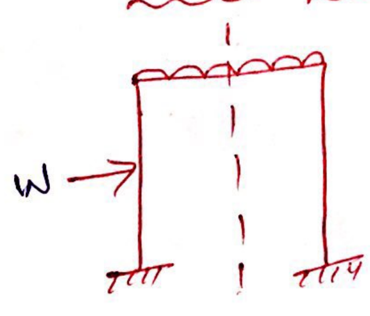


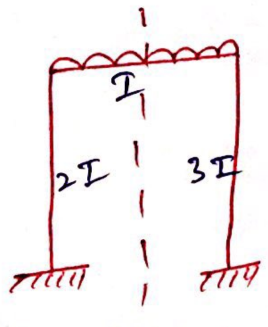
This is called sway frame, because due to unsymmetrical loading.



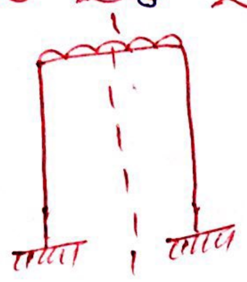
This is called non sway frame, because the deflection is resistant by support 'c'.

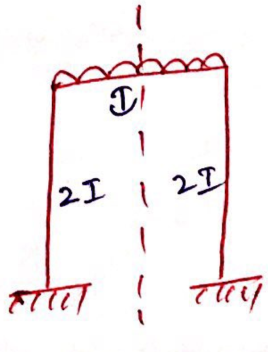
Sway frame

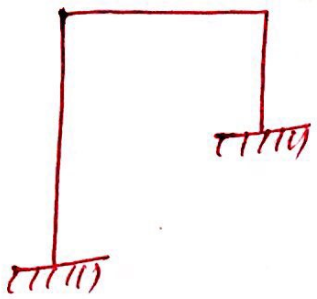


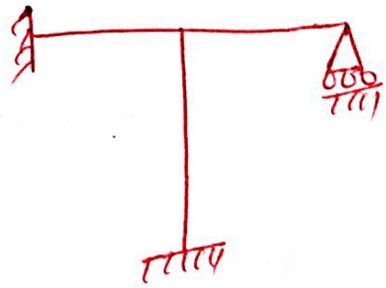


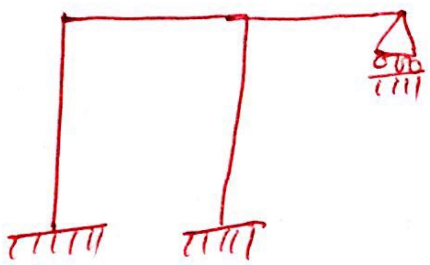
Non sway frame

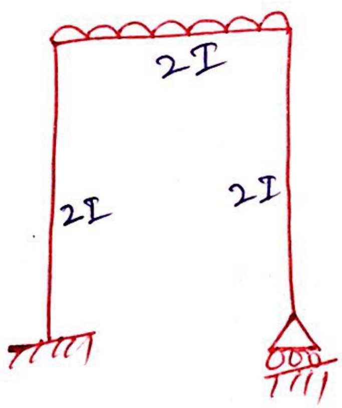


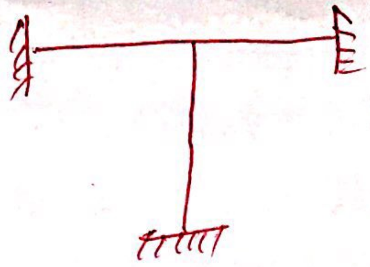


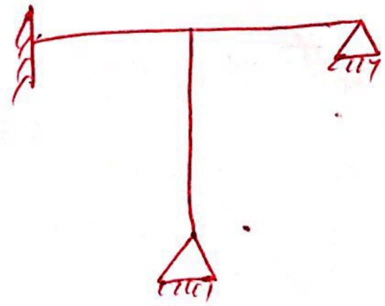


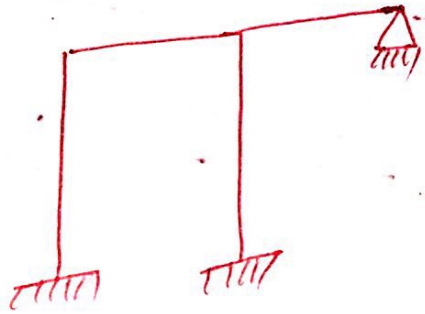


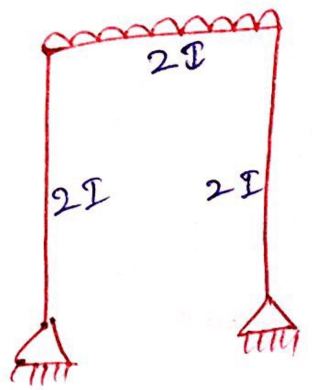



2I
2I
2I



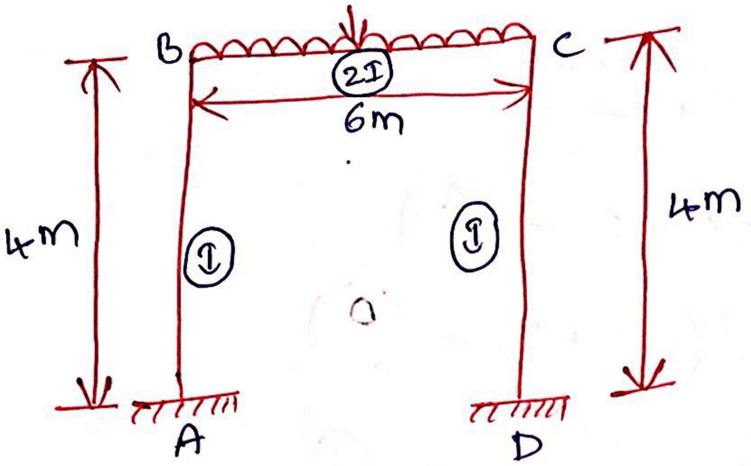





2I
2I
2I

Non sway problem :-

(1) Analyse the frame shown in below fig. by SDM. Draw BMD.



Ⓐ fixed end moments :-

$$M_{F_{AB}} = M_{F_{BA}} = M_{F_{CD}} = M_{F_{DC}} = 0$$

$$M_{F_{BC}} = \frac{-wl^2}{12} = \frac{-40 \times 6^2}{12} = -120 \text{ KN.m.}$$

$$M_{F_{CB}} = \frac{wl^2}{12} = \frac{40 \times 6^2}{12} = 120 \text{ KN.m.}$$

Apply slope deflection equation :-

$$M_{AB} = M_{F_{AB}} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$ $\left(\begin{matrix}\theta_A = 0, \text{ fixed} \\ \Delta = 0\end{matrix}\right)$.

$$= 0 + \frac{2EI}{4}\left(2 \times 0 + \theta_B - \frac{3 \times 0}{4}\right)$$

$$M_{AB} = 0.5\, EI\theta_B \rightarrow (1)$$

$$M_{BA} = M_{F_{BA}} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2EI}{4}\left(2\theta_B + 0 - \frac{3 \times 0}{4}\right)$$

$$M_{BA} = EI\theta_B \rightarrow (2)$$

$$M_{BC} = M_{F_{BC}} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\Delta}{l}\right) \qquad (\Delta = 0)$$

$$= -120 + \frac{2E(2I)}{6}\left(2\theta_B + \theta_C - \frac{3\times0}{6}\right)$$

$$M_{BC} = -120 + \frac{4EI\theta_B}{3} + \frac{2EI\theta_C}{3} \rightarrow (3)$$

$$M_{CB} = M_{F_{CB}} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\Delta}{l}\right)$$

$$= 120 + \frac{2E(2I)}{6}\left(2\theta_C + \theta_B - \frac{3(0)}{l}\right)$$

$$M_{CB} = 120 + \frac{2EI\theta_B}{3} + \frac{4EI\theta_C}{3} \rightarrow (4)$$

$$M_{CD} = M_{F_{CD}} + \frac{2EI}{l}\left(2\theta_C + \theta_D - \frac{3\Delta}{l}\right) \qquad \left(\begin{matrix}\theta_D = 0, \text{fixed} \\ \Delta = 0\end{matrix}\right)$$

$$= 0 + \frac{2EI}{4}\left(2\theta_C + 0 - \frac{3\times0}{4}\right)$$

$$M_{CD} = EI\theta_C \rightarrow (5)$$

$$M_{DC} = M_{F_{DC}} + \frac{2EI}{l}\left(2\theta_D + \theta_C - \frac{3\Delta}{l}\right)$$

$$= 0 + \frac{2EI}{4}\left(2\times0 + \theta_C - \frac{3\times0}{4}\right)$$

$$M_{DC} = 0.5EI\theta_C \rightarrow (6)$$

Joint equilibrium equations :—

Joint 'B'

$$M_{BA} + M_{BC} = 0$$

$$EI\theta_B - 120 + \frac{4EI\theta_B}{3} + \frac{2EI\theta_C}{3} = 0 \rightarrow (7)$$

Joint 'C'

$$M_{CB} + M_{CD} = 0$$

$$120 + \frac{2EI\theta_B}{3} + \frac{4EI\theta_C}{3} + EI\theta_C = 0 \rightarrow (8)$$

Solving eq^ns (7) & (8)

$$\boxed{\begin{aligned} EI\theta_B &= 72 \\ EI\theta_C &= -72 \end{aligned}}$$

Final Moments :—

Sub. $EI\theta_B$ & $EI\theta_C$ values in eq^ns (1), (2), (3), (4), (5), (6)

$M_{AB} = 0.5\, EI\theta_B = 0.5(72) = 36$ KN.m.

$M_{BA} = EI\theta_B = 72$ KN.m.

$M_{BC} = -120 + \frac{4EI\theta_B}{3} + \frac{2EI\theta_C}{3} = -120 + \frac{4(72)}{3} + \frac{2(-72)}{3} = -72$ KN.m.

$M_{CB} = 120 + \frac{2EI\theta_B}{3} + \frac{4EI\theta_C}{3} = 120 + \frac{2(72)}{3} + \frac{4(-72)}{3} = 72$ KN.m.

$M_{CD} = EI\theta_C = -72$ KN.m.

$M_{DC} = 0.5\, EI\theta_C = 0.5(-72) = -36$ KN.m.

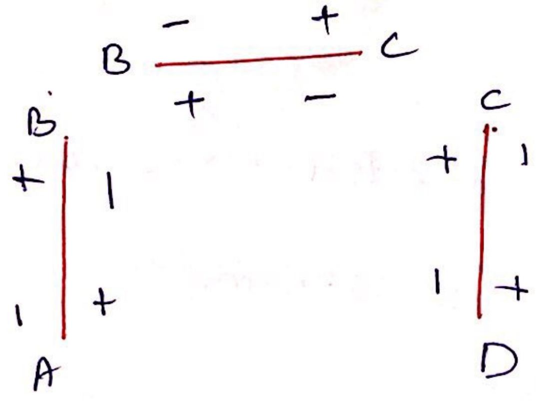
Maximum Bending Moment :—

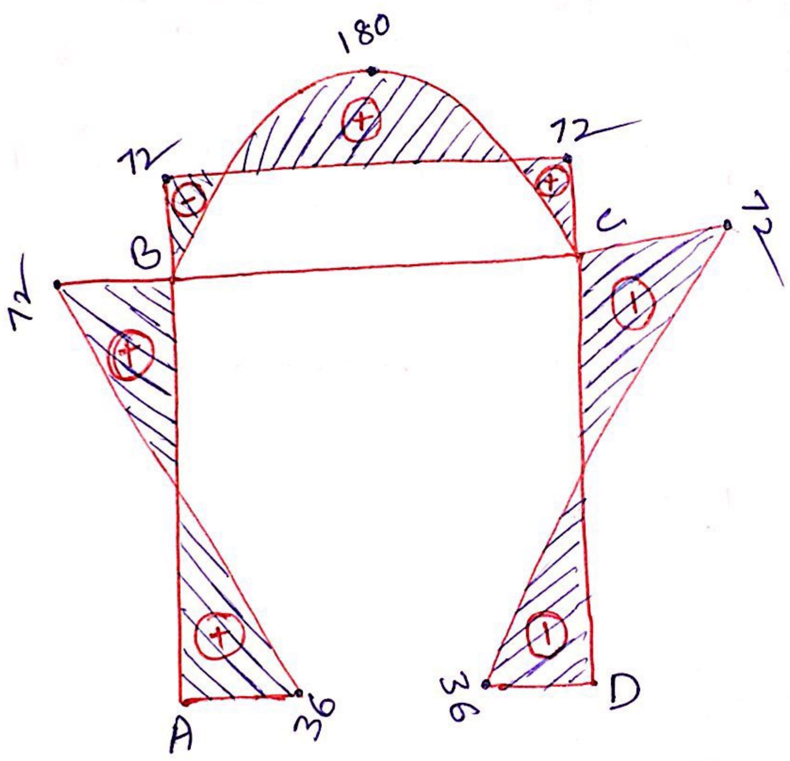
Max. BM, AB = 0

$$\text{Max. BM, BC} = \frac{wl^2}{8} = \frac{40(6)^2}{8} = 180 \text{ KN.m.}$$

Max. BM, CD = 0

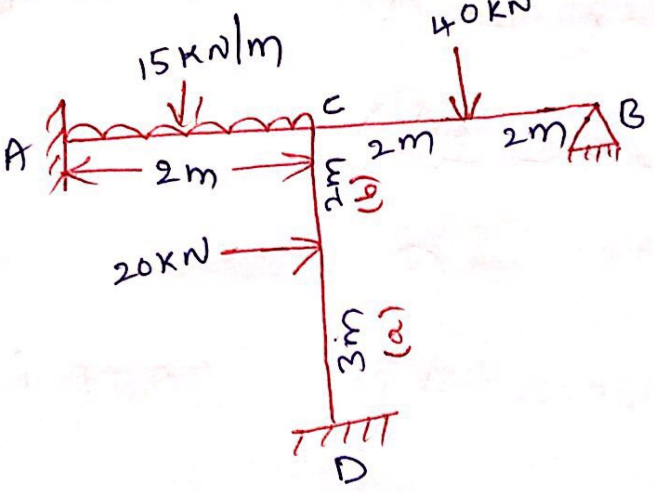
Final BMD :—





Non Sway Problem by SDM :—

(i) Analyse the frame and draw BMD by slope deflection method.



Ⓐ Fixed end moments :—

$$M_{F_{AC}} = \frac{-wl^2}{12} = \frac{-15 \times (2)^2}{12} = -5\ KN.m.$$

$$M_{F_{CA}} = \frac{wl^2}{12} = \frac{15 \times (2)^2}{12} = 5\ KN.m.$$

$$M_{F_{CB}} = \frac{-wl}{8} = \frac{-40 \times (4)}{8} = -20\ KN.m.$$

$$M_{F_{BC}} = \frac{wl}{8} = \frac{40 \times (4)}{8} = 20\ KN.m.$$

$$M_{F_{DC}} = \frac{-wab^2}{l^2} = \frac{-20 \times (3) \times (2)^2}{(5)^2} = -9.6\ KN.m.$$

$$M_{F_{CD}} = \frac{wa^2b}{l^2} = \frac{20 \times (3)^2 \times (2)}{(5)^2} = 14.4\ KN.m.$$

Apply slope deflection equation :—

$$M_{AC} = M_{F_{AC}} + \frac{2EI}{l}\left(2\theta_A + \theta_c - \frac{3\Delta}{l}\right)$$ $(\theta_A = 0, \text{fixed})$ $(\Delta = 0)$

$$M_{AC} = -5 + EI\theta_c \rightarrow ①$$

$$M_{CA} = M_{F_{CA}} + \frac{2EI}{l}\left(2\theta_c + \theta_A - \frac{3\Delta}{l}\right)$$

$$M_{CA} = 5 + 2EI\theta_c \rightarrow ②$$

$$M_{CB} = M_{F_{CB}} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\Delta}{l}\right)$$

$$M_{CB} = -20 + \frac{EI\theta_B}{2} + EI\theta_C \rightarrow (3)$$

$$M_{BC} = M_{F_{BC}} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\Delta}{l}\right)$$

$$M_{BC} = 20 + EI\theta_B + \frac{EI\theta_C}{2} \rightarrow (4)$$

$$M_{CD} = M_{F_{CD}} + \frac{2EI}{l}\left(2\theta_C + \theta_D - \frac{3\Delta}{l}\right) \qquad (\theta_D = 0, \text{fixed})$$

$$M_{CD} = 14.4 + \frac{4EI\theta_C}{5} \rightarrow (5)$$

$$M_{DC} = M_{F_{DC}} + \frac{2EI}{l}\left(2\theta_D + \theta_C - \frac{3\Delta}{l}\right)$$

$$M_{DC} = -9.6 + \frac{2EI\theta_C}{5} \rightarrow (6)$$

Apply joint equilibrium equation :—

Joint 'C'

$$M_{CA} + M_{CD} + M_{CB} = 0$$

$$5 + 2EI\theta_C + 14.4 + \frac{4EI\theta_C}{5} + \left(-20 + \frac{EI\theta_B}{2} + EI\theta_C\right) = 0$$

$$\frac{EI\theta_B}{2} + EI\theta_C\left(2 + \frac{4}{5} + 1\right) = -5 - 14.4 + 20 \rightarrow (7)$$

Joint 'B'

$$M_{BC} = 0$$

$$20 + EI\theta_B + \frac{EI\theta_C}{2} = 0$$

$$EI\theta_B + \frac{EI\theta_C}{2} = -20 \rightarrow (8)$$

Solving eqlls (7) & (8)

$$\boxed{\begin{aligned} EI\theta_B &= -21.49 \\ EI\theta_c &= 2.98 \end{aligned}}$$

Final moments :—

Sub. $EI\theta$, $EI\theta_c$ values in eqlls (1), (2), (3), (4), (5), (6).

$M_{AC} = -2.02$ KN.m.

$M_{CA} = 10.96$ KN.m.

$M_{CB} = -27.76$ KN.m.

$M_{BC} = 0$

$M_{CD} = 16.78$ KN.m.

$M_{DC} = -8.4$ KN.m.

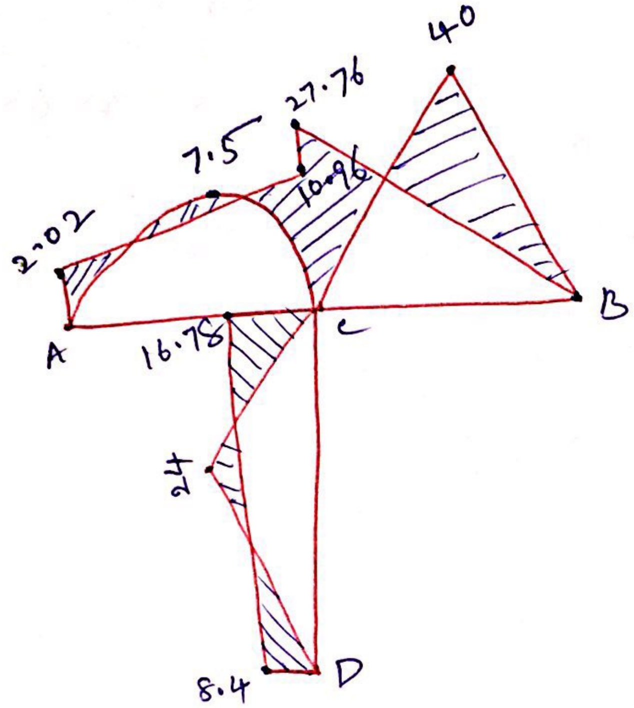
Maximum BMD :—

Max. BM, AC $= \frac{wl^2}{8} = \frac{15 \times (2)^2}{8} = 7.5$ KN.m.

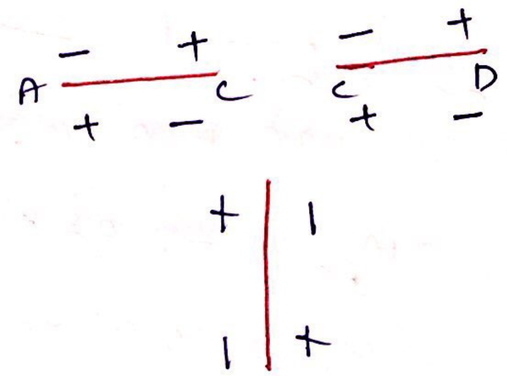
Max. BM, CB $= \frac{Wl}{4} = \frac{40 \times 4}{4} = 40$ KN.m.

Max. BM, DC $= \frac{Wab}{l} = \frac{20 \times (3) \times (2)}{5} = 24$ KN.m.

Final BMD :—



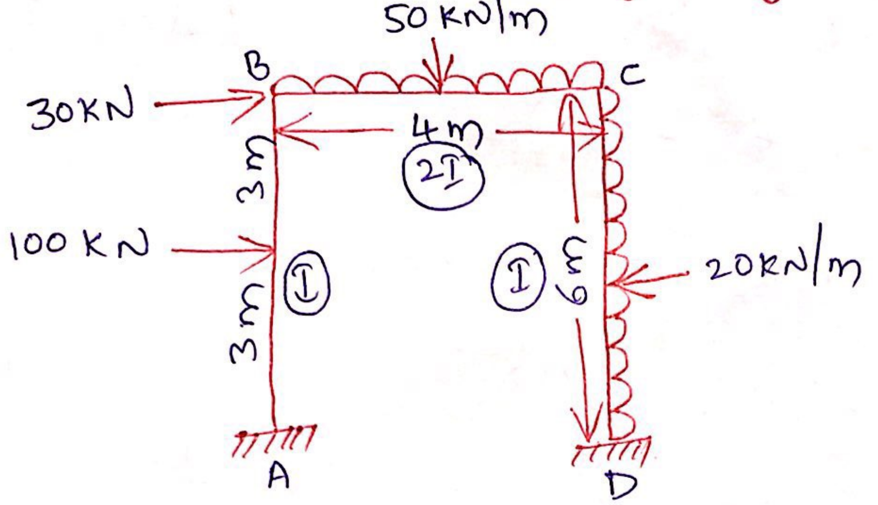
Sign convention



Sway Analysis :—

① Analyse the portal frame by using slope deflection method.

