Unit-4

Elasticity, Creep & Shrinkage – Modulus of elasticity – Dynamic modulus of elasticity – Poisson's ratio – Creep of concrete – Factors influencing creep – Relation between creep & time – Nature of creep – Effects of creep – Shrinkage – types of shrinkage.

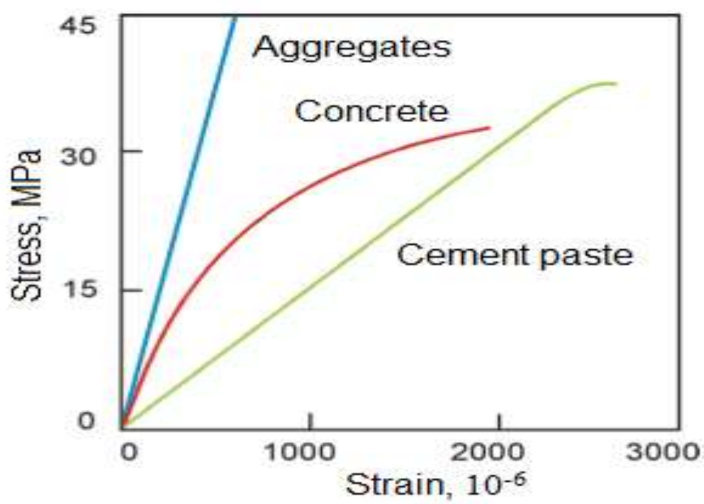
Concept-1

Modulus of Elasticity

Modulus of Elasticity (also called Elastic Modulus or Young's Modulus) is defined as the ratio of stress to strain within the elastic limit of a material. It measures the stiffness of a material how much it deforms under load.

Formula:

$E = \frac{\text{Stress}}{\text{Strain}} = \frac{\sigma}{\varepsilon}$



Aggregates (blue line) exhibit a steep, almost linear stress-strain relationship, indicating high stiffness and low deformability. This means they can resist high stress with very little strain.

Cement paste (green curve) shows a relatively flatter and more curved line, reflecting its lower stiffness and higher deformability compared to aggregates. Cement paste deforms significantly even under moderate stress, indicating that it is the weaker and more ductile component in concrete.

Concrete (red curve), which is a composite of both cement paste and aggregates, displays an intermediate behavior. Initially, it follows a linear trend (dominated by aggregate stiffness), then gradually curves as the cement paste begins to deform more significantly. The non-linear portion reflects microcracking and plastic deformation in the matrix.

This graph highlights that the overall behavior of concrete in compression is governed by the interaction between the stiff, brittle aggregates and the weaker, more deformable cement paste. The composite nature gives concrete both strength and some ductility, although the strain capacity remains limited compared to ductile materials like steel.

i. Modulus of Elasticity (Static) of concrete

The modulus of elasticity (E) of concrete is a measure of its stiffness or resistance to elastic deformation under applied load. When concrete is subjected to axial loading, it exhibits a linear stress-strain relationship up to a certain limit—this region is referred to as the elastic zone. The slope of this linear part is called the modulus of elasticity. It indicates how much a unit strain occurs for each unit stress and is essential in the design of structural members that must limit deflection.

The value of E depends on the concrete's compressive strength, aggregate properties, and age. In practice, it is determined using compressive testing equipment on cylindrical or cube specimens fitted with strain gauges. As per IS 456:2000, the modulus of elasticity for concrete can be approximated by the formula:

$$E = 5000\sqrt{f_{ck}}$$

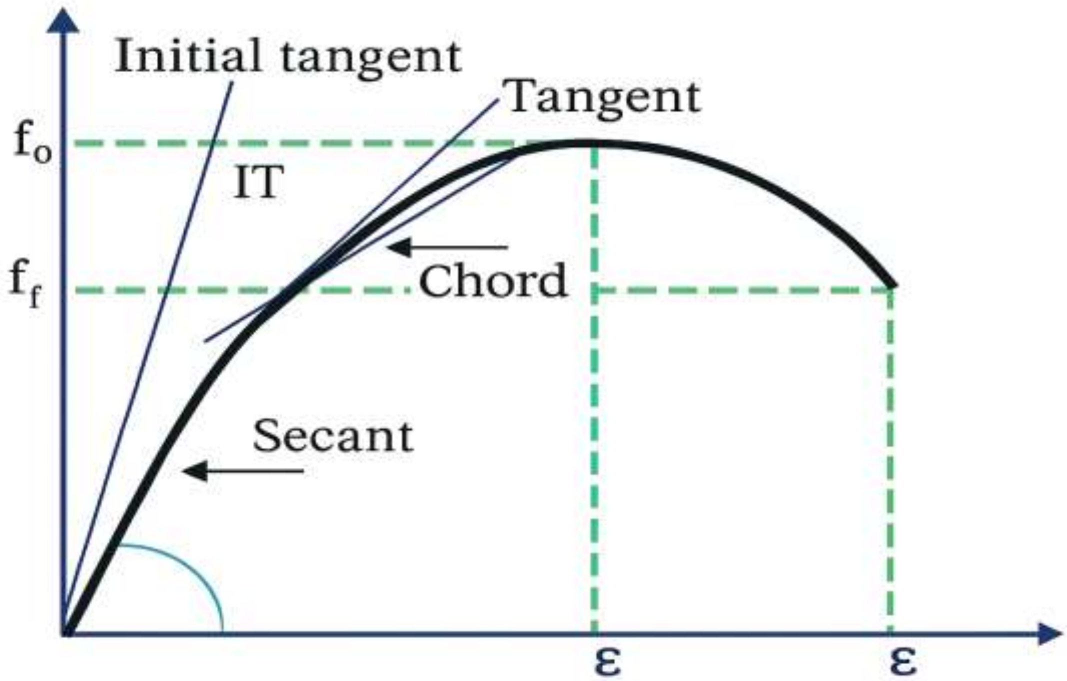
Where:

E is the static modulus of elasticity in MPa

f_{ck} is the characteristic compressive strength of concrete (MPa)

This value is crucial in the calculation of deflections in beams and columns under service loads.

Types of Modulus of Elasticity in Concrete



Concrete does not behave as a perfectly elastic material. Its stress-strain relationship is nonlinear, especially beyond the initial loading stage. Due to this, various types of modulus of elasticity are used to represent the stiffness of concrete under different conditions. The main types include Initial Tangent Modulus, Tangent Modulus, Secant Modulus, and Chord Modulus. Each has a unique definition and application in structural analysis and design.

Initial Tangent Modulus

The Initial Tangent Modulus is defined as the slope of the stress-strain curve drawn at the origin (the starting point of loading). It represents the initial stiffness of concrete when the load is first applied and before any microcracks or inelastic behavior begin.

$$E_{\text{initial}} = \left(\frac{d\sigma}{d\varepsilon}\right)_{\varepsilon=0}$$

This modulus is mainly used in theoretical studies, research, and finite element modeling to represent the very early elastic behavior of concrete. However, it is not practical for structural design because concrete quickly deviates from linear behavior as stress increases.

Tangent Modulus

The Tangent Modulus is the slope of the tangent drawn at any specific point on the stress-strain curve. It reflects how concrete behaves at that exact stress level and shows how quickly stress changes with respect to strain.

$E_{\text{tangent}} = \frac{d\sigma}{d\varepsilon}$(at a specific point)

This modulus becomes particularly important in nonlinear analysis, especially near the peak stress, where the curve begins to flatten due to internal microcracking. It is commonly used in advanced computational simulations such as finite element analysis (FEA) where detailed modeling of stress-strain behavior is required.

Secant Modulus

The Secant Modulus is a practical and widely used measure in engineering. It is defined as the slope of a line joining the origin (0,0) to a specific point on the stress-strain curve, often taken at 0.33 or 0.45 times the ultimate compressive strength of concrete.

$E_{\text{secant}} = \frac{\sigma}{\varepsilon}$

A widely accepted formula in Indian Standards (IS 456:2000) to estimate this modulus is:

$E_c = 5000\sqrt{f_{ck}}$

Where:

E_c is the modulus of elasticity in MPa,

f_{ck} is the characteristic compressive strength of concrete in MPa.