(19)



④ Fixed end moments :—

$$M_{F_{AB}} = \frac{-wl}{8} = \frac{-100\times6}{8} = -75 \text{ KN·m.}$$

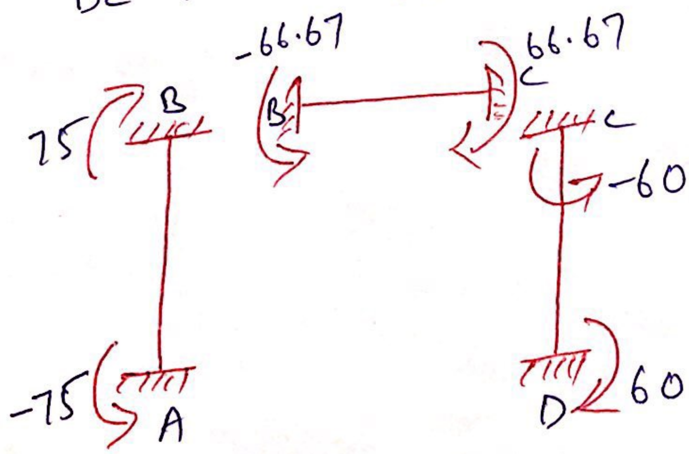
$$M_{F_{BA}} = \frac{wl}{8} = \frac{100\times6}{8} = 75 \text{ KN·m.}$$

$$M_{F_{BC}} = \frac{-wl^2}{12} = \frac{-50\times(4)^2}{12} = -66.67 \text{ KN·m.}$$

$$M_{F_{CB}} = \frac{wl^2}{12} = \frac{50\times(4)^2}{12} = 66.67 \text{ KN·m.}$$

$$M_{F_{CD}} = \frac{-wl^2}{12} = \frac{-20\times(6)^2}{12} = -60 \text{ KN·m.}$$

$$M_{F_{DC}} = \frac{wl^2}{12} = \frac{20\times(6)^2}{12} = 60 \text{ KN·m.}$$



Apply slope deflection equation for each span :—

$$M_{AB} = M_{F_{AB}} + \frac{2EI}{l}\left(2\theta_A + \theta_B - \frac{3\Delta}{l}\right)$$ $(\theta_A = 0, \text{fixed})$ $(\Delta = ?)$ $(\theta_B = ?)$

$$= -75 + \frac{2EI}{6}\left(2\times 0 + \theta_B - \frac{3\Delta}{6}\right)$$

$$M_{AB} = 0.33EI\theta_B - 0.167EI\Delta - 75 \rightarrow (1)$$

$$M_{BA} = M_{F_{BA}} + \frac{2EI}{l}\left(2\theta_B + \theta_A - \frac{3\Delta}{l}\right)$$ $(\theta_A = 0, \text{fixed}, \Delta = ?)$ $(\theta_B = ?)$

$$= 75 + \frac{2EI}{6}\left(2\theta_B + 0 - \frac{3\Delta}{6}\right)$$

$$M_{BA} = 0.67EI\theta_B - 0.167EI\Delta + 75 \rightarrow (2)$$

$$M_{BC} = M_{F_{BC}} + \frac{2EI}{l}\left(2\theta_B + \theta_C - \frac{3\Delta}{l}\right)$$ $(\theta_B = ?, \theta_C = ?, \Delta = 0)$

$$= -66.67 + \frac{2E(2I)}{4}\left(2\theta_B + \theta_C - \frac{3(0)}{4}\right)$$

$$M_{BC} = 2EI\theta_B + EI\theta_C - 66.67 \rightarrow (3)$$

$$M_{CB} = M_{F_{CB}} + \frac{2EI}{l}\left(2\theta_C + \theta_B - \frac{3\Delta}{l}\right)$$ $(\theta_B = ?, \theta_C = ?, \Delta = 0)$

$$= 66.67 + \frac{2E(2I)}{4}\left(2\theta_C + \theta_B - \frac{3(0)}{4}\right)$$

$$M_{CB} = EI\theta_B + 2EI\theta_C + 66.67 \rightarrow (4)$$

$$M_{CD} = M_{F_{CD}} + \frac{2EI}{l}\left(2\theta_C + \theta_D - \frac{3\Delta}{l}\right)$$ $(\theta_D = 0, \text{fixed}, \theta_C = ?, \Delta = ?)$

$$= -60 + \frac{2EI}{6}\left(2\theta_C + 0 - \frac{3\Delta}{6}\right)$$

$$M_{CD} = 0.67EI\theta_C - 0.167EI\Delta - 60 \rightarrow (5)$$

$$M_{DC} = M_{F_{DC}} + \frac{2EI}{l}\left(2\theta_D + \theta_c - \frac{3\Delta}{l}\right)$$

$$\left(\begin{array}{l}\theta_D = 0, \text{ fixed} \\ \theta_c = ? \\ \Delta = ?\end{array}\right)$$

$$= 60 + \frac{2EI}{6}\left(2\times 0 + \theta_c - \frac{3\Delta}{6}\right)$$

$$M_{DC} = 0.33\,EI\theta_c - 0.167\,EI\Delta + 60 \rightarrow (6)$$

Apply joint equilibrium equation :-

Joint 'B' $\Rightarrow$ $M_{BA} + M_{BC} = 0$

$$0.67\,EI\theta_B - 0.167\,EI\Delta + 75 + (2EI\theta_B + EI\theta_c - 66.67) = 0$$

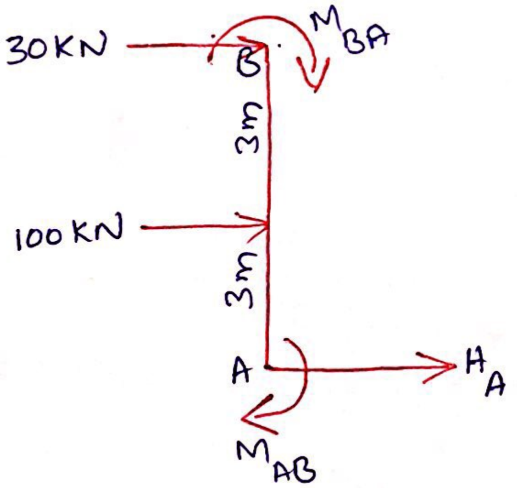
$$2.67\,EI\theta_B + EI\theta_c - 0.167\,EI\Delta = -8.33 \rightarrow (7)$$

Joint 'c' $\Rightarrow$ $M_{CB} + M_{CD} = 0$

$$EI\theta_B + 2EI\theta_c + 66.67 + (0.67\,EI\theta_c - 0.167\,EI\Delta - 60) = 0$$

$$EI\theta_B + 2.67\,EI\theta_c - 0.167\,EI\Delta = -6.67 \rightarrow (8)$$

Apply shear condition (i.e. Horizontal equilibrium) :-

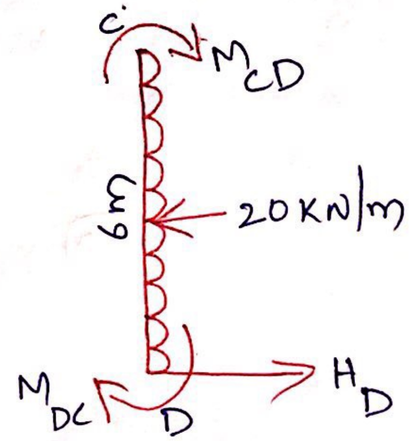


$$\Sigma M_B = 0$$

$$M_{AB} + M_{BA} - (H_A \times 6) - (100 \times 3) = 0$$

$$M_{AB} + M_{BA} - 300 = 6H_A$$

$$\frac{M_{AB} + M_{BA} - 300}{6} = H_A$$

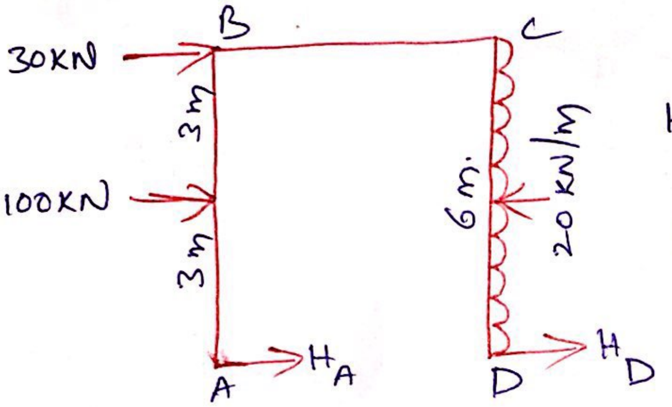


$$\Sigma M_C = 0$$

$$M_{CD} + M_{DC} - (H_D \times 6) + (20 \times 6 \times \tfrac{6}{2}) = 0$$

$$M_{CD} + M_{DC} + 360 = H_D \times 6$$

$$\frac{M_{CD} + M_{DC} + 360}{6} = H_D$$



$$\Sigma f_x = 0$$

$$H_A + H_D + 100 + 30 - (20 \times 6) = 0$$

$$H_A + H_D + 130 - 120 = 0$$

$$\boxed{H_A + H_D + 10 = 0}$$

$$\left(\frac{M_{AB} + M_{BA} - 300}{6}\right) + \left(\frac{M_{CD} + M_{DC} + 360}{6}\right) + 10 = 0$$

$$\frac{M_{AB} + M_{BA} - 300 + M_{CD} + M_{DC} + 360 + 60}{6} = 0$$

$$M_{AB} + M_{BA} + M_{CD} + M_{DC} + 120 = 0$$

$$(0.33 EI\theta_B - 0.167 EI\Delta - 75) + (0.67 EI\theta_B - 0.167 EI\Delta + 75)$$
$$+ (0.67 EI\theta_C - 0.167 EI\Delta - 60) + (0.33 EI\theta_C - 0.167 EI\Delta + 60) + 120 = 0$$

$$EI\theta_B + EI\theta_C - 0.668 EI\Delta = -120 \rightarrow 9$$

Scanned with CamScanner

$$2.67\,EI\theta_B + EI\theta_C - 0.167\,EI\Delta = -8.33 \rightarrow (7)$$

$$EI\theta_B + 2.67\,EI\theta_C - 0.167\,EI\Delta = -6.67 \rightarrow (8)$$

$$EI\theta_B + EI\theta_C - 0.668\,EI\Delta = -120 \rightarrow (9)$$

Solving eqns (7), (8), (9), then.

21

$$\boxed{\begin{aligned} EI\theta_B &= 6.6 \\ EI\theta_C &= 7.59 \\ EI\Delta &= 200.89 \end{aligned}}$$

$EI\theta_B$, $EI\theta_C$, $EI\Delta$ values sub. in eqns (1), (2), (3), (4), (5), (6)

$$M_{AB} = 0.33\,EI\theta_B - 0.167\,EI\Delta - 75$$

$$= 0.33(6.6) - 0.167(200.89) - 75$$

$$\boxed{M_{AB} = -106.37\ \text{KN.m.}}$$

$$M_{BA} = 0.67\,EI\theta_B - 0.167\,EI\Delta + 75$$

$$= 0.67(6.6) - 0.167(200.89) + 75$$

$$\boxed{M_{BA} = 45.87\ \text{KN.m.}}$$

$$M_{BC} = 2EI\theta_B + EI\theta_C - 66.67$$

$$= 2(6.6) + 7.59 - 66.67$$

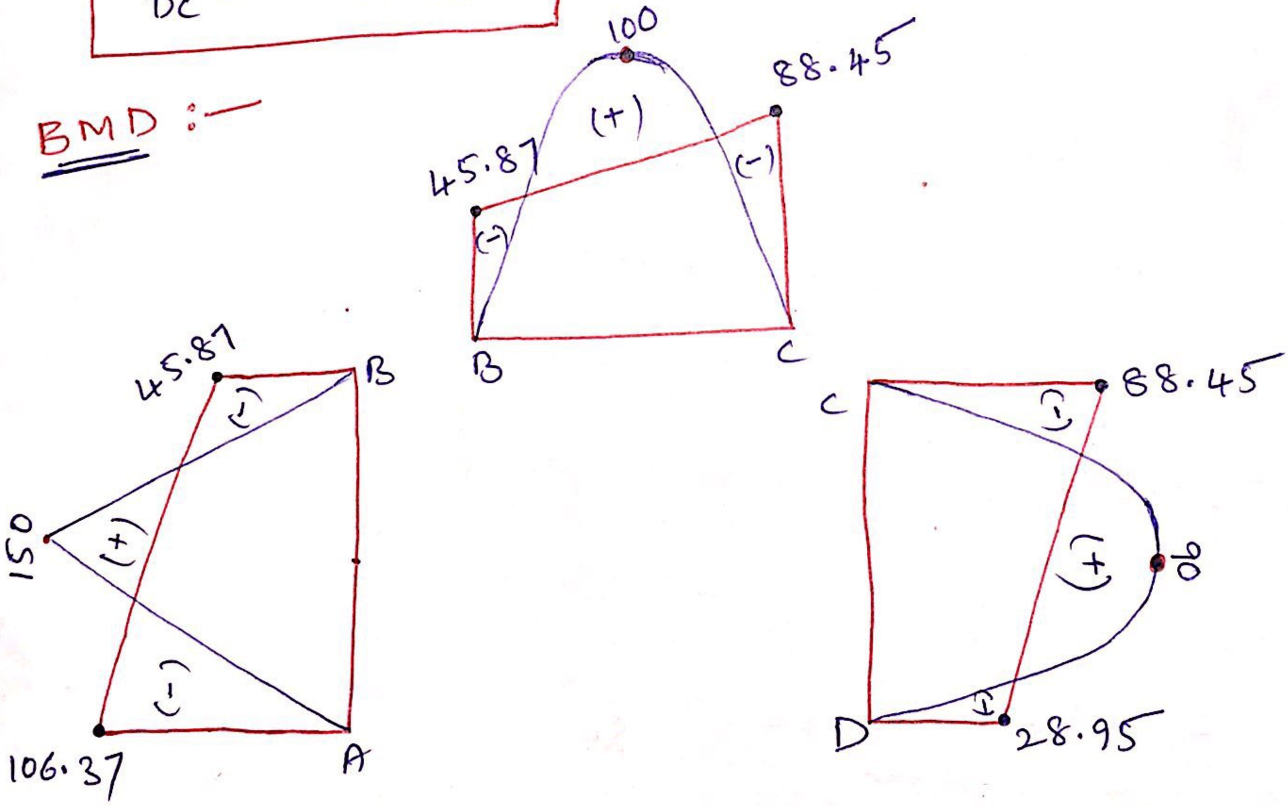
$$\boxed{M_{BC} = -45.87\ \text{KN.m.}}$$

$$M_{CB} = EI\theta_B + 2EI\theta_C + 66.67$$
$$= 6.6 + 2(7.59) + 66.67$$
$$\boxed{M_{CB} = 88.45 \text{ KN.m.}}$$

$$M_{CD} = 0.67 EI\theta_C - 0.167 EI\Delta - 60$$
$$= 0.67(7.59) - 0.167(200.89) - 60$$
$$\boxed{M_{CD} = -88.45 \text{ KN.m.}}$$

$$M_{DC} = 0.33 EI\theta_C - 0.167 EI\Delta + 60$$
$$= 0.33(7.59) - 0.167(200.89) + 60$$
$$\boxed{M_{DC} = 28.95 \text{ KN.m.}}$$

BMD :-



$$\text{Max. BM, } AB = \frac{wl}{4} = \frac{100 \times 6}{4} = 150 \text{ KN.m.}$$
$$\text{Max. BM, } BC = \frac{wl^2}{8} = \frac{50 \times (4)^2}{8} = 100 \text{ KN.m.}$$
$$\text{Max. BM, } CD = \frac{wl^2}{8} = \frac{20 \times (6)^2}{8} = 90 \text{ KN.M.}$$

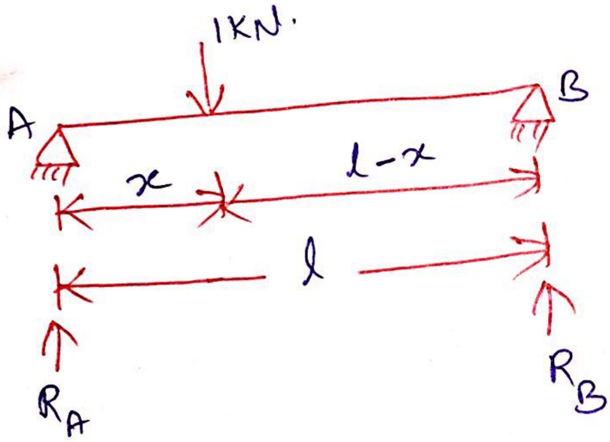
UNIT – V

Moving loads and Influence Line Diagram

ML and ILD

Introduction :– ILD represents a graph of functions like reactions, shear force, bending moment for various location of unit load under the span to calculate the given quantity.

ILD for reaction :–



$$\Sigma M_A = 0;\ (R_A \times 0) + (1 \times x) - (R_B \times l) = 0$$

$$1 \times x = R_B \times l$$

$$\boxed{R_B = \frac{x}{l}}$$

$$\Sigma f_x = 0;\ R_A + R_B - 1 = 0$$

$$R_A + \frac{x}{l} - 1 = 0$$

$$\boxed{R_A = \frac{l - x}{l}}$$

ILD for R_A :—

$$R_A = \frac{l-x}{l}$$

At point 'A', $x = 0$

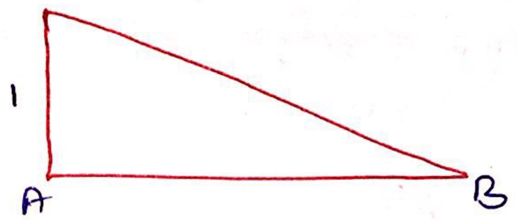
$$R_A = \frac{l-0}{l}$$

$$\boxed{R_A = 1}$$

At point 'B', $x = l$.

$$R_A = \frac{l-l}{l}$$

$$\boxed{R_A = 0}$$



ILD for R_B :—

$$R_B = \frac{x}{l}$$

At point 'A', $x = 0$

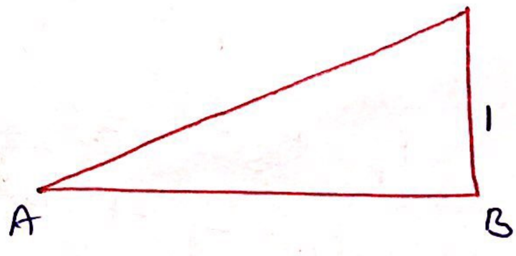
$$R_B = \frac{0}{l}$$

$$\boxed{R_B = 0}$$

At point 'B', $x = l$

$$R_B = \frac{l}{l}$$

$$\boxed{R_B = 1}$$



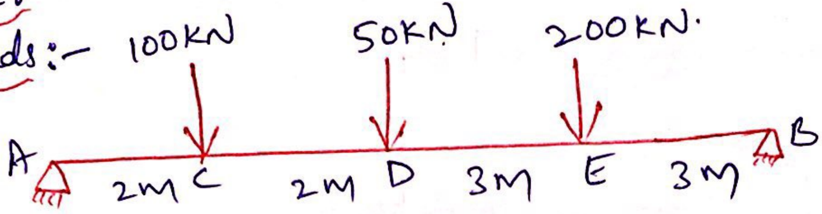
Uses of ILD :— Case (1) :—

(1) To calculate the given function under given set of loading.

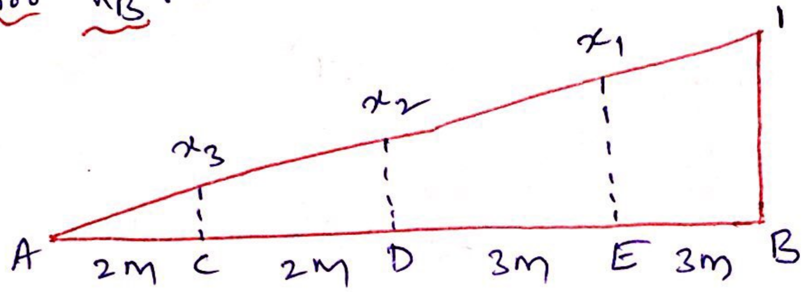
i.e. To calculate the reaction at 'B' under given set of loading.

Consider

Point loads :—



ILD for R_B :—



Here minimum ordinate is '0' at 'A'

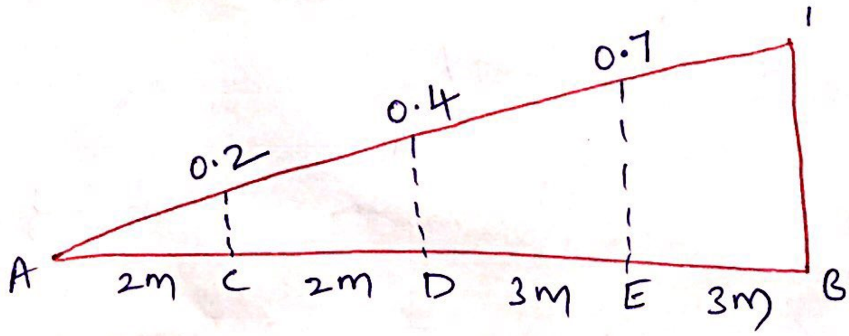
maximum ordinate is '1' at 'B'

From similar triangle concept $\left[\dfrac{\text{perpendicular}}{\text{Base}}\right]$

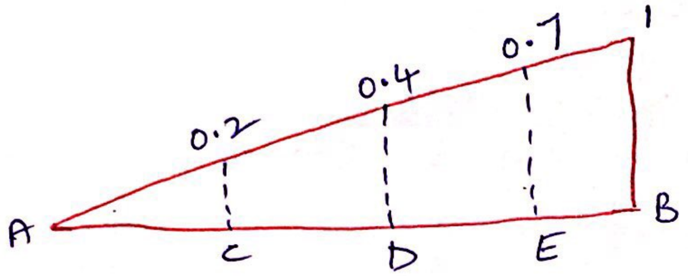
(i) $\dfrac{1}{10} = \dfrac{x_1}{7} \Rightarrow \boxed{x_1 = 0.7}$

(ii) $\dfrac{1}{10} = \dfrac{x_2}{4} \Rightarrow \boxed{x_2 = 0.4}$

(iii) $\dfrac{1}{10} = \dfrac{x_3}{2} \Rightarrow \boxed{x_3 = 0.2}$



Reaction at 'B' :—

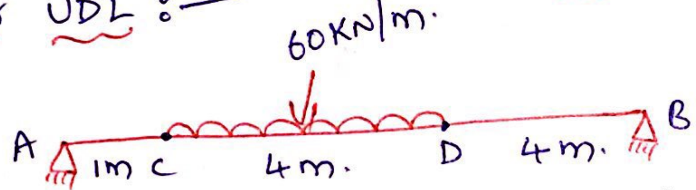


$$R_B = (100 \times 0.2) + (50 \times 0.4) + (200 \times 0.7)$$

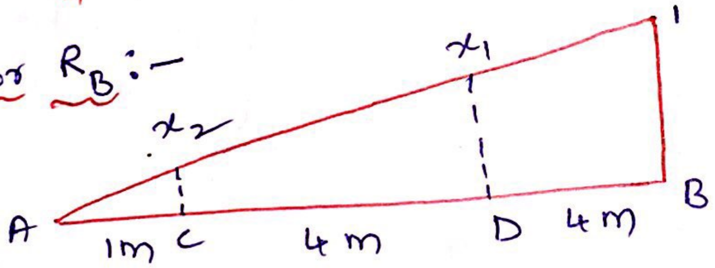
$$\boxed{R_B = 180\ KN.}$$

Calculate reaction at 'B' :—

Consider UDL :—



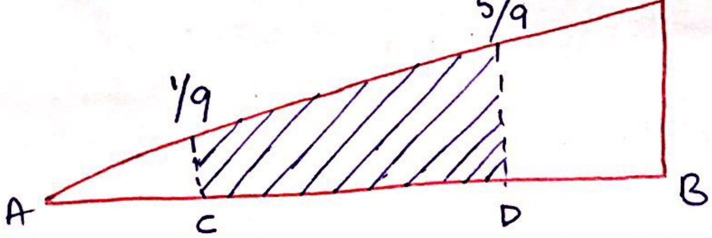
ILD for R_B :—



From similar triangle concept

$$\frac{1}{9} = \frac{x_1}{5} \Rightarrow \boxed{x_1 = \frac{5}{9}}$$

$$\frac{1}{9} = \frac{x_2}{1} \Rightarrow \boxed{x_2 = \frac{1}{9}}$$



(Area of shaded region)

R_B = Intensity of UDL × Area of ILD under UDL. (3)

$= 60 \times \left(\frac{1}{9} + \frac{5}{9}\right) \times \frac{4}{2}$

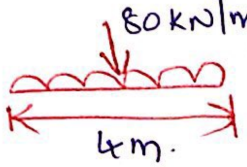
Area of Trapezoidal $= \frac{(a+b)}{2} \times h$.

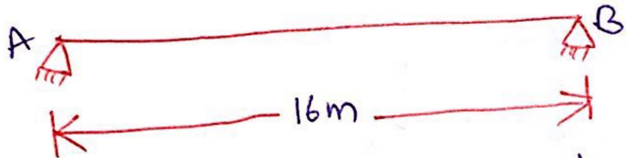
$\boxed{R_B = 80\ KN.}$

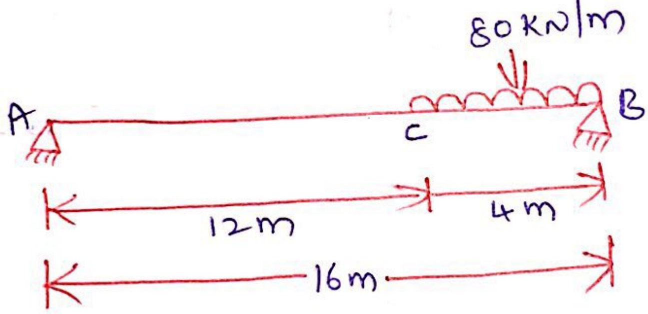
case (2) :—

(2) To locate the position of the live load to get the maximum value of the quantity.

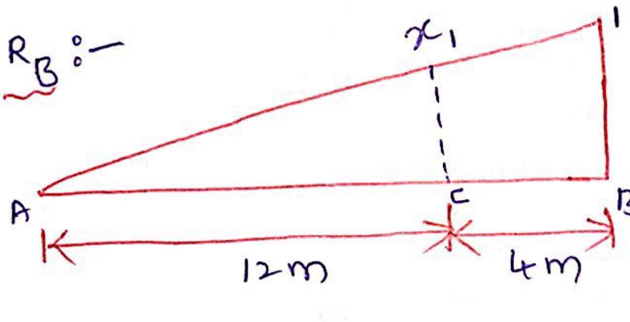
Calculate Reaction at 'B'?



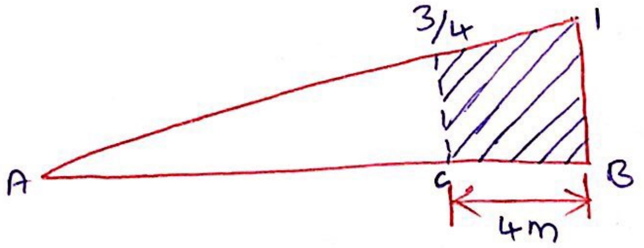




ILD for R_B :—



from similar Triangle $\frac{1}{16} = \frac{x_1}{12} \Rightarrow x_1 = \frac{12}{16} \Rightarrow \boxed{x_1 = \frac{3}{4}}$

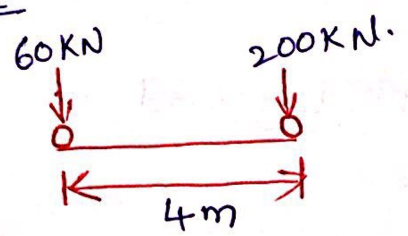


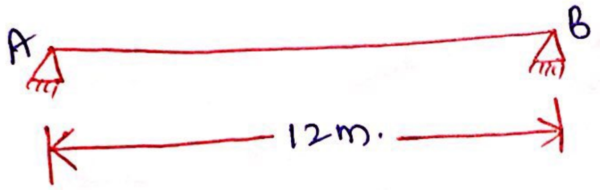
R_B = Intensity of UDL × Area of ILD under UDL.

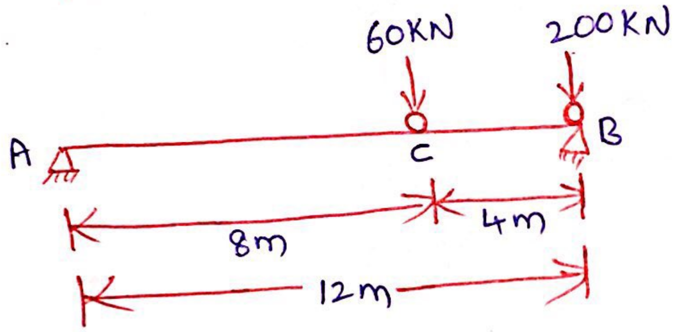
$$= 80 \times \left(\frac{3}{4}+1\right) \times \frac{4}{2}$$

$$\boxed{R_B = 280\ KN.}$$

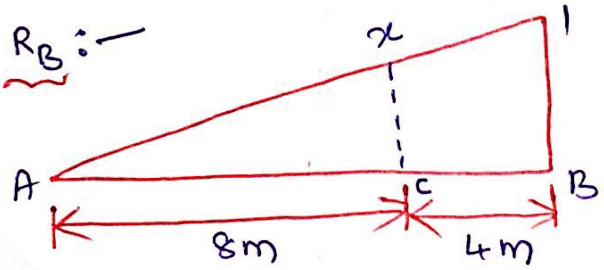
Wheel load :— calculate Reaction at 'B' :—





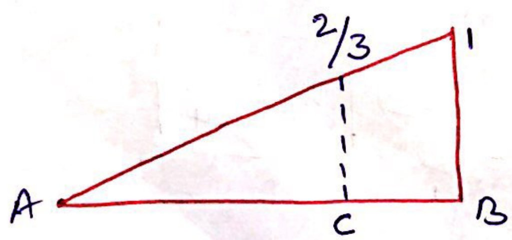


ILD for R_B :—



from similar triangle; $\frac{1}{12} = \frac{x}{8} \Rightarrow x = \frac{8}{12}$

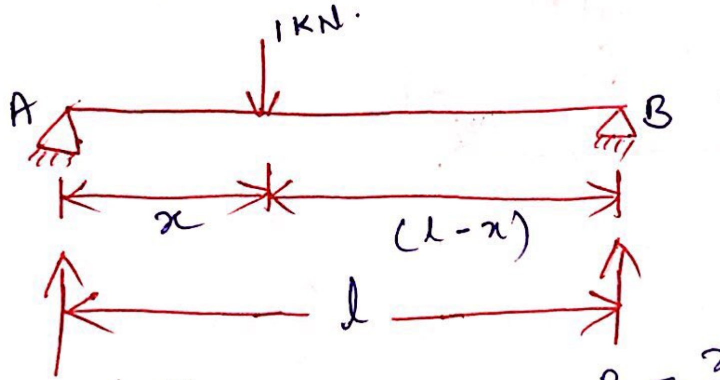
$$\boxed{x = \frac{2}{3}}$$



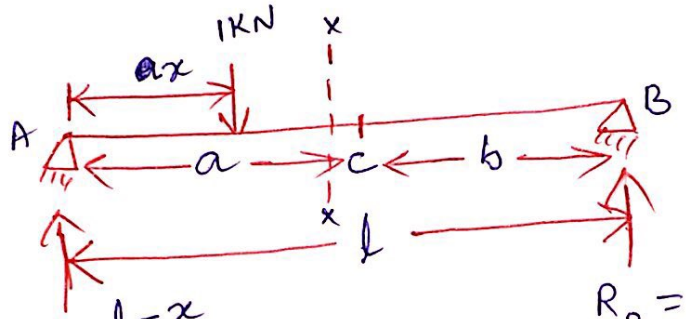
$$R_B = \left(60 \times \frac{2}{3}\right) + (200 \times 1)$$

$$\boxed{R_B = 240\ KN.}$$

ILD for shear force :- calculate shear force at 'c' :- (4)



$R_A = \frac{l-x}{l}$ $R_B = \frac{x}{l}$



$R_A = \frac{l-x}{l}$ $R_B = \frac{x}{l}$

shear force at c :-

span AC (left portion)

span CB (right portion)

when unit load is on left portion :-

$$(SF)_c = R_A - 1$$

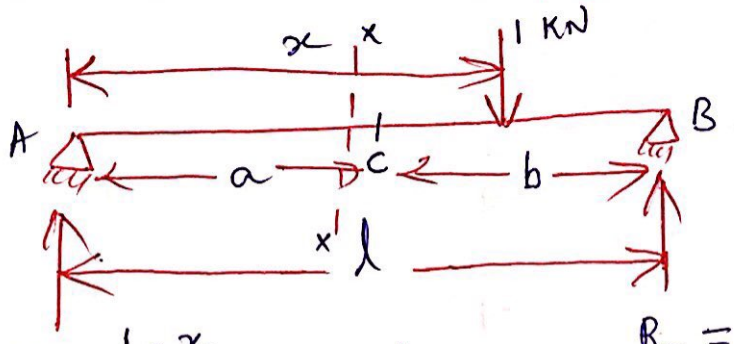
$$= \left(\frac{l-x}{l}\right) - 1 = \frac{l-x-l}{l}$$

$$\boxed{(SF)_c = \frac{-x}{l}}$$

where $x = a$

$$\boxed{(SF)_c = \frac{-a}{l}}$$

When unit load is on right portion :—



$R_A = \frac{l-x}{l}$ $R_B = \frac{x}{l}$

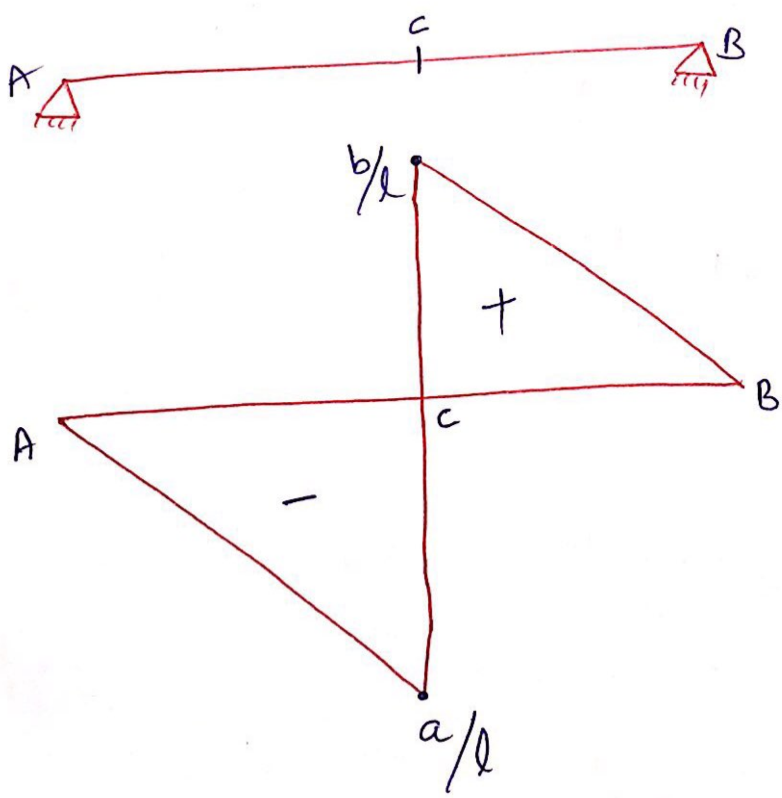
$$\boxed{(SF)_c = \frac{l-x}{l}}$$

where $x = a$

$$(SF)_c = \frac{l-a}{l}$$

$$\boxed{(SF)_c = \frac{b}{l}}$$

Shear force diagram :— ILD for $(SF)_c$:—



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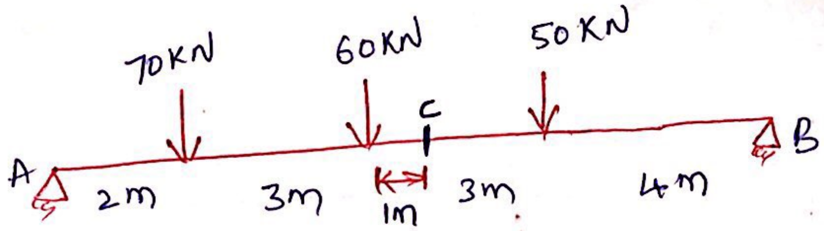
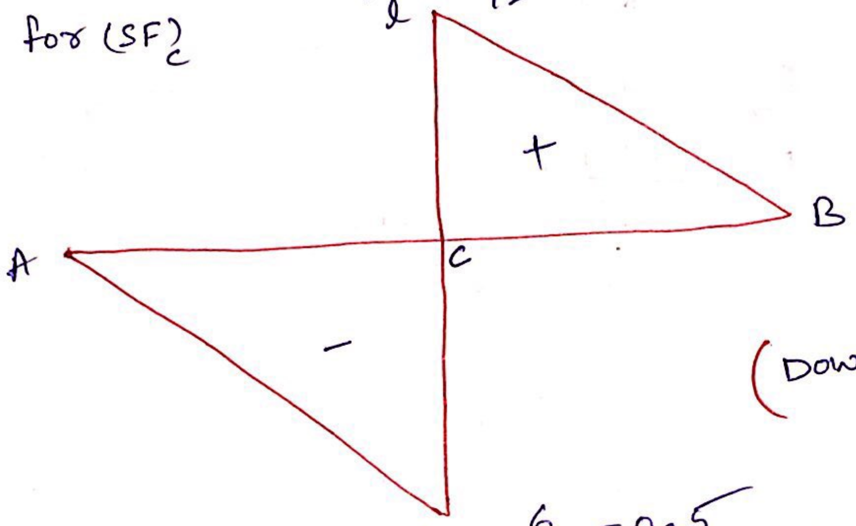


Calculate $(SF)_C$

(upward ordinate is $\frac{b}{l}$)

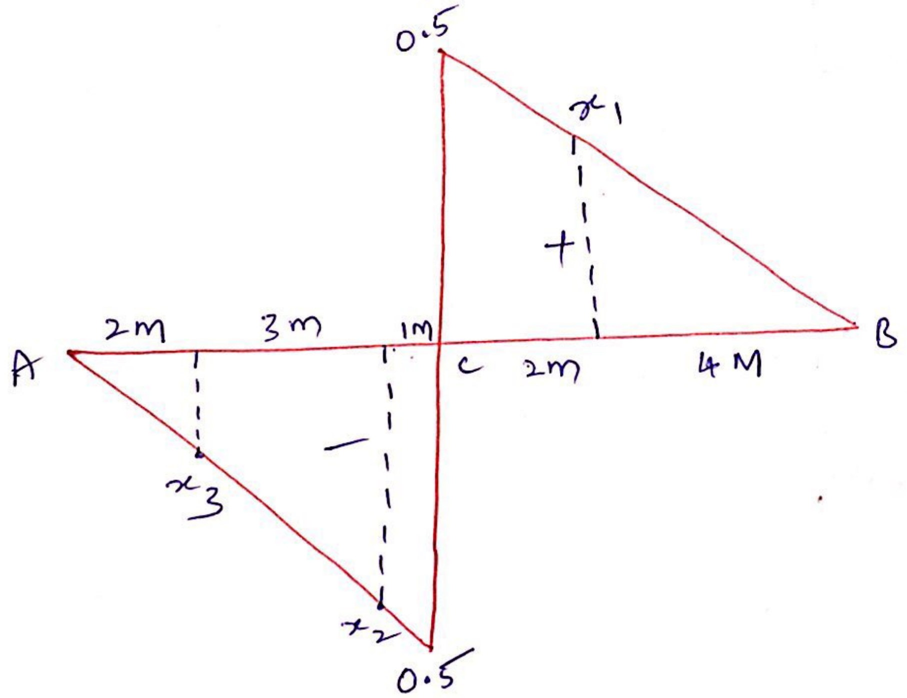
$$\frac{b}{l} = \frac{6}{12} = 0.5$$

(A) ILD for $(SF)_C$



(Downward ordinate is $\frac{a}{l}$)

$$\frac{a}{l} = \frac{6}{12} = 0.5$$



From similar triangle concept:

upper triangle: $\frac{0.5}{6} = \frac{x_1}{4}$

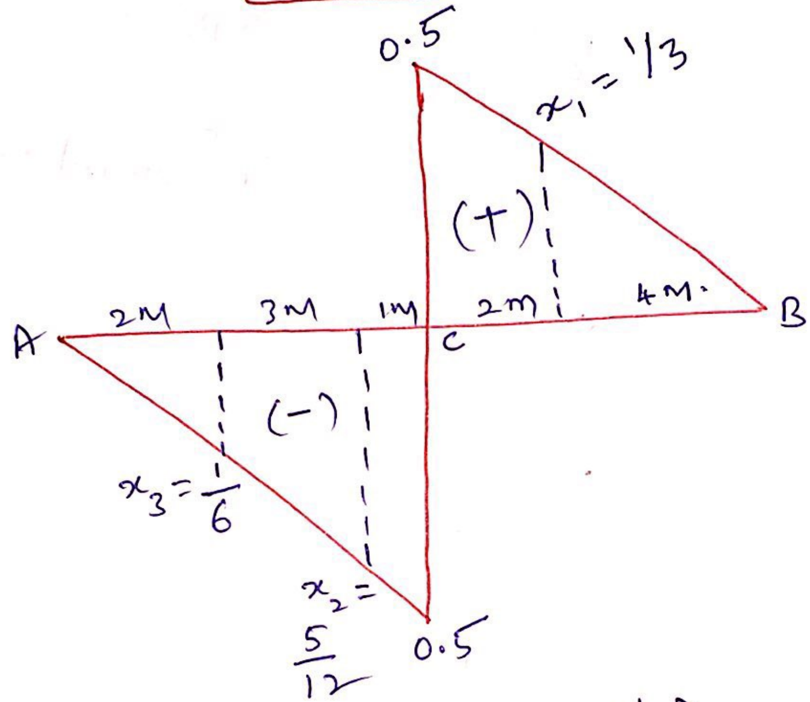
$$\boxed{x_1 = \frac{1}{3}}$$

Lower triangle: $\frac{0.5}{6} = \frac{x_2}{5}$

$$\boxed{x_2 = \frac{5}{12}}$$

$$\frac{0.5}{6} = \frac{x_3}{2}$$

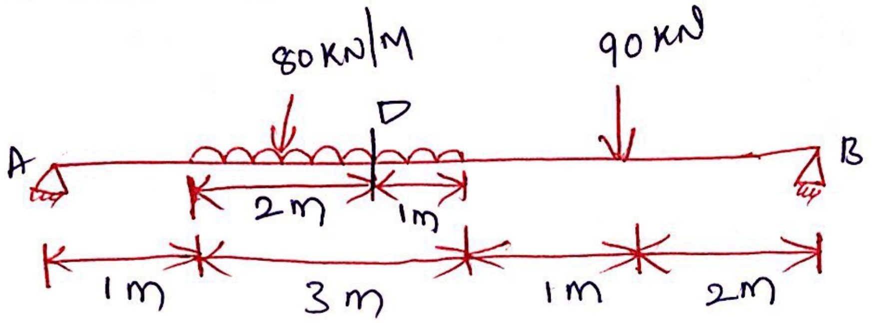
$$\boxed{x_3 = \frac{1}{6}}$$

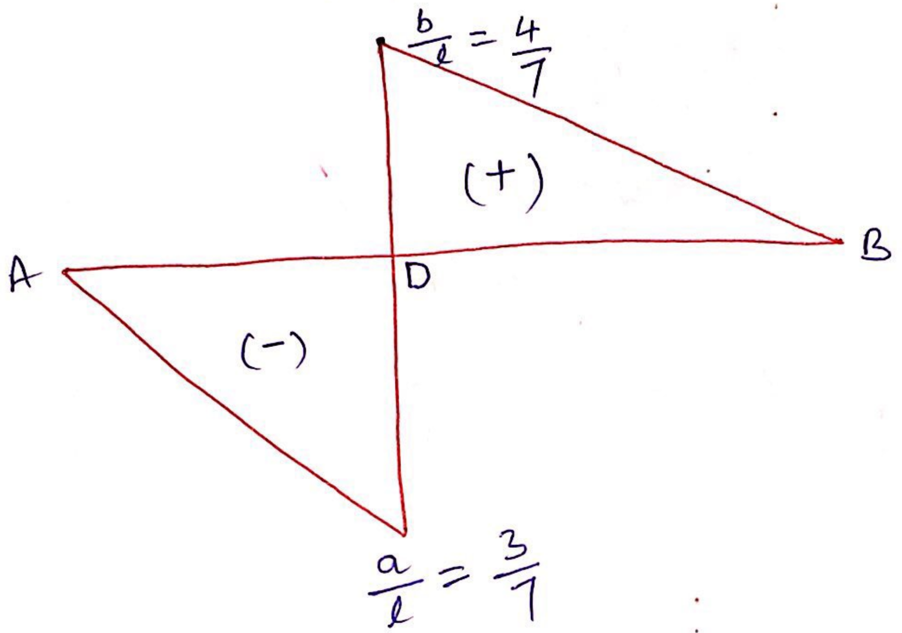


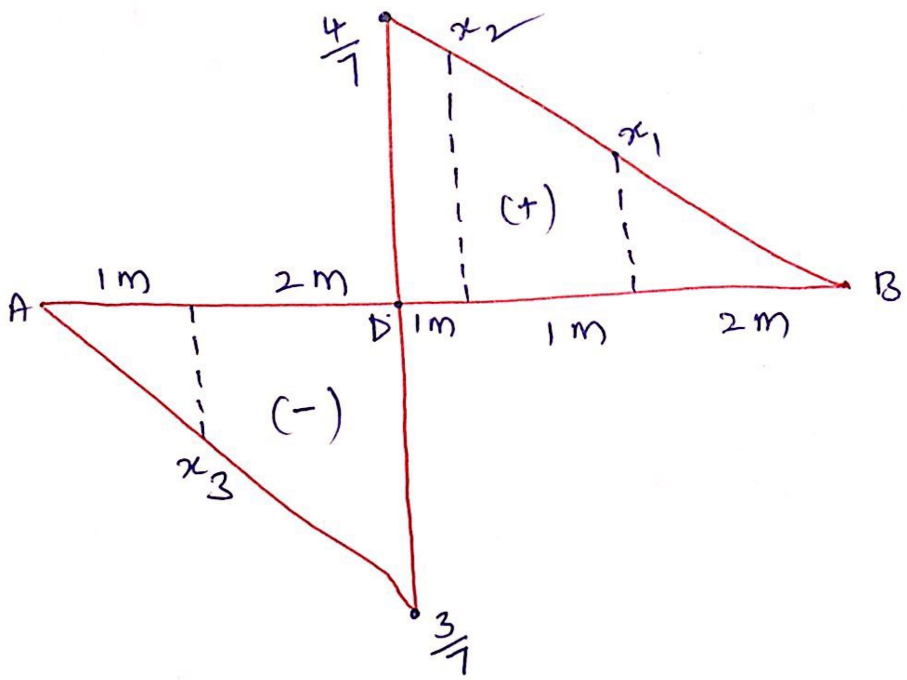
$$(SF)_c = 70\left(\frac{-1}{6}\right) + 60\left(\frac{-5}{12}\right) + 50\left(\frac{1}{3}\right)$$

$$\boxed{(SF)_c = -20\,KN.}$$

2) Shear force at D.







From similar triangle concept :

upper triangle : $\frac{4/7}{4} = \frac{x_2}{3}$

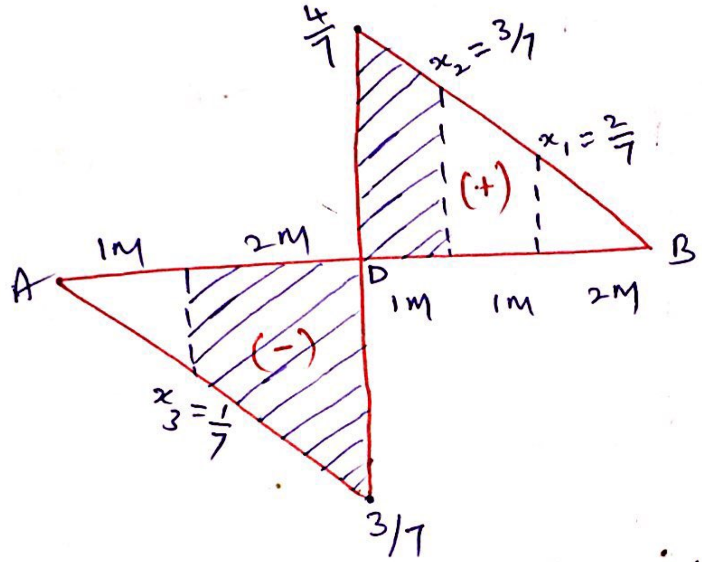
$$\boxed{x_2 = \frac{3}{7}}$$

$$\frac{4/7}{4} = \frac{x_1}{2}$$

$$\boxed{x_1 = \frac{2}{7}}$$

Lower triangle: $\frac{3/7}{3} = \frac{x_3}{1}$

$$\boxed{x_3 = \frac{1}{7}}$$

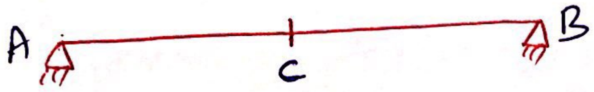


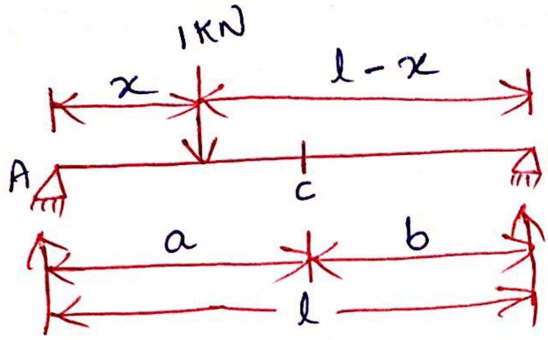
$$(SF)_D = 80\left[\left(-\frac{1}{7}\right) + \left(-\frac{3}{7}\right)\right] \times \frac{2}{2} + 80 \times \left(\frac{4}{7} + \frac{3}{7}\right) \cdot \frac{1}{2} + \left(90 \times \frac{2}{7}\right)$$

$$\boxed{(SF)_D = 20 \text{ KN.}}$$

ILD for Bending Moment :—

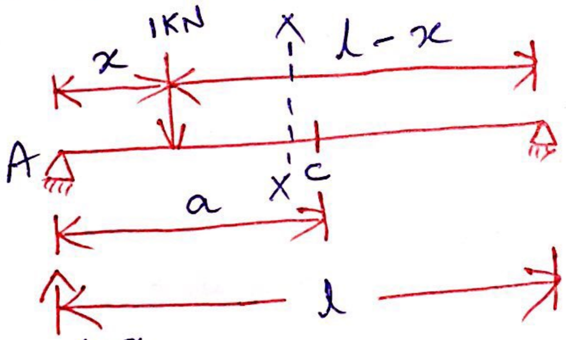
ILD for M_C ?





$R_A = \frac{l-x}{l}$ $\qquad$ $R_B = \frac{x}{l}$

when unit load is in left portion i.e. AC :—



$R_A = \frac{l-x}{l}$

$$M_C = (R_A \times a) - 1(a-x)$$

$$= \left(\frac{l-x}{l}\right)a - (a-x)$$

$$= \frac{al - ax}{l} - (a-x)$$

$$= \frac{\cancel{al} - ax - \cancel{al} + xl}{l}$$

$$= \frac{xl - ax}{l}$$

$$a + b = l$$

$$b = (l - a)$$

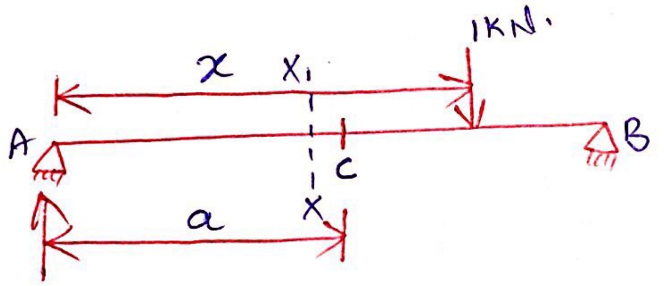
$$= \frac{x(l-a)}{l}$$

$$\boxed{M_c = \frac{b \cdot x}{l}}$$

at $x = a$

$$\boxed{M_c = \frac{ab}{l}}$$

when unit load is in right portion i.e. CB :—



$$R_A = \frac{l - x}{l}$$

$$M_c = R_A \times a$$

$$= \left(\frac{l-x}{l}\right)a$$

at point 'c', $x = a$

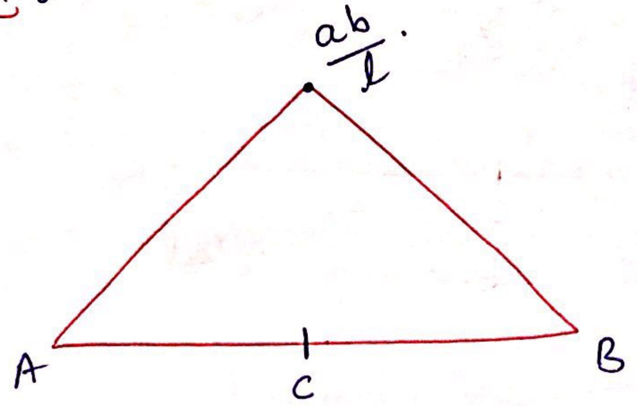
$$M_c = \frac{(l-a)a}{l}$$

$$a + b = l$$

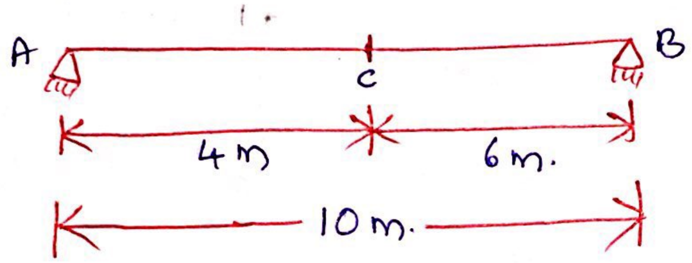
$$b = (l - a)$$

$$\boxed{M_c = \frac{ab}{l}}$$

ILD for BM :—



Ex:-



Find moment at c ?

(A) $M_c = \frac{ab}{l}$

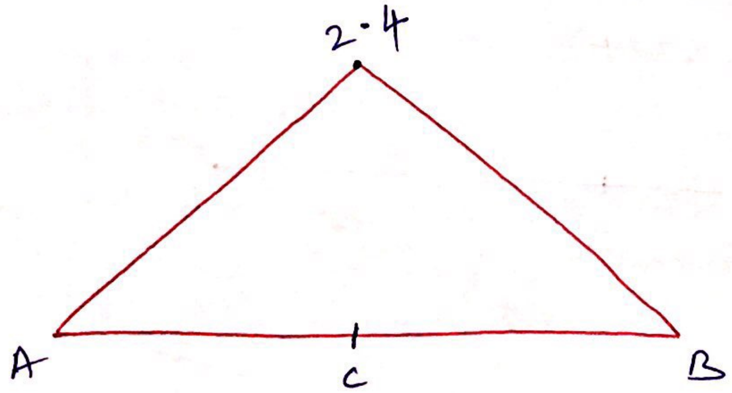
where: $a = 4m$

$b = 6m$

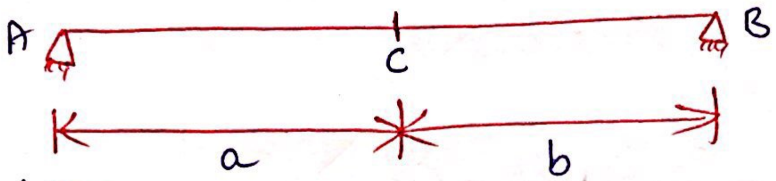
$l = 10m.$

$$M_c = \frac{4 \times 6}{10}$$

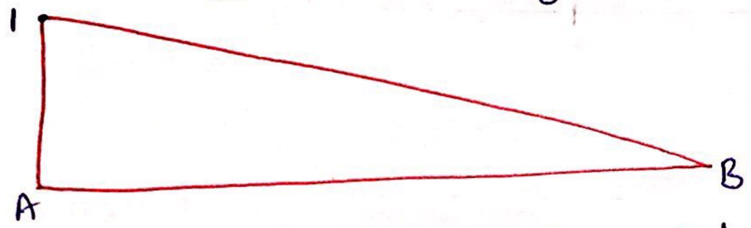
$$\boxed{M_c = 2.4}$$



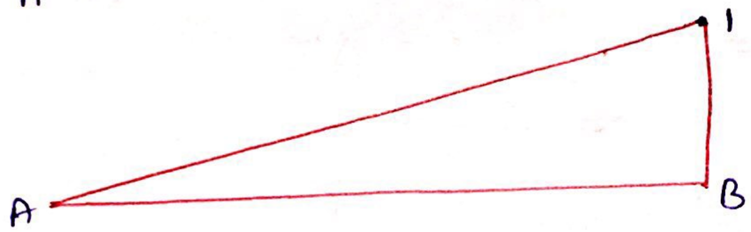
Final ILD for R_A, R_B, $(SF)_c$, $(BM)_c$:—



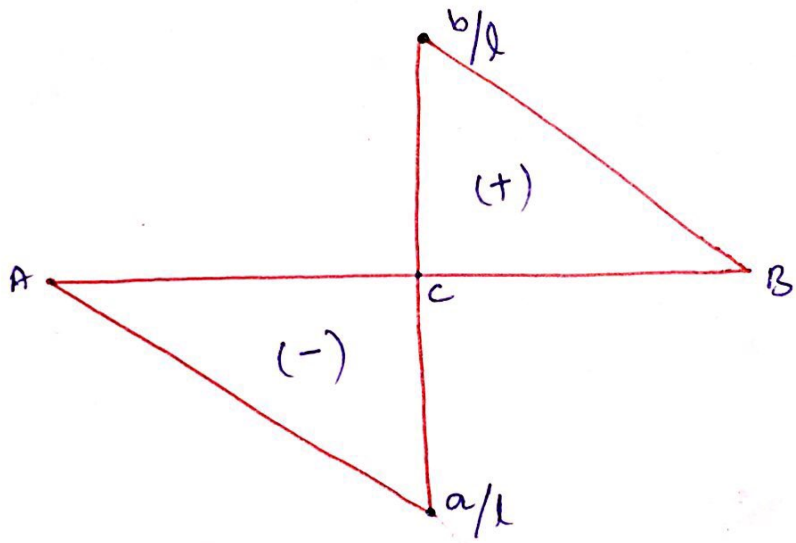
ILD for R_A



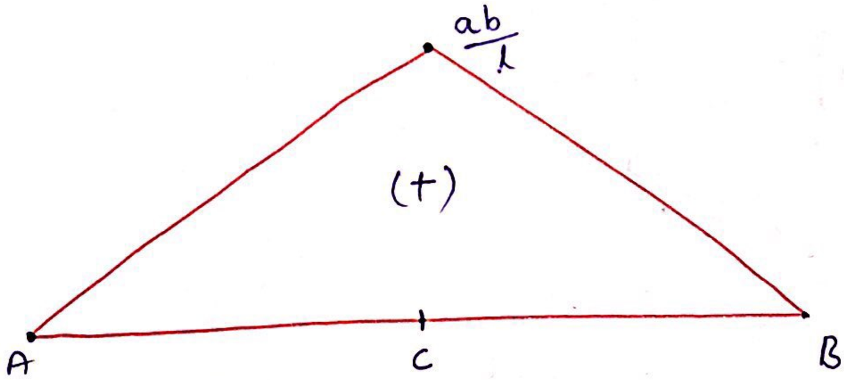
ILD for R_B



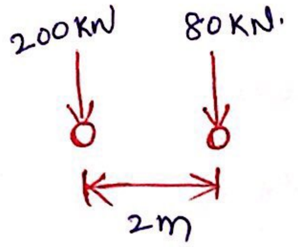
ILD for SF

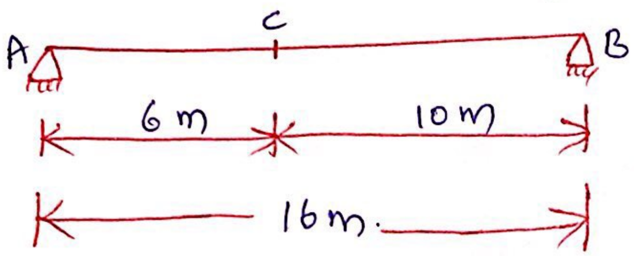


ILD for BM

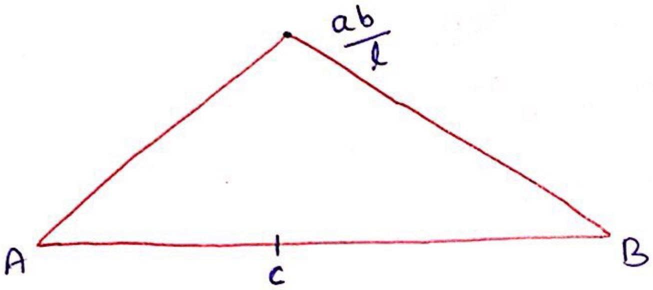


1. Find moment at 'c' ?





(A)

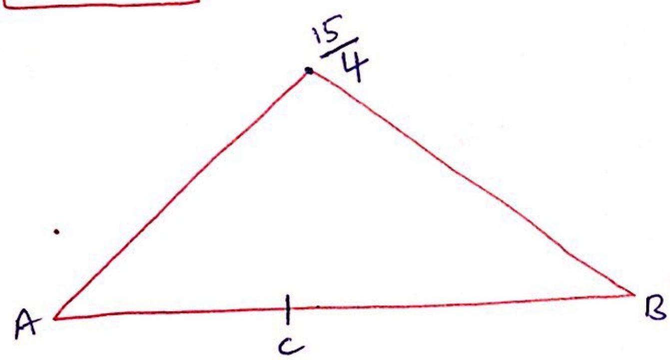


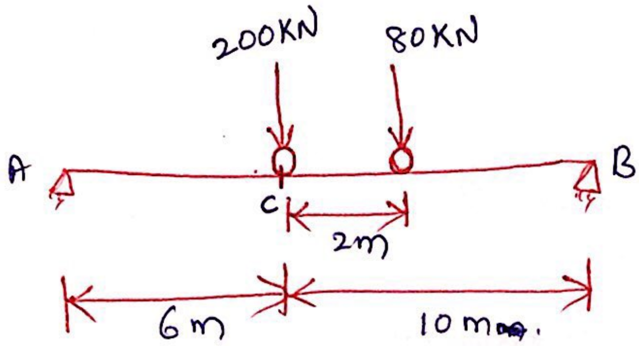
where: $a = 6m$, $b = 10m$, $l = 16m$.