For example, for M25 concrete:

$E_c = 5000 \times \sqrt{25} = 25{,}000$ MPa

This modulus is commonly used in structural design, especially in calculating short-term deflections, crack widths, stress analysis, and load-deformation behavior under service loads. It gives a good approximation of average stiffness up to working stress levels.

Chord Modulus

The Chord Modulus is similar to the secant modulus but is calculated between two selected points on the stress-strain curve, not necessarily starting from zero. It provides the average stiffness of concrete over a particular stress range, often between the initial and final working stress.

$E_{\text{chord}} = \frac{\sigma_2 - \sigma_1}{\varepsilon_2 - \varepsilon_1}$

This modulus is useful when initial deformation (like microcracking or creep) is not considered important, such as in serviceability analysis or fatigue studies. It provides a more accurate estimation of stiffness under repeated loading or when evaluating partially cracked concrete behavior.

The Initial Tangent and Tangent Moduli are useful in research and nonlinear modeling; Secant and Chord Moduli are more practical for everyday structural engineering applications. Among these, the Secant Modulus is the most commonly used in design codes and calculations.

ii. Dynamic Modulus of Elasticity (Ed)

Dynamic modulus is determined using non-destructive vibration methods like resonant frequency or ultrasonic pulse velocity. It measures stiffness under rapid cyclic loading and is typically higher than static moduli (tangent/secant) because there is no time for microcrack development.

$E_d = k \times E_s$(where k = 1.1 to 1.25)

Use Case: Used in pavement engineering, seismic analysis, and non-destructive evaluation.

Relationship between Moduli

Modulus Type	**Behavior Described**	**Typical Value Relation**
Initial Tangent Modulus (E_i)	Slope at origin	Highest (Ideal, unused in design)
Tangent Modulus (E_t)	Slope at any point	Varies with stress level
Secant Modulus (E_s)	Avg. slope to specific stress	Used in IS 456, practical design
Dynamic Modulus (E_d)	Stiffness via wave/vibration	10–25% higher than E_s

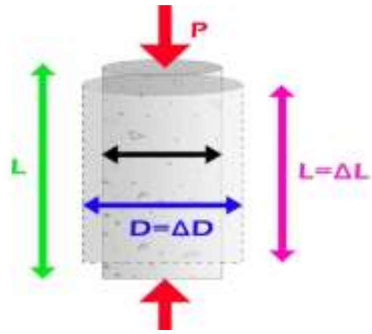
Concept -2

Poisson's Ratio

Poisson's ratio (ν) is defined as the ratio of lateral strain to axial strain when a material is subjected to uniaxial stress. When concrete is compressed vertically, it tends to expand laterally. Poisson's ratio quantifies this lateral expansion relative to vertical contraction.

$\nu = \frac{\text{Lateral Strain}}{\text{Axial Strain}}$

For concrete, Poisson's ratio generally lies between 0.15 and 0.25, depending on the type of aggregate, curing, and age. This property is especially important in structural analysis, particularly in finite element modeling and volumetric deformation calculations.



Significance and Impact of Poisson's Ratio in Concrete

Shear Deformation

Poisson's ratio plays a critical role in how concrete behaves under external loads. When concrete is subjected to axial deformation (either compression or tension), it tends to deform laterally due to Poisson's effect. This lateral expansion or contraction contributes to shear deformation, especially in structural members like beams and slabs, which are commonly subjected to bending. This affects the overall behavior and stability of the structure.

Control of Cracking

Concrete is known for its low tensile strength and susceptibility to cracking. Poisson's ratio influences the magnitude and direction of tensile stresses within concrete elements. These stresses directly affect where and how cracks form. By understanding Poisson's ratio, engineers can better predict crack patterns and adopt crack control measures during design and construction.

Load Transfer Mechanism

In composite structures (e.g., prestressed concrete beams), load transfer between different materials (like tendons and concrete) is significantly affected by Poisson's ratio. A proper understanding of this ratio helps in ensuring efficient load sharing, improving the overall performance of the structural system.