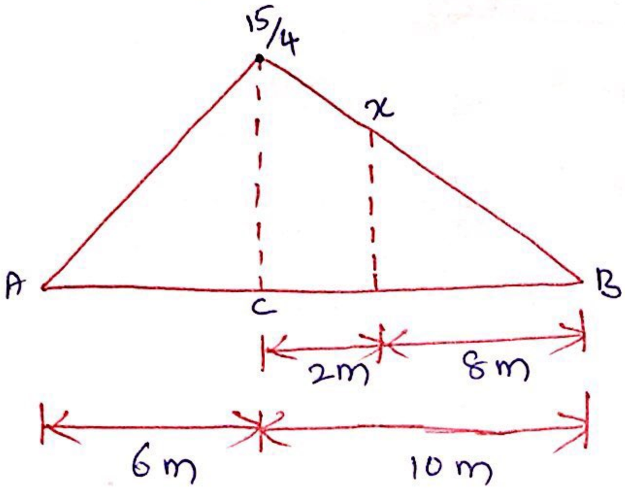
$$M_c = \frac{ab}{l}$$

$$= \frac{6 \times 10}{16}$$

$$\boxed{M_c = \frac{15}{4}}$$



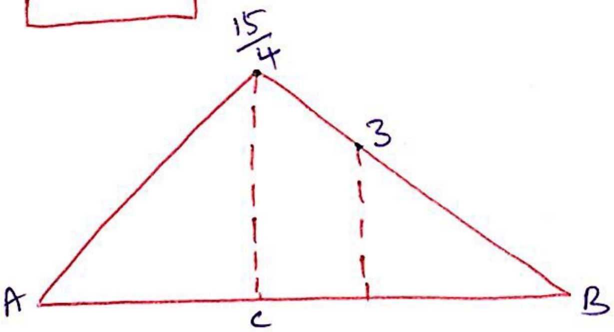




From similar triangle concept:

$$\frac{15/4}{10} = \frac{x}{8}$$

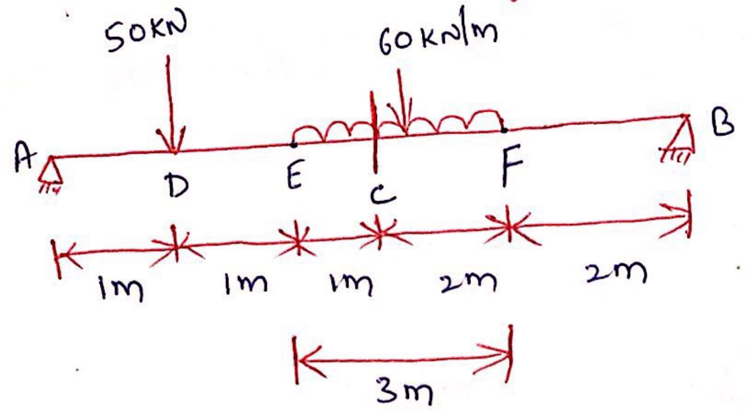
$$\boxed{x = 3}$$



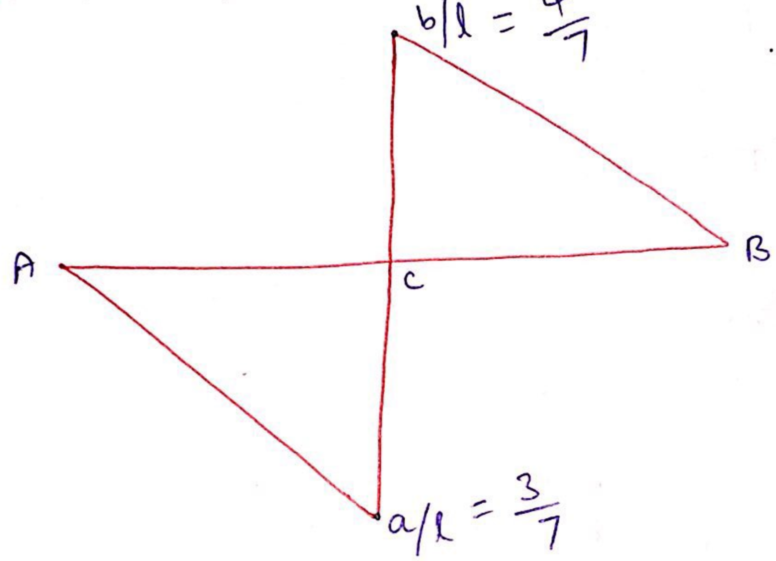
$$M_c = \left(200 \times \frac{15}{4}\right) + (80 \times 3)$$

$$\boxed{M_c = 990 \text{ KN.m.}}$$

(1) Find the shear force and bending moment at 'c'



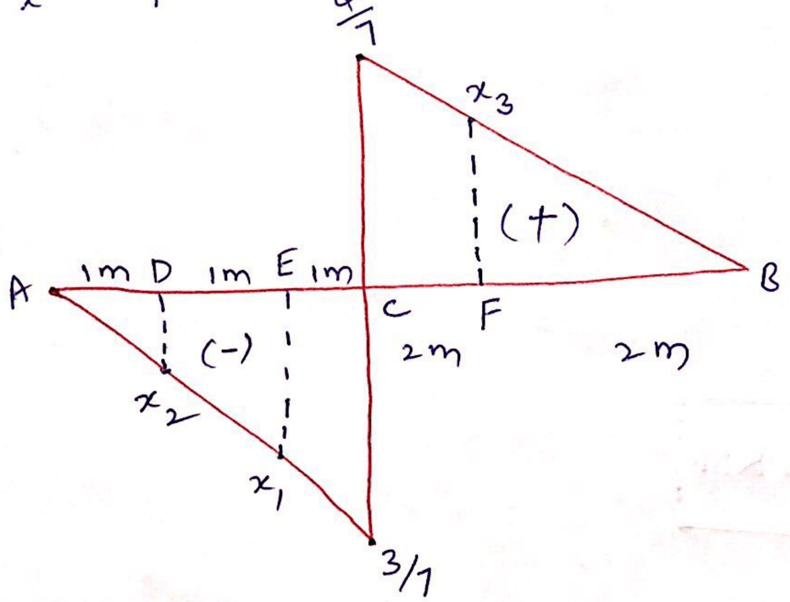
(A) ILD for shear force at 'c' :—



where: $a = 3m$, $b = 4m$, $l = 7m$.

$$\frac{a}{l} = \frac{3}{7}$$

$$\frac{b}{l} = \frac{4}{7}$$



From similar triangle concept :

$$\frac{4/7}{4} = \frac{x_3}{2}$$

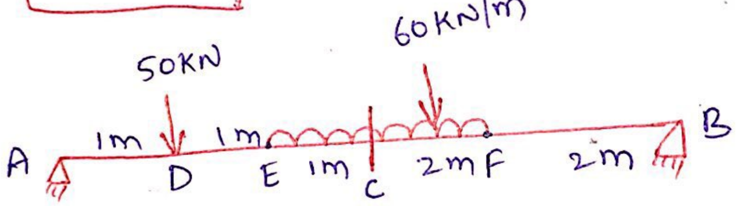
$$\boxed{x_3 = \frac{2}{7}}$$

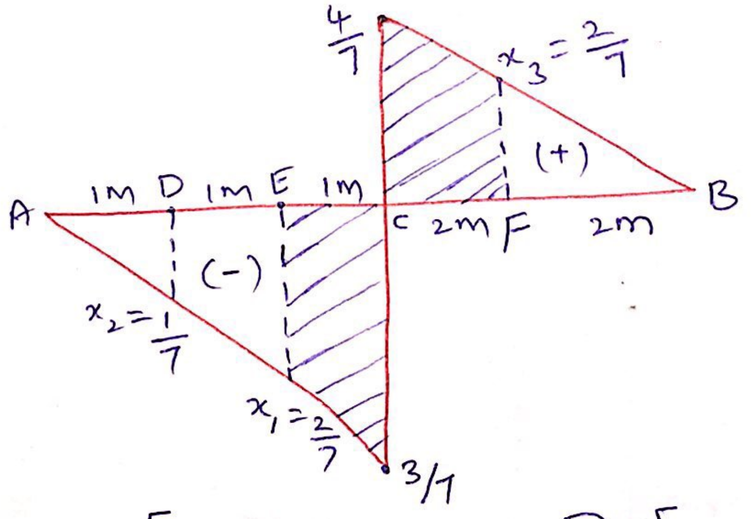
$$\frac{3/7}{3} = \frac{x_1}{2}$$

$$\boxed{x_1 = \frac{2}{7}}$$

$$\frac{3/7}{3} = \frac{x_2}{1}$$

$$\boxed{x_2 = \frac{1}{7}}$$



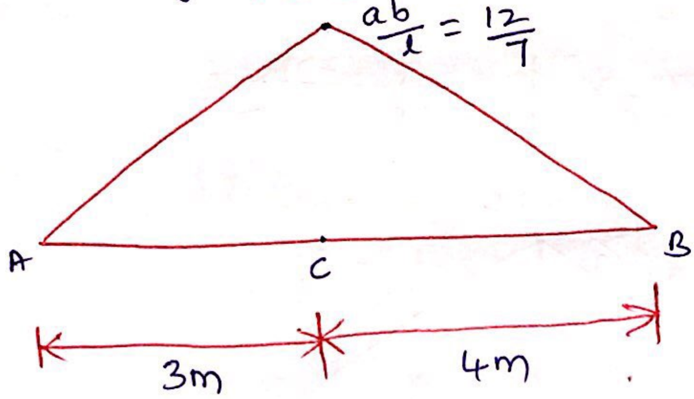


$(a+b)\cdot\frac{h}{2}$

$$(SF)_c = 50\times\left(\frac{-1}{7}\right) + \left[60\times\left(\frac{-2}{7}\right) + \left(\frac{-3}{7}\right)\times\frac{1}{2}\right] + \left[60\times\left(\frac{4}{7}\right) + \left(\frac{2}{7}\right)\times\cancel{\frac{2}{2}}\right]$$

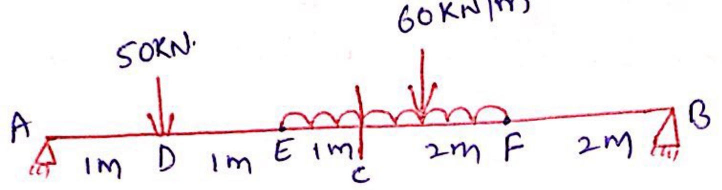
$$\boxed{(SF)_c = 29.46 \text{ KN.}}$$

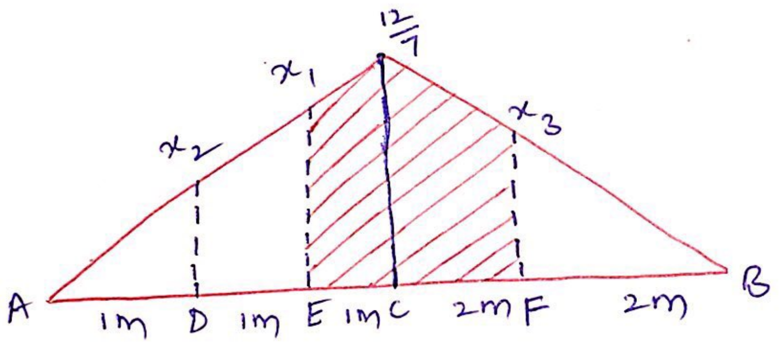
ILD for bending moment at 'c' :—



where : $a = 3m, b = 4, l = 7m.$

$$\frac{ab}{l} = \frac{3 \times 4}{7} = \frac{12}{7}$$



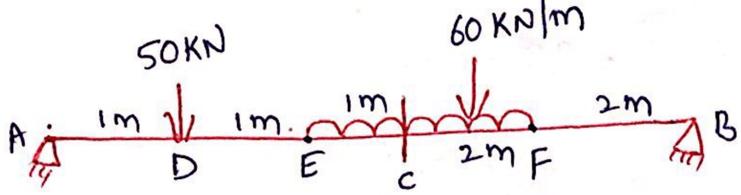


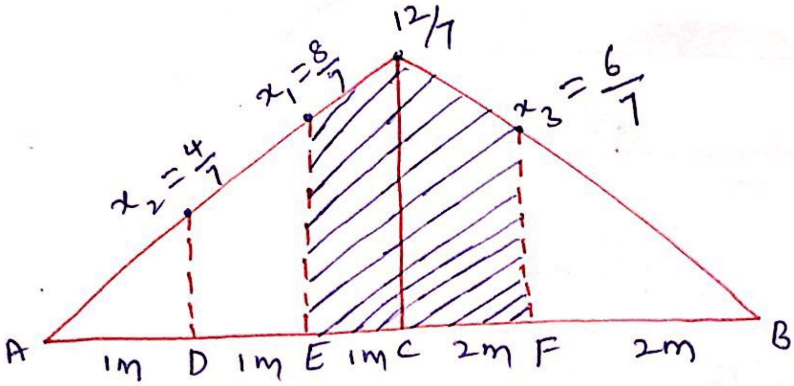
From similar triangle concept :

$$\frac{12/7}{3} = \frac{x_1}{2} \Rightarrow x_1 = \frac{\cancel{24}^{8}}{\cancel{21}_{7}} \Rightarrow \boxed{x_1 = \frac{8}{7}}$$

$$\frac{12/7}{3} = \frac{x_2}{1} \Rightarrow x_2 = \frac{\cancel{12}^{4}}{\cancel{21}_{7}} \Rightarrow \boxed{x_2 = \frac{4}{7}}$$

$$\frac{12/7}{4} = \frac{x_3}{2} \Rightarrow x_3 = \frac{\cancel{24}^{6}}{\cancel{28}_{7}} \Rightarrow \boxed{x_3 = \frac{6}{7}}$$



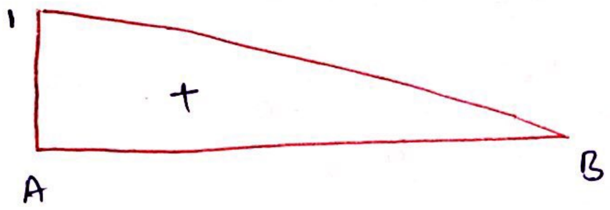


$$(BM)_c = \left(50 \times \frac{4}{7}\right) + \left[60 \times \left(\frac{8}{7} + \frac{12}{7}\right) \times \frac{1}{2}\right] + \left[60 \times \left(\frac{12}{7} + \frac{6}{7}\right) \times \frac{2}{2}\right]$$

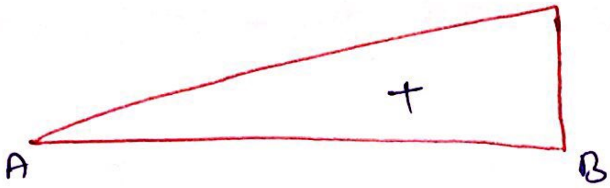
$$\boxed{(BM)_c = 268.2 \text{ KN.m.}}$$