Material Characterization

Poisson's ratio is a key input in material testing and modeling, such as triaxial or consolidation tests. Accurate measurement of this ratio helps in defining the mechanical behavior of concrete, which is vital for design and structural analysis.

Impacts of Poisson’s Ratio on Concrete Design

Deformation Characteristics

Poisson’s ratio influences how concrete expands or contracts laterally when subjected to axial loads. Under compression, concrete expands laterally; under tension, it contracts. This differential movement can lead to cracking, especially on the tension side of beams or slabs. Therefore, engineers must account for Poisson’s effect to ensure proper serviceability and safety.

Stress Distribution

Poisson’s ratio affects internal stress distributions within concrete elements. For example, in reinforced concrete beams, stress concentrations near edges or corners due to Poisson’s effect can initiate cracks and reduce the element's load-carrying capacity. Accurate analysis ensures better prediction of stress patterns and helps maintain structural integrity.

Shear Behavior

Shear strength is a crucial property in the design of concrete elements. Poisson’s ratio influences how shear forces are transferred across joints and interfaces. This affects the shear capacity of elements such as slabs, beams, and foundations. Proper detailing of shear reinforcement should consider the effects of Poisson’s ratio.

Dynamic and Seismic Analysis

In dynamic or seismic design, Poisson’s ratio significantly affects how stress waves travel through concrete. It influences the vibration characteristics and response to earthquakes. Accurate determination of Poisson’s ratio helps in modeling seismic behavior and ensuring structures can withstand dynamic loads.

Selection of Materials and Mix Design

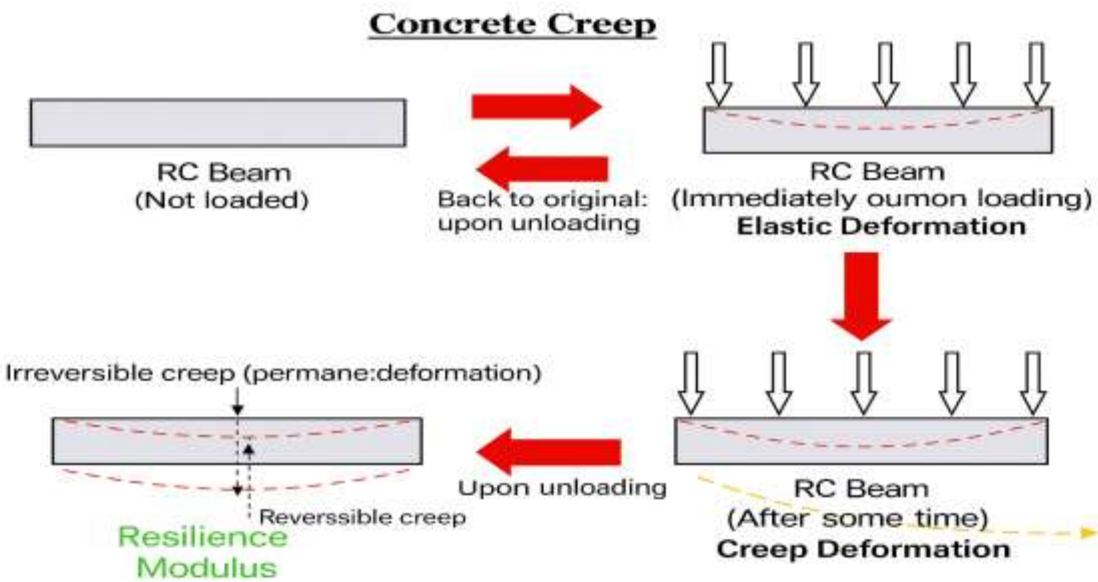
Since Poisson’s ratio depends on material properties, such as aggregate type, water-cement ratio, and curing conditions, it is essential to consider it when designing concrete mixes. This ensures the mix behaves as expected under load and contributes to optimal structural performance.

Poisson’s ratio is a vital material property that influences many aspects of concrete’s structural behavior, including deformation, stress distribution, crack development, shear transfer, and dynamic response. A clear understanding and proper consideration of this parameter are essential for designing safe, durable, and efficient concrete structures.

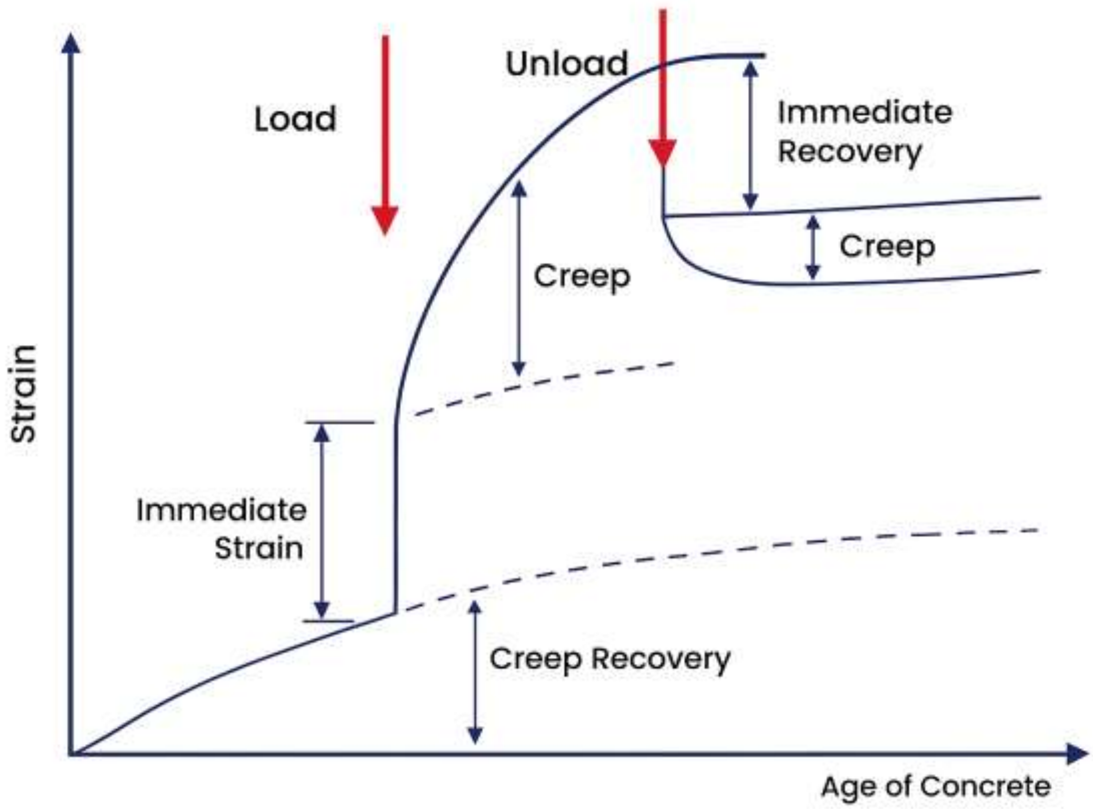
Concept -3

Creep of Concrete

Creep is the progressive, time-dependent deformation of hardened concrete under sustained load. Unlike elastic deformation, which is immediate and recoverable, creep increases with time and is mostly irreversible. It is more significant at early ages when the concrete is still hydrating and continues slowly over months or years.



Creep can lead to excessive deflection in long-span beams, loss of prestress, and redistribution of stresses in statically indeterminate structures. It is especially relevant in high-rise buildings, bridges, and prestressed concrete elements.



- The graph shows how concrete behaves over time when a load is applied and then removed.
- When the load is first applied, concrete immediately shortens—this is called immediate strain.
- As the load stays on, concrete slowly continues to deform over time. This time-dependent deformation is called creep.
- Alongside this, concrete also shrinks naturally due to drying, called shrinkage (even without load).
- When the load is removed, concrete partly springs back—this is called immediate recovery.
- Then, over time, it slowly regains a bit more shape, called creep recovery.
- However, it never fully returns to its original shape—some permanent deformation remains.
- This shows that concrete doesn't behave like a perfect elastic material—it has both time-dependent and irreversible behavior, important in long-term structural design.

Factors Influencing Creep

Aggregates:
The type and quality of aggregate significantly influence creep. Strong, dense aggregates reduce creep, while weak or porous ones increase it due to less internal resistance to deformation.