Effect of Rolling Loads :—

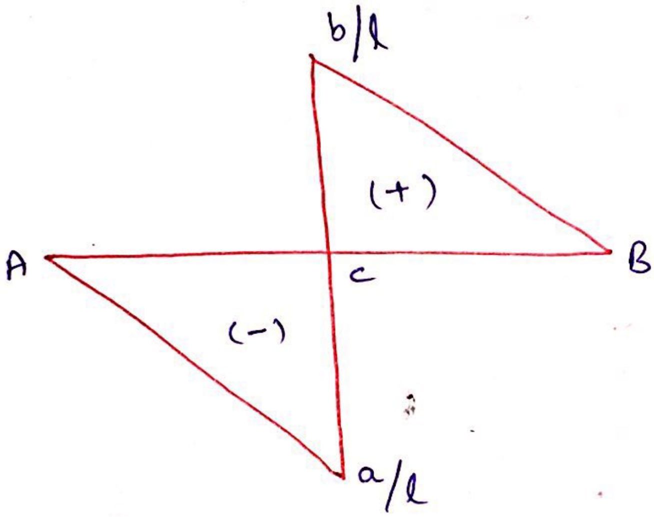
ILD for R_A



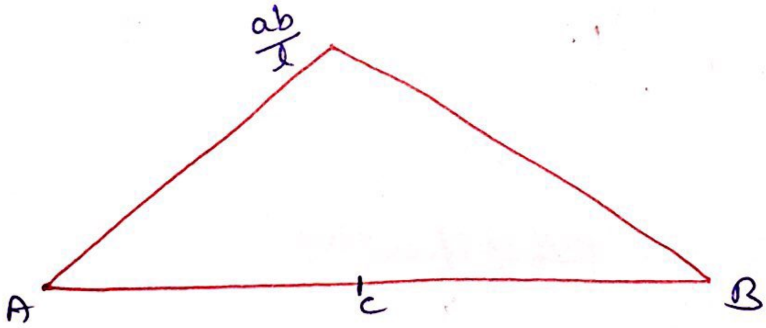
ILD for R_B



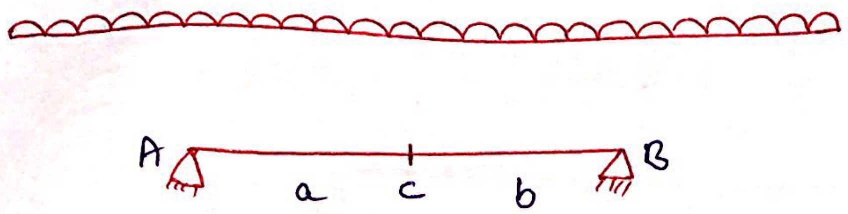
ILD for $(SF)_c$



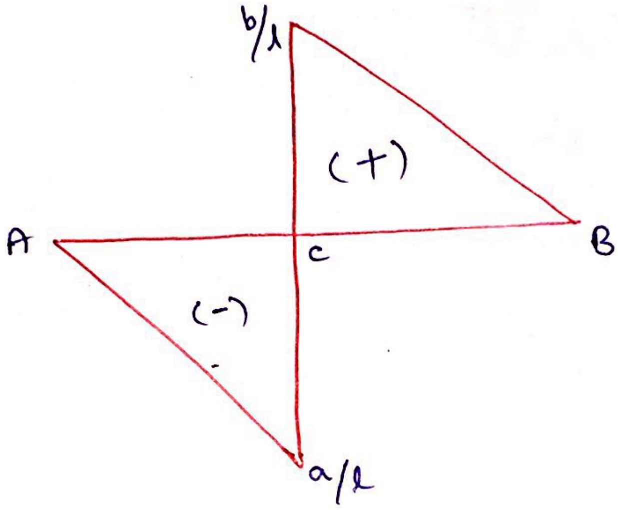
ILD for $(BM)_c$

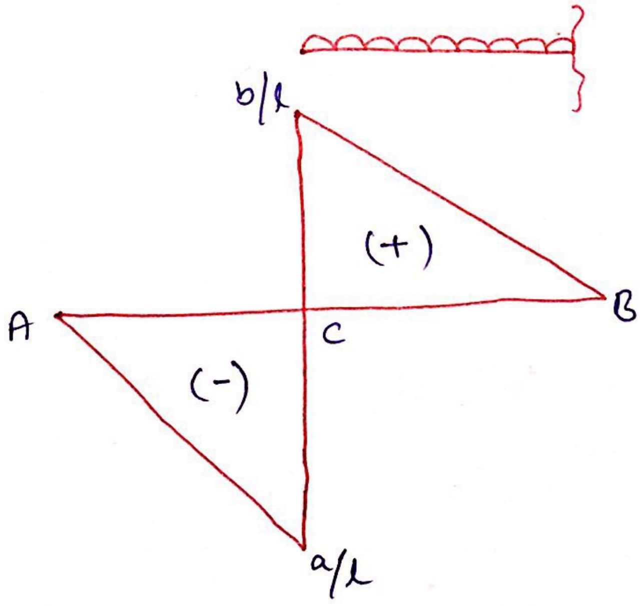


Case - I : when a UDL whose span greater than the span of the beam :—

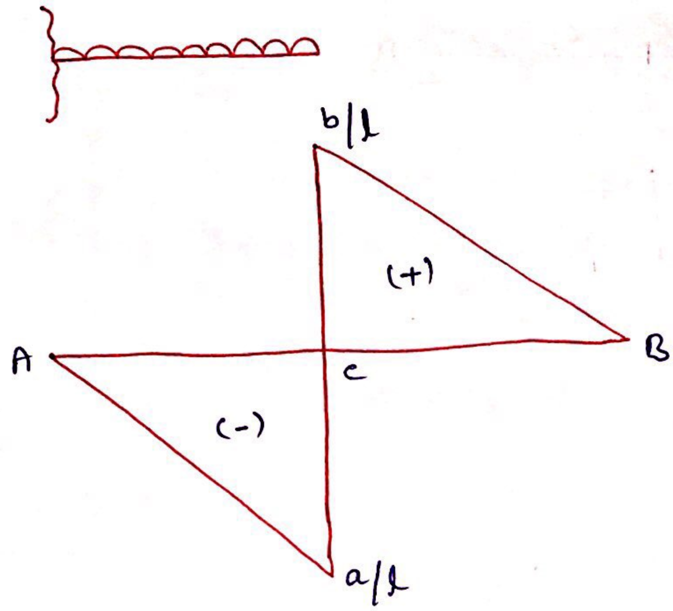


Maximum positive shear force at 'c' :—

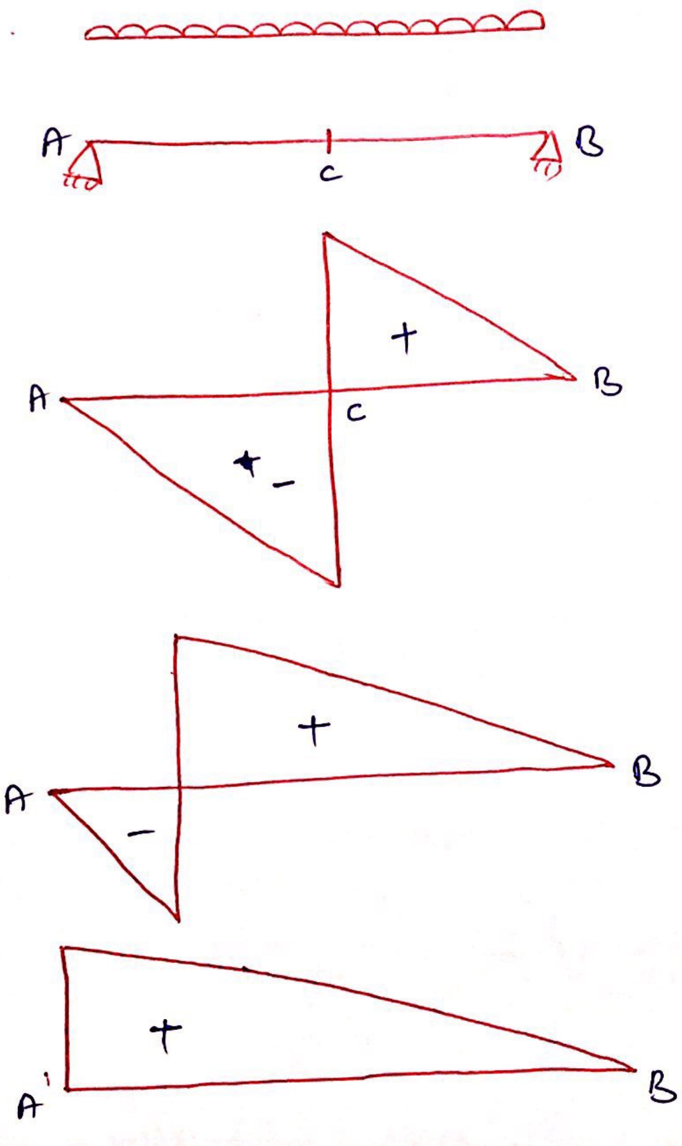


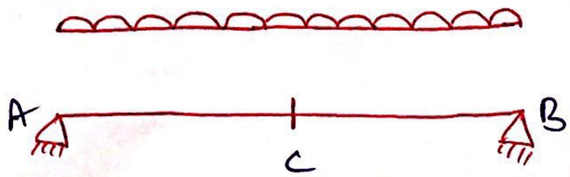


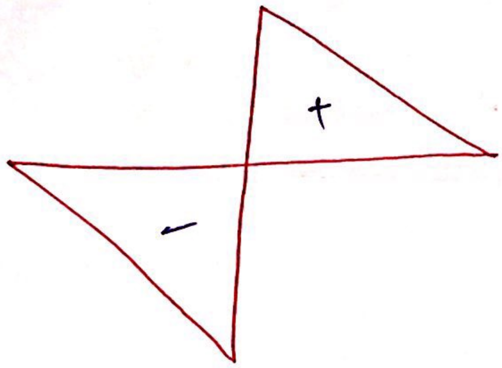
Maximum negative shear force at 'c' :—

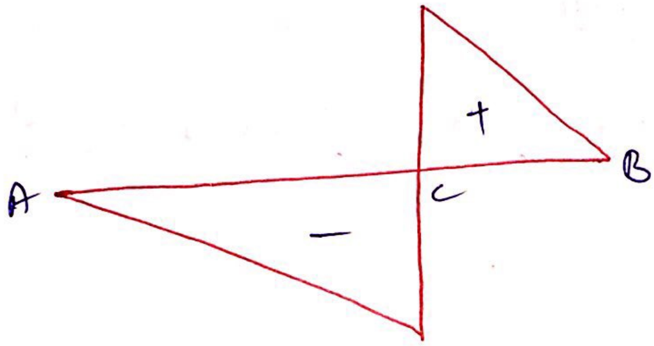


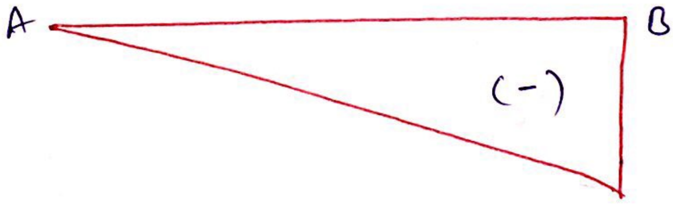
Absolute maximum positive/negative shear force at 'c' :—



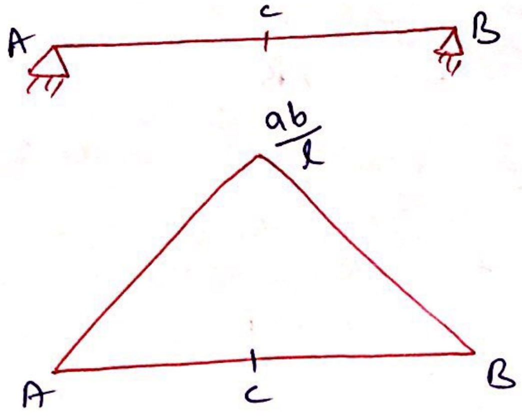

A
C
B


+
-

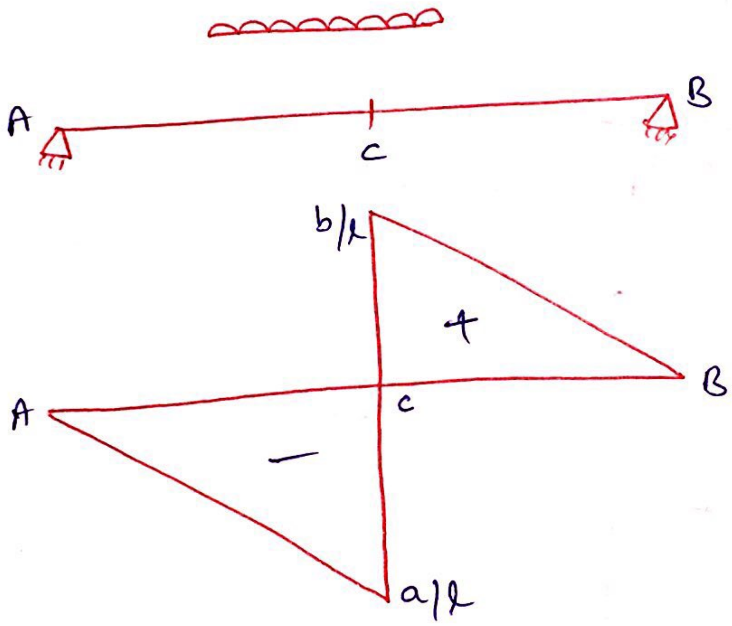

+
A
C
B
-


A
B
(-)

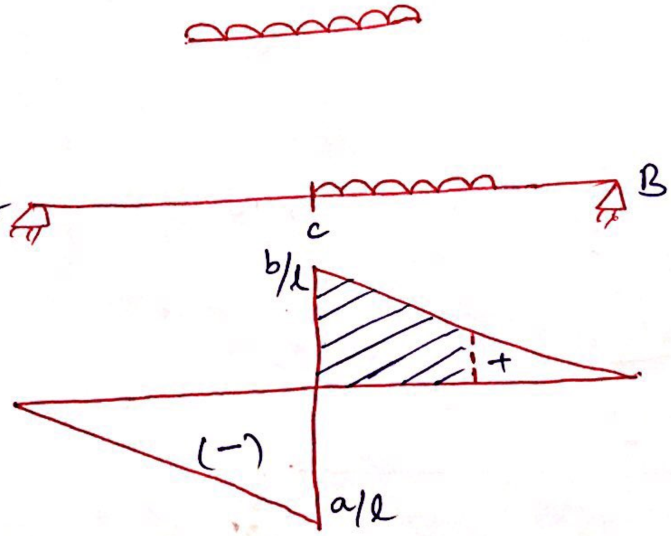
Bending moment at 'c' :—



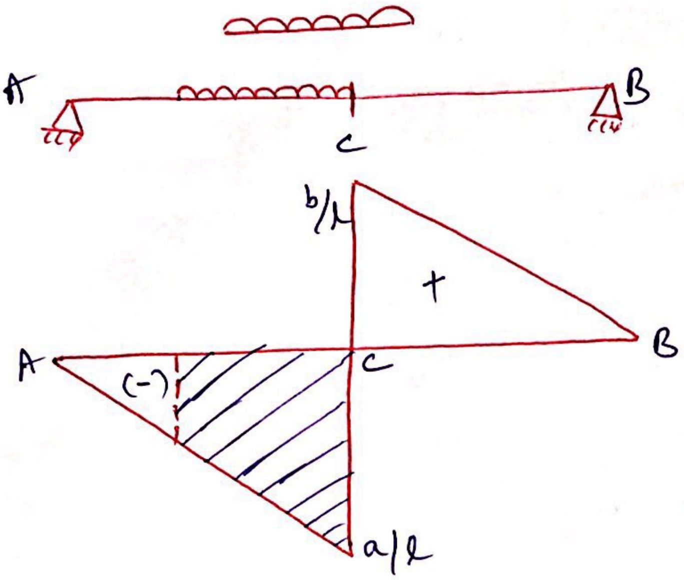
Case - II : UDL shorter than the span :—



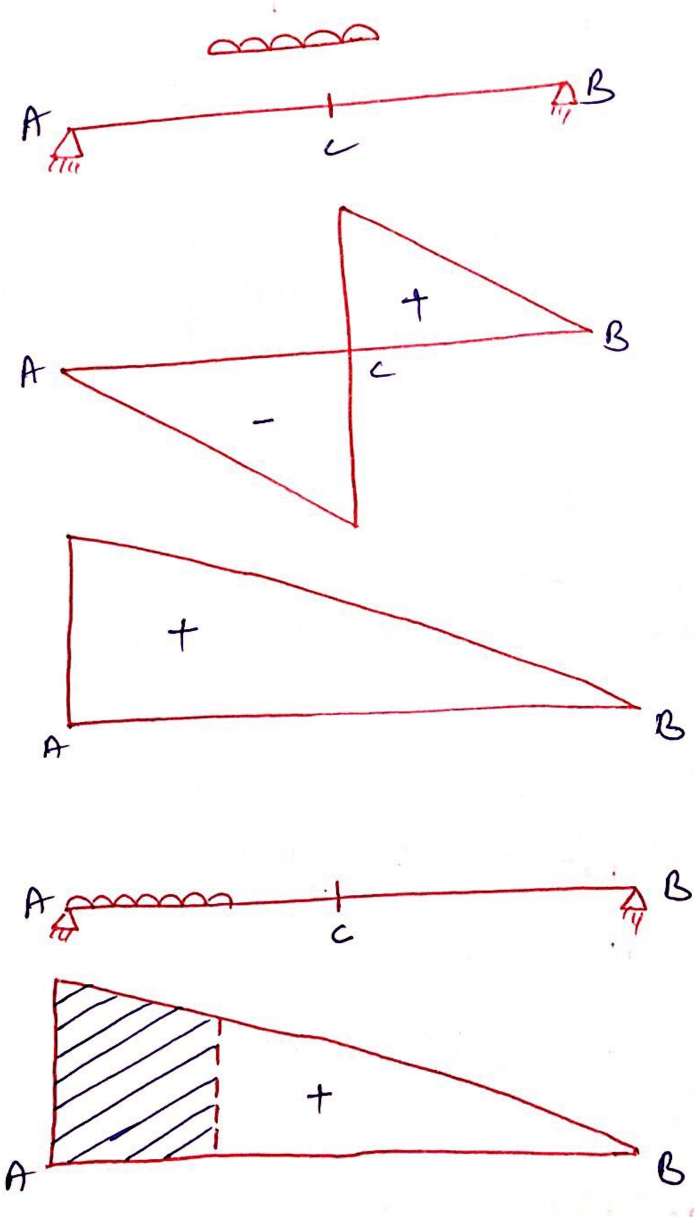
Maximum positive $(SF)_c$:—



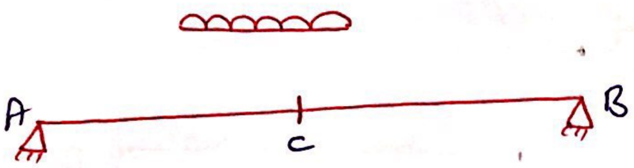
Maximum negative $(SF)_c$:—

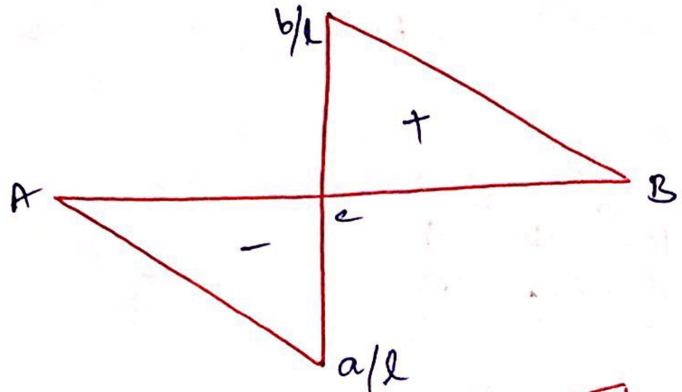


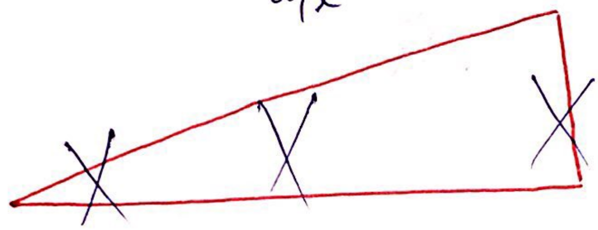
Absolute maximum positive $(SF)_c$:—

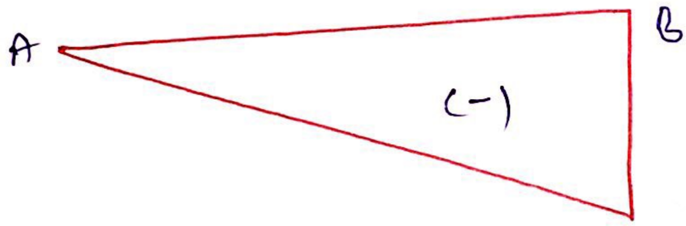


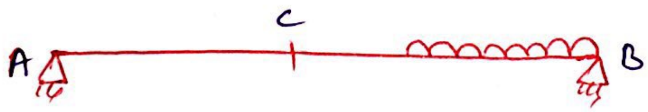
Absolute maximum negative $(SF)_c$:—

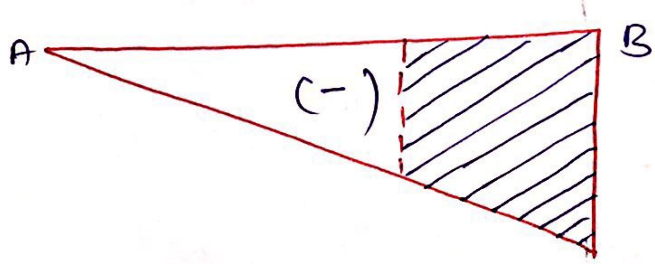




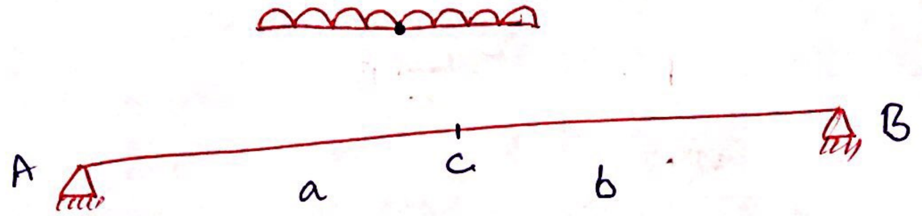


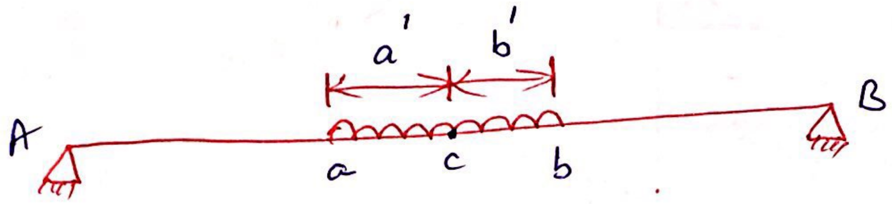


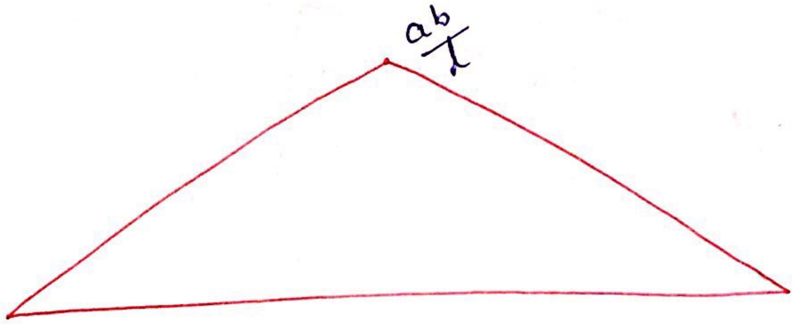




Case (iii) : Maximum Bending moment at 'c' :—

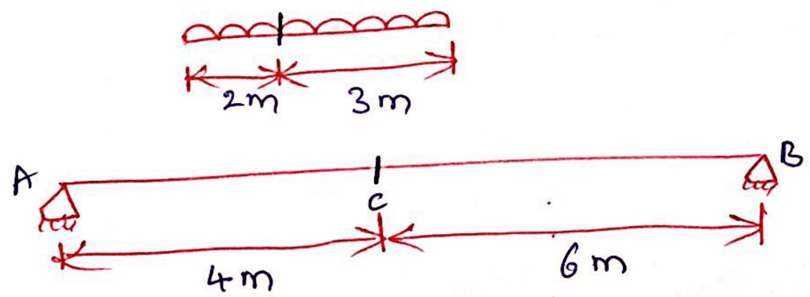


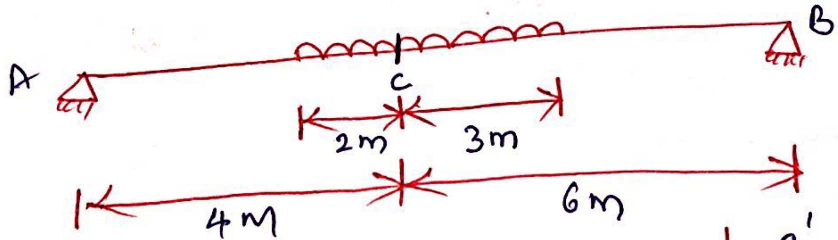




Max $\Rightarrow$ $\boxed{\dfrac{a'}{a} = \dfrac{b'}{b}}$

Ex:-





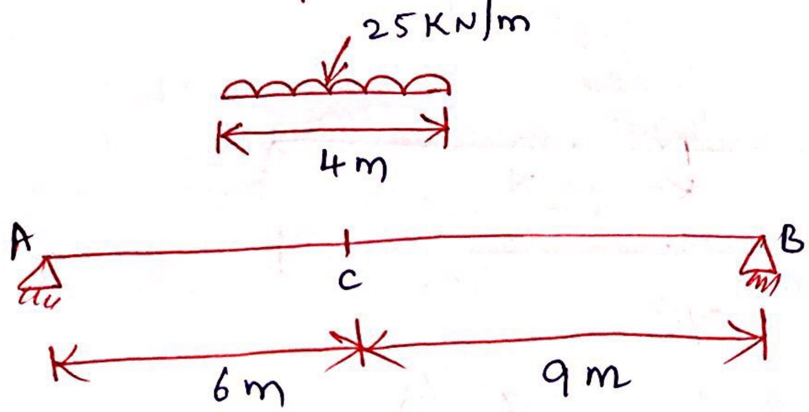
$a' = 2m,\ b' = 3m,\ a = 4m,\ b = 6m.$

$$\frac{a'}{a} = \frac{b'}{b}$$

$$\frac{2}{4} = \frac{3}{6} \Rightarrow \boxed{\frac{1}{2} = \frac{1}{2}}$$

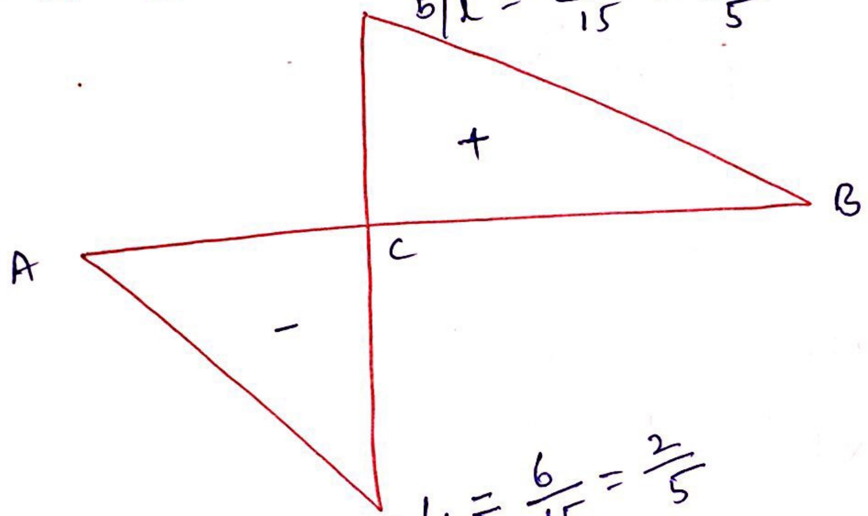
① Find the maximum +ve & −ve shear force at 'C' and maximum bending moment at 'C' ?