Mix Proportions:
A higher water-cement ratio and more cement paste result in more creep. Leaner mixes with more aggregate content tend to creep less due to better internal stability.

Age at Loading:
Younger concrete creeps more when loaded because the hydration process is still ongoing. Older, matured concrete shows less creep due to a stronger internal structure.

Curing Conditions:
Proper curing improves strength and reduces creep. Inadequate curing leads to weaker concrete with higher long-term deformation under load.

Cement Type:
Slow-hydrating or low-heat cements reduce creep. Some types like high-alumina cement may increase creep due to differences in hydration and microstructure.

Stress Level:
Higher sustained stress leads to more creep, especially when it exceeds 40–50% of the concrete's ultimate strength. Excessive stress can cause accelerated creep and even failure.

Volume-to-Surface Ratio:
Thicker sections lose less moisture and exhibit lower drying creep. Thin or slender sections dry faster and therefore experience higher creep.

Temperature:
Higher temperatures speed up hydration and creep. In hot climates, creep occurs more rapidly and needs to be considered in design.

Admixtures:
Chemical admixtures like plasticizers can increase creep by increasing paste content, while pozzolanic materials like fly ash may reduce it by densifying the matrix.

Types of Creep

Basic Creep:
This is the creep that occurs without moisture loss, purely due to sustained load in sealed conditions. It depends mainly on the concrete's internal structure and age.

Drying Creep:
Also known as Pickett Effect, it happens when concrete undergoes loading and moisture loss simultaneously. It results in greater deformation than basic creep.

Autogenous Creep:
Occurs in high-strength or low water-cement ratio concretes due to internal drying (self-desiccation), even in the absence of external drying or loading.

Effects of Creep

Increased Deflection:
Creep causes gradual deflection in beams and slabs over time, which can affect serviceability if not properly accounted for in design.

Loss of Prestress:
In prestressed concrete, creep leads to a reduction in the prestressing force, impacting the structural performance and design life.

Cracking Risk:
Uneven or differential creep within a structure can lead to internal stresses, causing cracking, especially in rigid or restrained elements.

Stress Redistribution:
Creep can be beneficial in some cases, as it helps redistribute stress concentrations, especially in continuous structures or under thermal loads.

Joint Movements:
Over time, creep can cause expansion or movement in joints and supports, which needs to be considered in structural detailing.

Creep is a time-dependent property that affects long-term performance of concrete structures. Understanding its causes and effects is essential for safe, durable, and efficient structural design.

Relation Between Creep and Time

Creep develops rapidly in the first few weeks and then slows down. The strain due to creep does not remain constant but increases logarithmically with time. A useful parameter is the creep coefficient, defined as:

$$\phi = \frac{\text{Creep strain}}{\text{Initial elastic strain}}$$

The total strain under load becomes:

$$\text{Total strain} = \text{Elastic strain} + \text{Creep strain}$$

Typically, creep strain may be 2 to 3 times the initial elastic strain depending on loading age and environment.

Benefits of Creep in Concrete

While creep is generally viewed as a long-term deformation that must be controlled, it also offers certain structural benefits when properly accounted for in design. These positive effects help enhance the performance, safety, and durability of concrete structures under sustained loads.

Stress Redistribution:
One of the major benefits of creep is its ability to redistribute internal stresses within a concrete structure. In statically indeterminate structures such as continuous beams and frames, creep allows highly stressed regions to gradually transfer load to less stressed areas. This stress equalization helps prevent sudden failures and enhances structural safety by reducing stress concentrations over time.

Improved Bonding in Prestressed Concrete:
In prestressed concrete, creep of the concrete surrounding the tendons results in gradual transfer of stress, which enhances the bonding between the tendons and the concrete. This slow redistribution contributes to better composite action and helps ensure that the prestressing force is applied more uniformly.

Reduces Shrinkage Stresses:
Since both creep and shrinkage are time-dependent behaviors, creep can help compensate for or balance shrinkage-induced stresses in concrete. This is especially helpful in reducing early-age cracking in thick sections or high-performance concrete mixes.

While excessive creep can be harmful if not considered in design, a moderate and predictable amount of creep is actually beneficial for the structural behavior of concrete. It aids in stress redistribution, crack control, and load accommodation. Engineers use these advantages to enhance the serviceability, safety, and longevity of structures.

Concept -4

Shrinkage in Concrete

Shrinkage is the reduction in volume of concrete over time, even without the application of external loads. This phenomenon is primarily due to the loss of moisture from the concrete or changes in the internal structure during hydration and hardening. If this shrinkage is restrained either internally (by aggregates) or externally (by supports or adjacent elements) it can cause cracks, which may affect the durability and aesthetics of the structure. Shrinkage is a critical concern, especially in slabs, walls, and pavements, and must be considered during design and construction.

Types of Shrinkage in Concrete

Plastic Shrinkage:
This occurs within the first few hours after placing concrete, when it is still in a plastic (unset) state. It results from the rapid evaporation of water from the surface, especially under hot, dry, or windy conditions. If the rate of evaporation exceeds the rate at which

water can rise from the bottom, surface tension pulls particles together, causing cracks to appear. These cracks are usually shallow but can weaken the concrete surface.

Drying Shrinkage:
This is the most common type of shrinkage and occurs after the concrete has hardened. As the concrete loses moisture to the surrounding environment over time, the paste volume contracts, leading to shrinkage. If the concrete is restrained during this process, cracks can form. Drying shrinkage is especially important in thin sections like slabs, and its effects continue for months or even years after construction.

Autogenous Shrinkage:
Autogenous shrinkage happens due to self-desiccation, where water inside the concrete is consumed during hydration without any external moisture loss. This is particularly significant in low water-cement ratio mixes and high-strength concrete. Although the overall magnitude is smaller than drying shrinkage, it can lead to internal cracking in dense concrete with limited permeability.

Thermal Shrinkage:
When concrete undergoes a temperature drop, particularly after the heat generated during hydration, it tends to contract. If this thermal contraction is restrained (e.g., by formwork or adjacent structural parts), it can cause cracks. Thermal shrinkage is more prominent in mass concrete structures like dams and large foundations.

Carbonation Shrinkage:
This long-term shrinkage is caused by the reaction between carbon dioxide in the atmosphere and calcium hydroxide in the concrete. This chemical reaction leads to minor volume reductions. Although slow, carbonation shrinkage can contribute to the overall cracking of exposed concrete surfaces.

Factors Influencing Shrinkage in Concrete

Water-Cement Ratio:
A higher water-cement ratio results in more capillary pores and higher moisture content. As this water evaporates during drying, greater shrinkage occurs. Therefore, leaner mixes with lower w/c ratios generally show less shrinkage.

Cement Content:
Higher cement content increases the amount of paste, which is the part of the concrete most prone to shrinkage. This leads to greater drying and autogenous shrinkage, especially in rich mixes.

Aggregate Characteristics:
Aggregates restrain the shrinkage of the surrounding paste. Larger, well-graded, and stiffer aggregates reduce shrinkage by occupying more volume and providing internal resistance. Conversely, lightweight or porous aggregates allow more paste movement, increasing shrinkage.

Curing Conditions:
Proper and timely curing helps maintain moisture in the concrete and reduces shrinkage, especially plastic and drying shrinkage. Inadequate curing accelerates water loss, making the concrete more prone to early-age cracks.

Environmental Conditions:
High temperatures, low humidity, and strong winds increase the rate of water evaporation, especially during the early stages of concrete hardening. These conditions intensify both plastic and drying shrinkage.

Size and Shape of Member:
Thin sections or members with a high surface area-to-volume ratio lose moisture faster, leading to greater shrinkage. Slabs, pavements, and thin walls are particularly vulnerable to shrinkage-related cracking.

Shrinkage and creep are two key long-term behaviors of concrete. While shrinkage mainly results in volume reduction due to moisture or chemical changes, creep provides beneficial stress relief under sustained loads. Understanding the types and causes of shrinkage, along with the positive role of creep, is essential for designing concrete structures that are durable, crack-resistant, and structurally efficient.