(16)



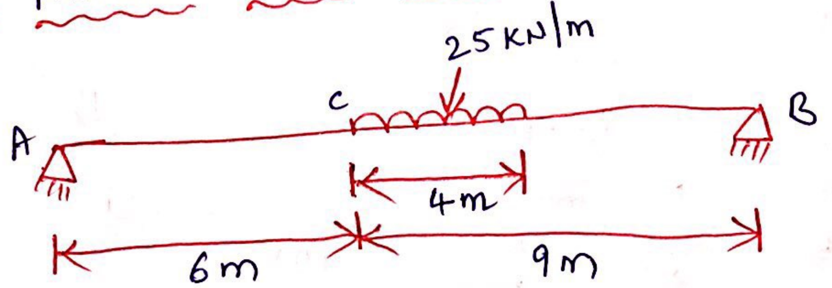
Ⓐ ILD for shear force :—

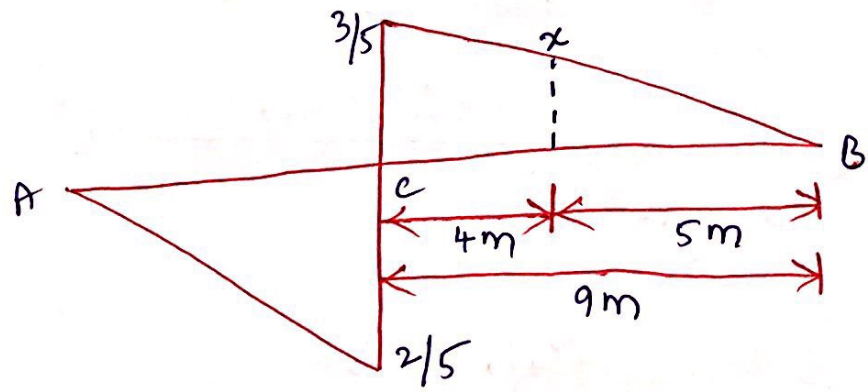
$b/l = \frac{9}{15} = \frac{3}{5}$



$a/l = \frac{6}{15} = \frac{2}{5}$

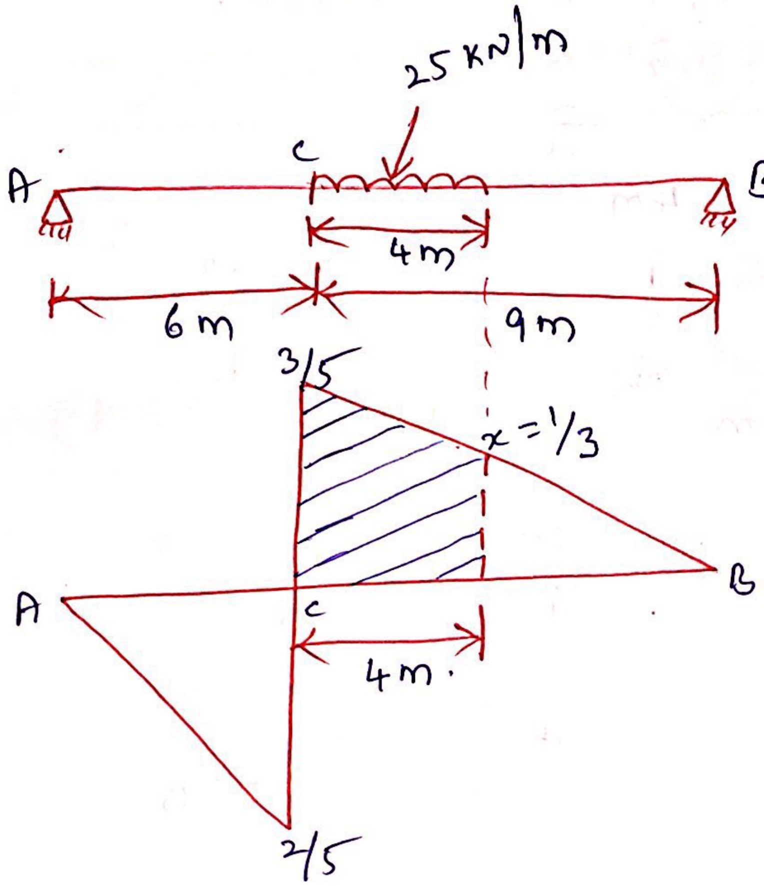
Maximum positive shear force at 'C' :—





Scanned with CamScanner

$$\frac{3/5}{9} = \frac{x}{5} \Rightarrow x = \frac{1}{3}$$

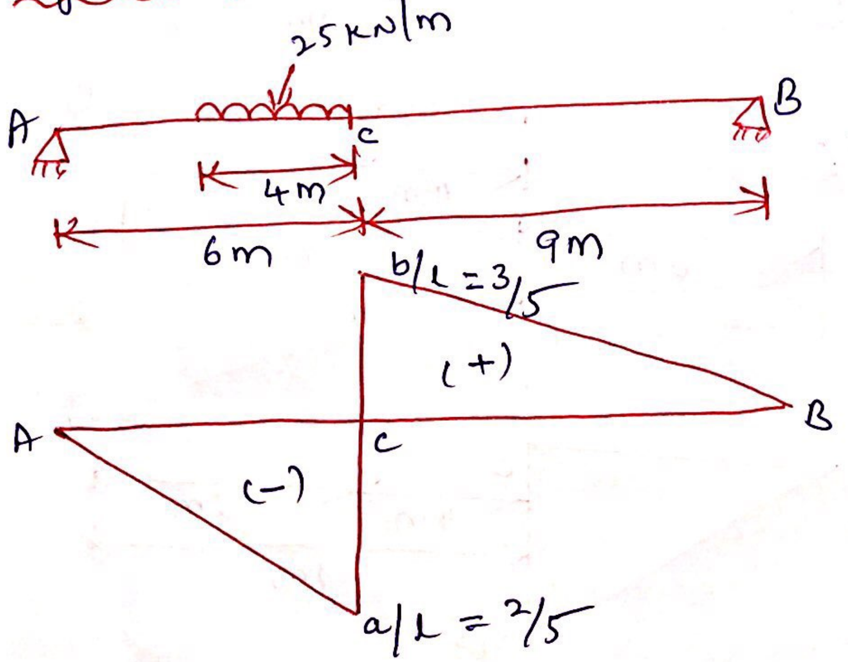


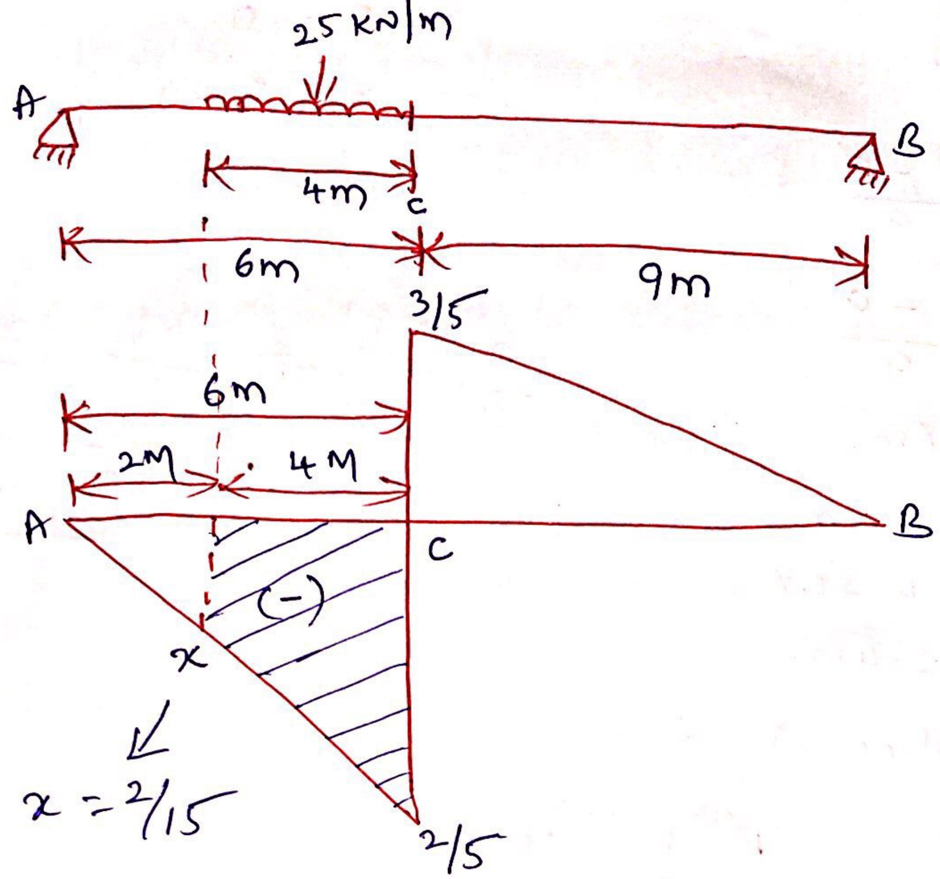
Maximum positive shear force at 'c' is

$$(SF)_c = 25 \times \left(\frac{3}{5} + \frac{1}{3}\right) \times \frac{4}{2}$$

$$(SF)_c = 46.67 \text{ kN}.$$

Maximum negative shear force at 'c' :—





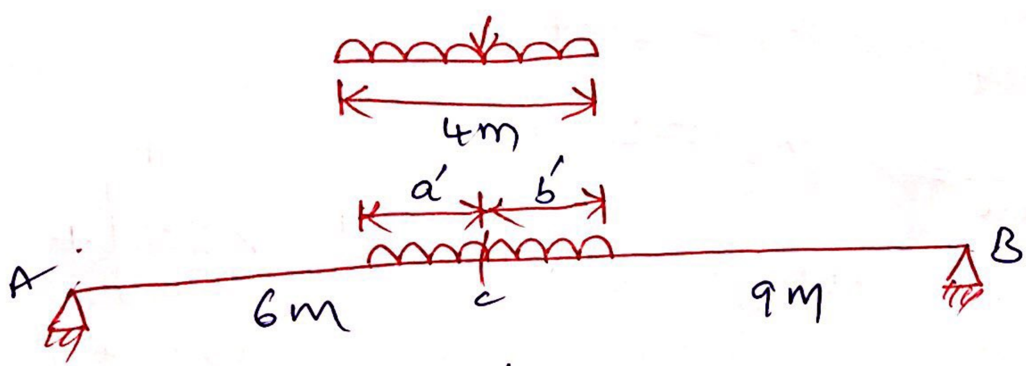
$$\frac{2/5}{6} = \frac{x}{2} \Rightarrow x = 2/15$$

Maximum negative shear force at 'c' :-

$$(SF)_c = -25 \times \left(\frac{2}{15} + \frac{2}{5}\right) \times \frac{4}{2}$$

$$(SF)_c = -26.65\ KN.$$

Maximum bending moment at 'c' :-



where: $\frac{a'}{a} = \frac{b'}{b}$

$$\frac{a'}{6} = \frac{b'}{9}$$

$$\frac{a'}{6} = \frac{l' - a'}{9}$$

$$\frac{a'}{6} = \frac{4 - a'}{9}$$

$$a' = 1.6 m.$$

Where: $b' = l' - a'$

$= 4 - 1.6$

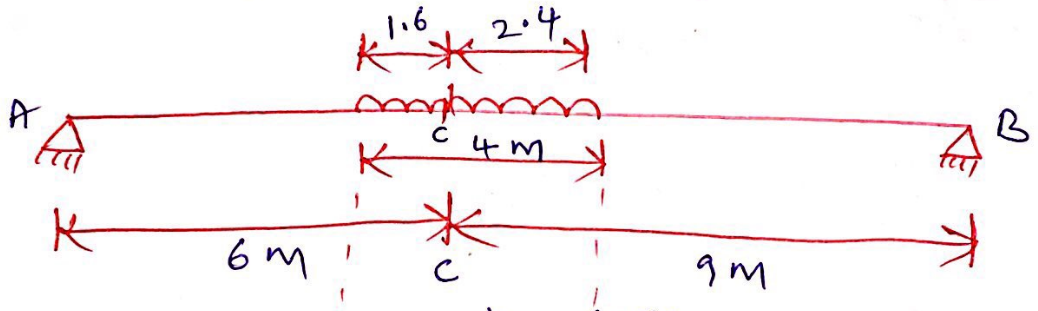
$b' = 2.4 m.$

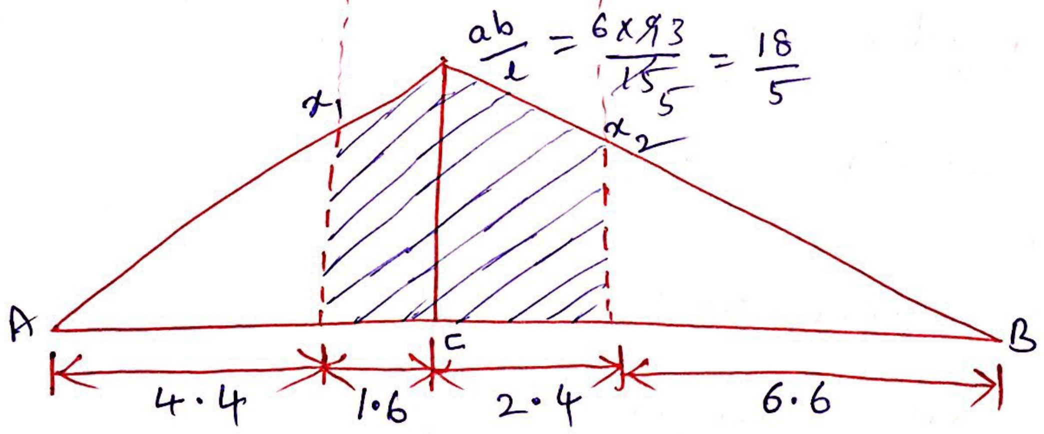
$$\left[\begin{array}{l} a' + b' = l' \\ b' = l' - a' \\ l' = 4 m. \end{array}\right]$$

$$\frac{a'}{a} = \frac{b'}{b} \Rightarrow \frac{1.6}{6} = \frac{2.4}{9}$$

$$\boxed{0.267 = 0.267}$$

ILD for bending at C:





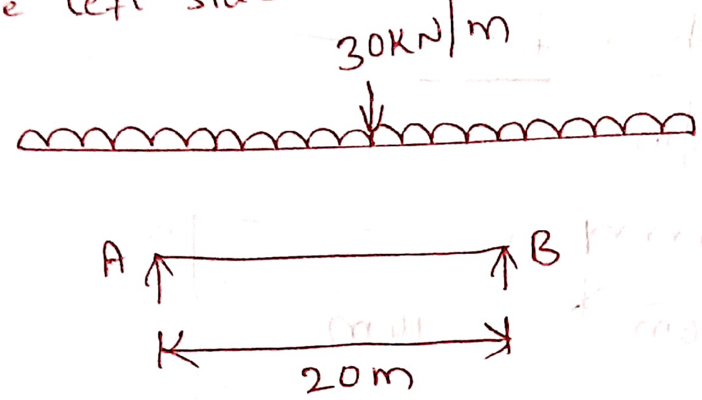
$$\frac{18/5}{6} = \frac{x_1}{4.4} \Rightarrow x_1 = 2.64$$

$$\frac{18/5}{9} = \frac{x_2}{6.6} \Rightarrow x_2 = 2.64$$

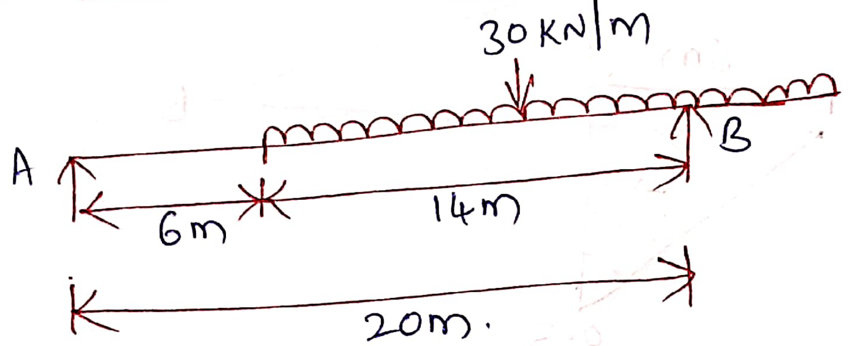
Max. BM at C; $(BM)_c = \left[25 \times \left(2.64 + \frac{18}{5}\right) \times \frac{1.6}{2}\right] + \left[25 \times \left(\frac{18}{5} + 2.64\right) \times \frac{2.4}{2}\right]$

$(BM)_c = 312$ KN.m.

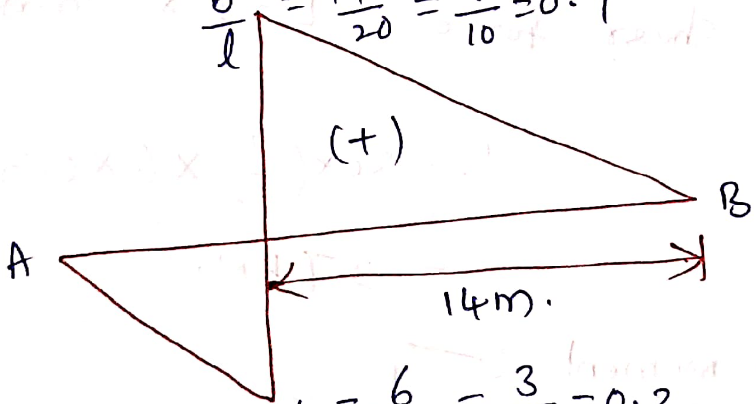
(1) A UDL of 30KN/m longer than span rolls over a girder of 20m. span from left to right. Draw ILD for maximum shear force and bending moment at a section 6m. from the left side. (1)



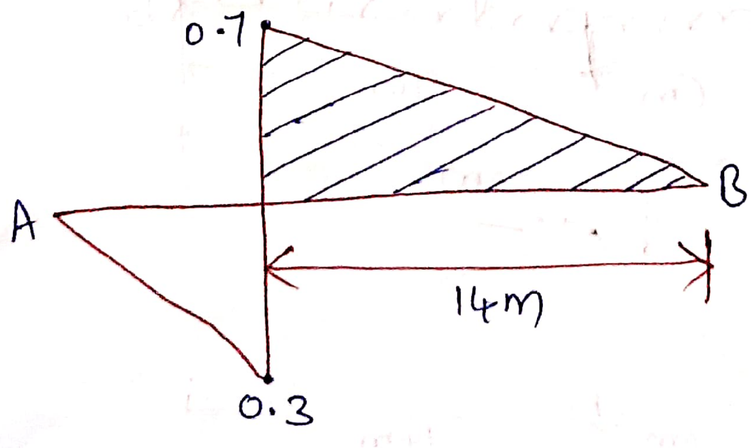
(A) Maximum positive shear force :—



$\frac{b}{l} = \frac{14}{20} = \frac{7}{10} = 0.7$



$a/l = \frac{6}{20} = \frac{3}{10} = 0.3$

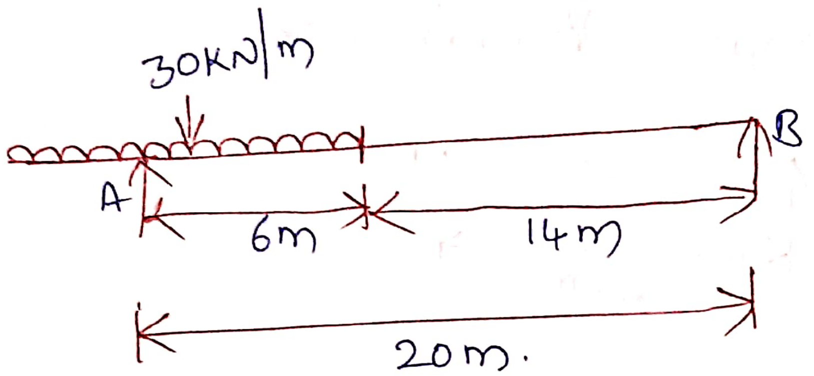


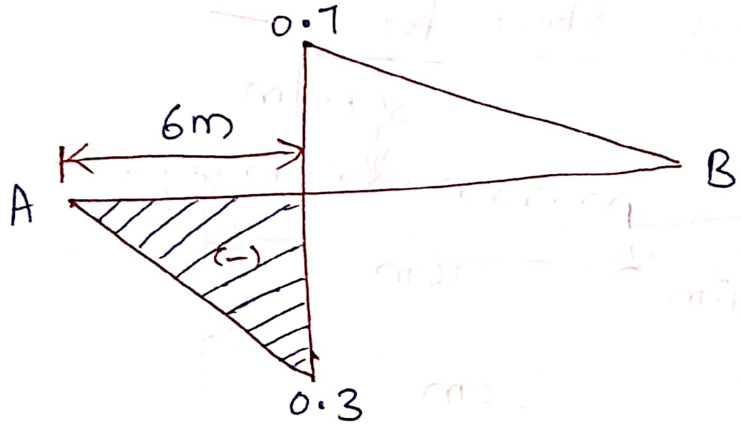
maximum positive shear force = Load x ordinate

$$= 30 \times \left(\frac{1}{2} \times 14 \times 0.7\right)$$

$$= 147\ KN.$$

maximum negative shear force :—



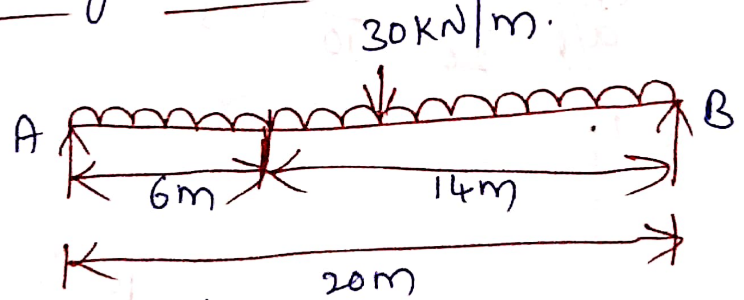


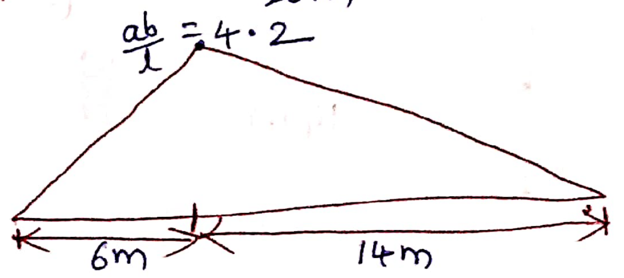
maximum negative shear force = Load x ordinate

$$= 30 \times \left(\frac{1}{2} \times 6 \times 0.3\right)$$

$$= 27\ KN.$$

Maximum bending moment :—

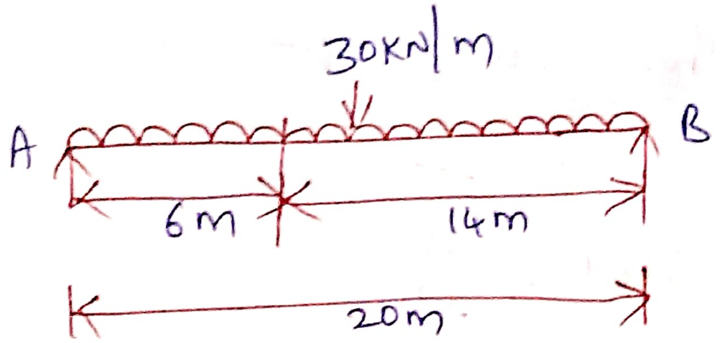


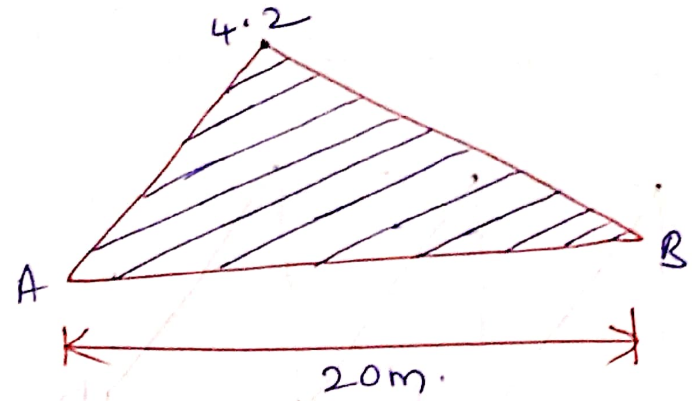


$a = 6$
$b = 14$
$l = 20$

$$\frac{ab}{l} = \frac{6 \times 14}{20}$$

$$= 4.2$$





Maximum bending moment = Load × ordinate

$= 30 \times (\frac{1}{2} \times 20 \times 4.2)$

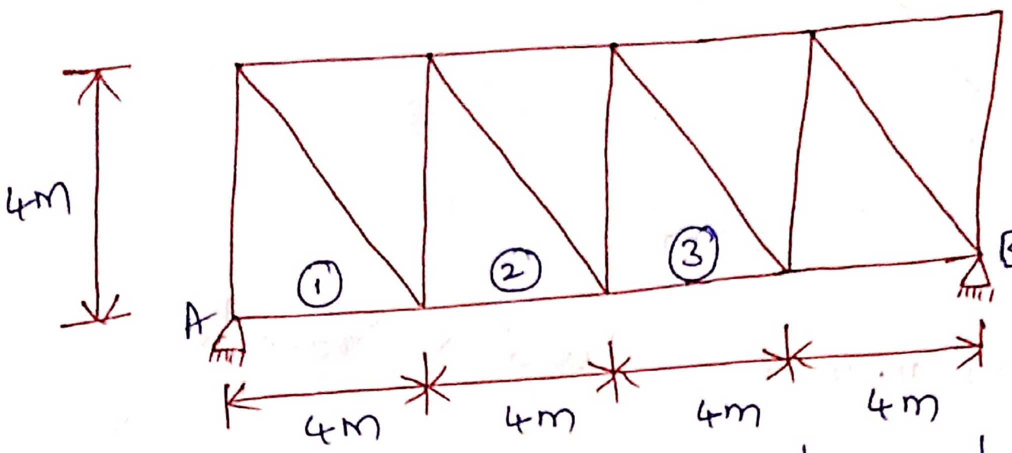
$= 1260$ KN.m.

Influence Lines for Trusses :-

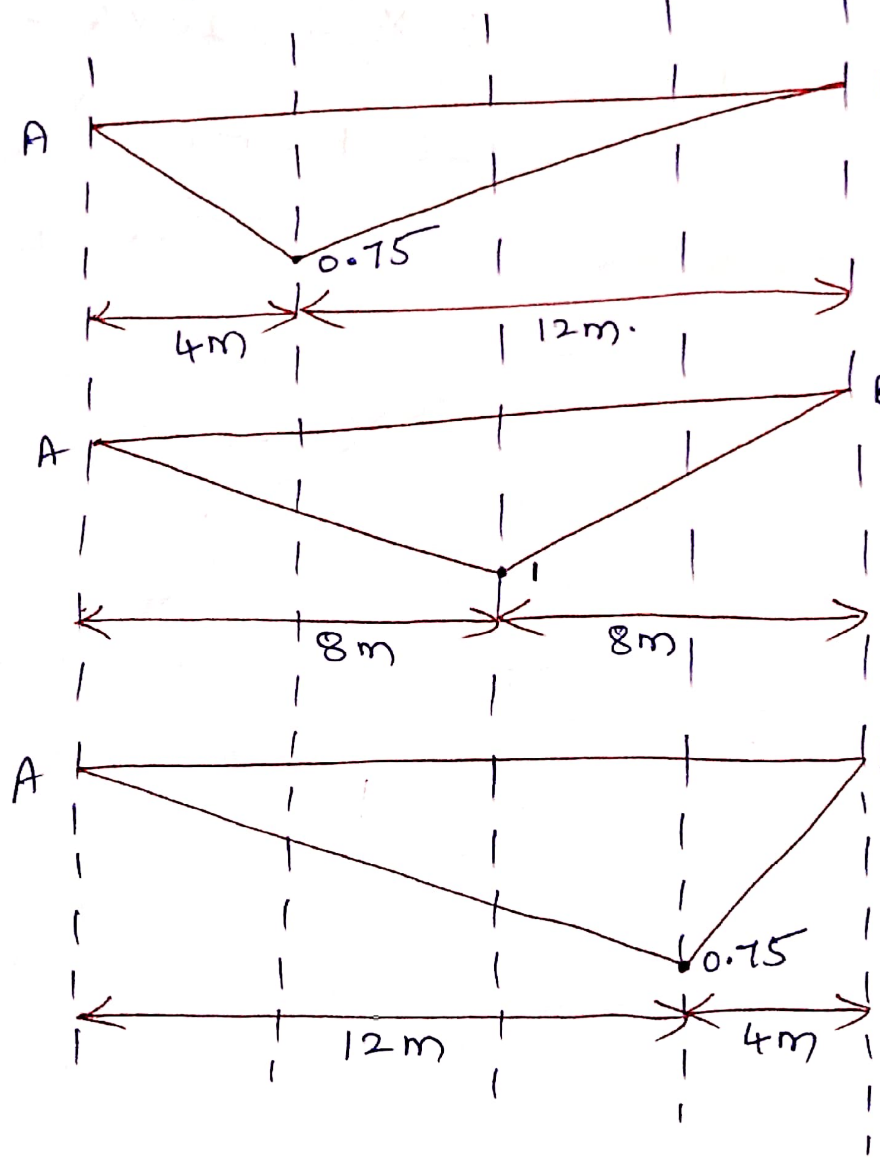
ILD for bottom chord :-

$$\text{ILD for bottom chord} = \frac{ab}{HL}$$

① Draw the ILD for members ①, ②, ③ in given truss.



(A)



B ILD ①

$$\frac{ab}{HL} = \frac{4 \times 12}{4 \times 16} = 0.75$$

B ILD ②

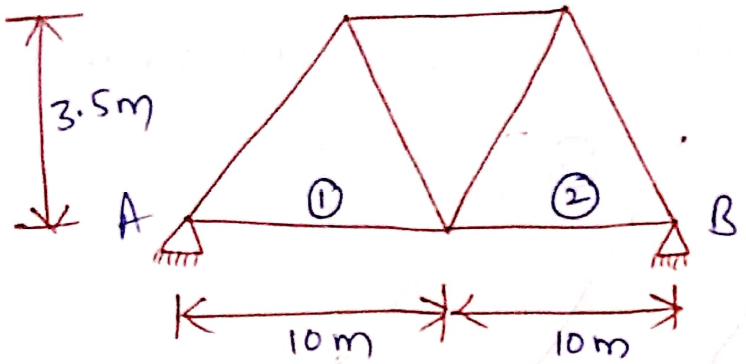
$$\frac{ab}{HL} = \frac{8 \times 8}{4 \times 16} = 1$$

B ILD ③

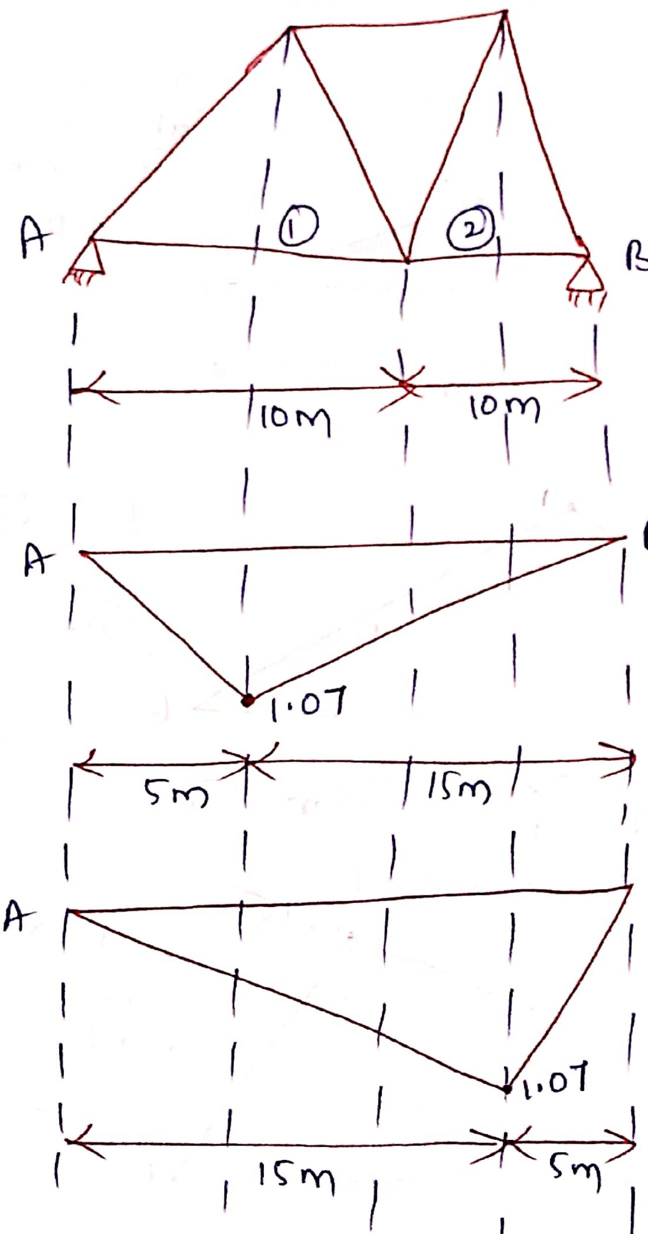
$$\frac{ab}{HL} = \frac{12 \times 4}{4 \times 16} = 0.75$$

(2) Draw the ILD for members (1), (2) in given truss.

(3)



(A)



ILD (1)

$$\frac{ab}{HL} = \frac{5 \times 15}{3.5 \times 20} = 1.07$$

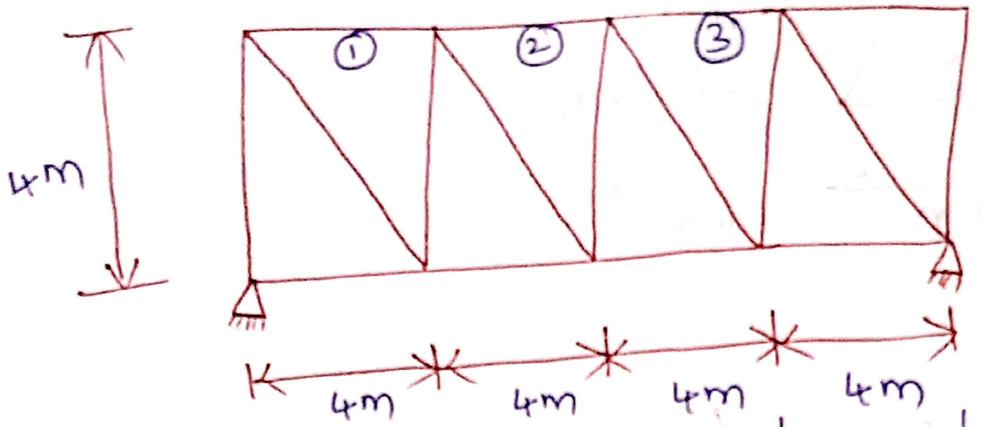
ILD (2)

$$\frac{ab}{HL} = \frac{15 \times 5}{3.5 \times 20} = 1.07$$

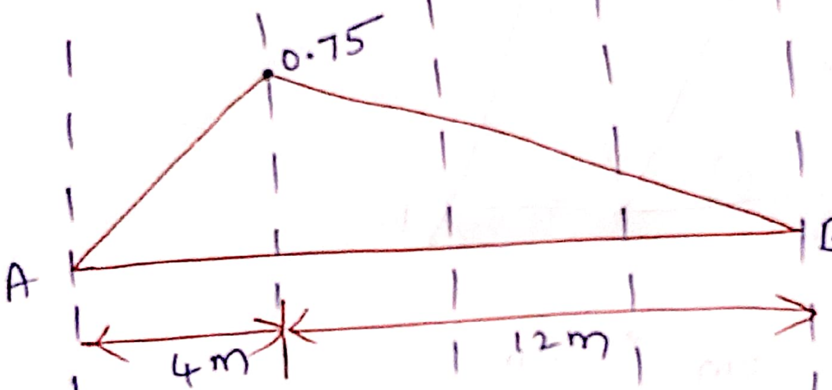
ILD for Top/upper chord :—

ILD for upper chord $= \frac{ab}{HL}$

(1) Draw the ILD for members (1), (2), (3) in given truss.

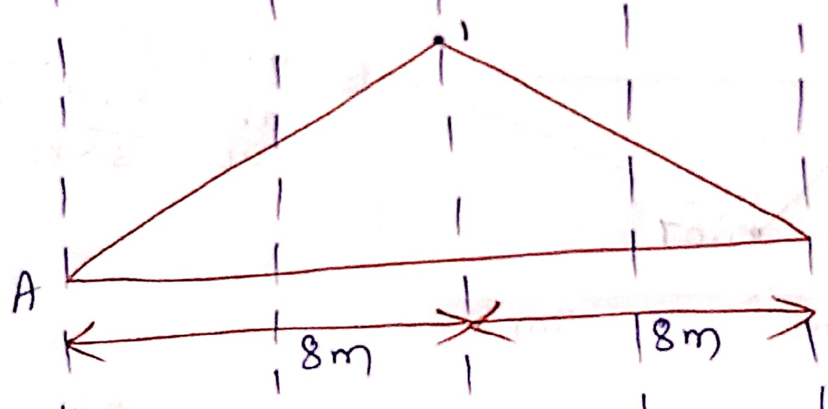


(A)



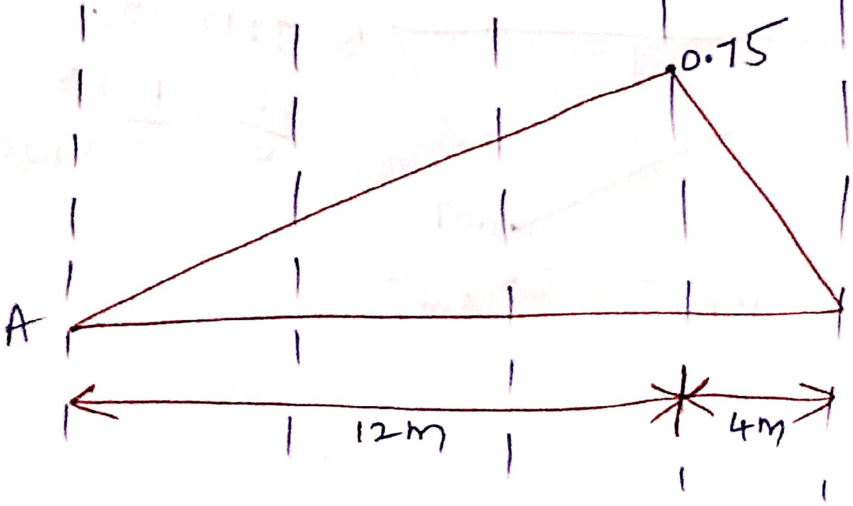
ILD (1)

$$\frac{ab}{HL} = \frac{4\times 12}{4\times 16} = 0.75$$



ILD (2)

$$\frac{ab}{HL} = \frac{8\times 8}{4\times 16} = 1$$

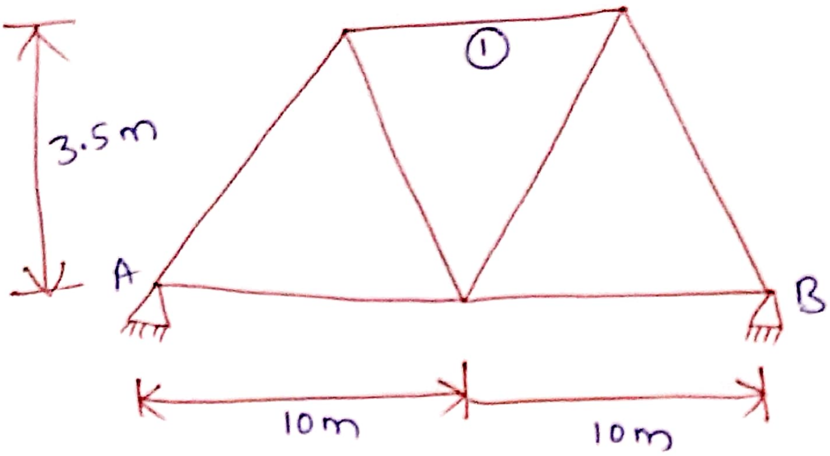


ILD (3)

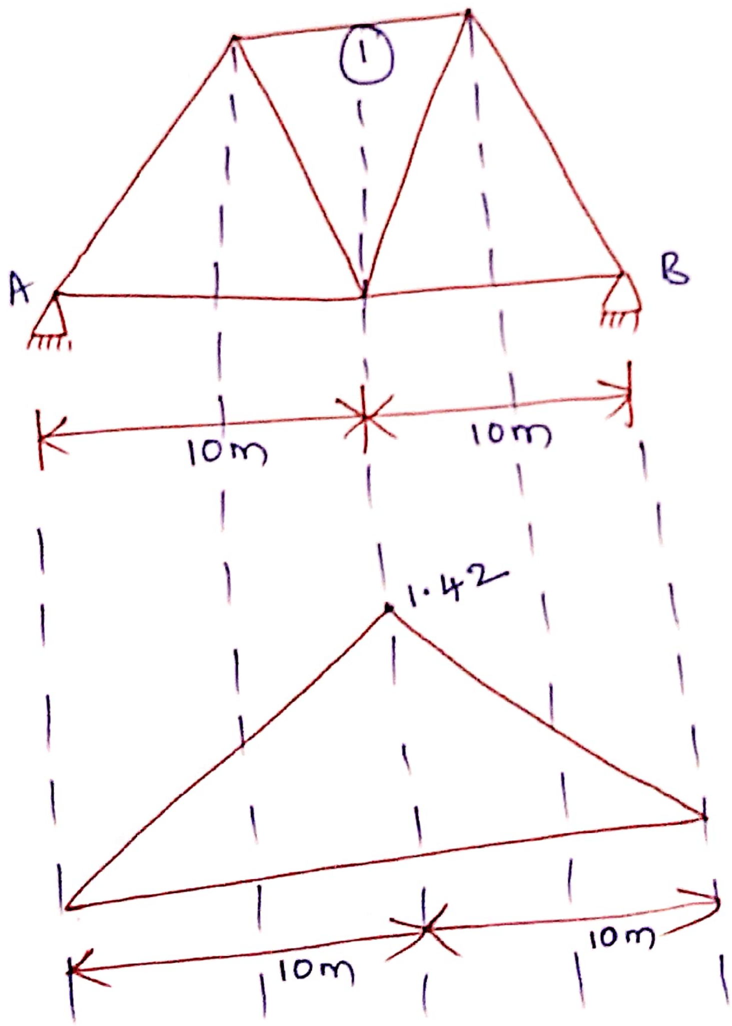
$$\frac{ab}{HL} = \frac{12\times 4}{4\times 16} = 0.75$$

(2) Draw the ILD for member (1) in given truss.

(4)



(A)



ILD (1)

$$\frac{ab}{HL} = \frac{10 \times 10}{3.5 \times 20}$$

$$= 1.42$$