Fluid mechanics

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SYLLABUS:

UNIT – I

Properties of Fluid

Distinction between a fluid and a solid; Properties of fluids – Viscosity, Newton law of viscosity; vapour pressure, boiling point, cavitation; surface tension, capillarity, Bulk modulus of elasticity, compressibility.

Fluid Statics

Fluid Pressure: Pressure at a point, Pascals law, Hydrostatic law, Piezometer, U-Tube Manometer, Single Column Manometer, U-Tube Differential Manometer, Micromanometers. Pressure gauges, Hydrostatic pressure and force: horizontal, vertical and inclined surfaces.

UNIT - II

Fluid Kinematics

Classification of fluid flow: steady and unsteady flow; uniform and non-uniform flow; laminar and turbulent flow; rotational and irrotational flow; compressible and incompressible flow; ideal and real fluid flow; One, two- and three-dimensional flows; Streamline, path line, streak line and stream tube; stream function, velocity potential function , flow net , One, two- and three-dimensional continuity equations in Cartesian coordinates applications.

Fluid Dynamics

Surface and Body forces -Euler's and Bernoulli's equation; Momentum equation. correction factors. Bernoulli's equation to real fluid flows.

UNIT - III

Flow Measurement in Pipes

Practical applications of Bernoulli's equation: venturi meter, orifice meter and pitot tube, applications of Momentum equations; Forces exerted by fluid flow on pipe bend, sudden enlargement in pipes.

Flow Over Notches & Weirs

Flow through rectangular; triangular and trapezoidal notches and weirs; End contractions; Velocity of approach. Broad crested weir.

UNIT – IV

Flow through Pipes

Reynolds experiment, Reynolds number, Loss of head through pipes, Darcy-Wiesbatch equation, minor losses, total energy line, hydraulic grade line, Pipes in series, equivalent pipes, pipes in parallel, siphon, branching of pipes, three reservoir problem, power transmission through pipes. Analysis of pipe networks: Hardy Cross method and EPA NET, water hammer in pipes and control measures.

UNIT - V

Laminar & Turbulent Flow

Laminar flow through circular pipes, and fixed parallel plates.

Boundary Layer Concepts

Prandtl contribution, Assumption and concept of boundary layer theory. Boundary-layer thickness, displacement, momentum & energy thickness concepts of laminar and turbulent boundary layers on a flat plate; Laminar sub-layer, smooth and rough boundaries. Local and average friction coefficients. Separation and Control. Drag and Lift and types of drag, magnus effect.

Book:

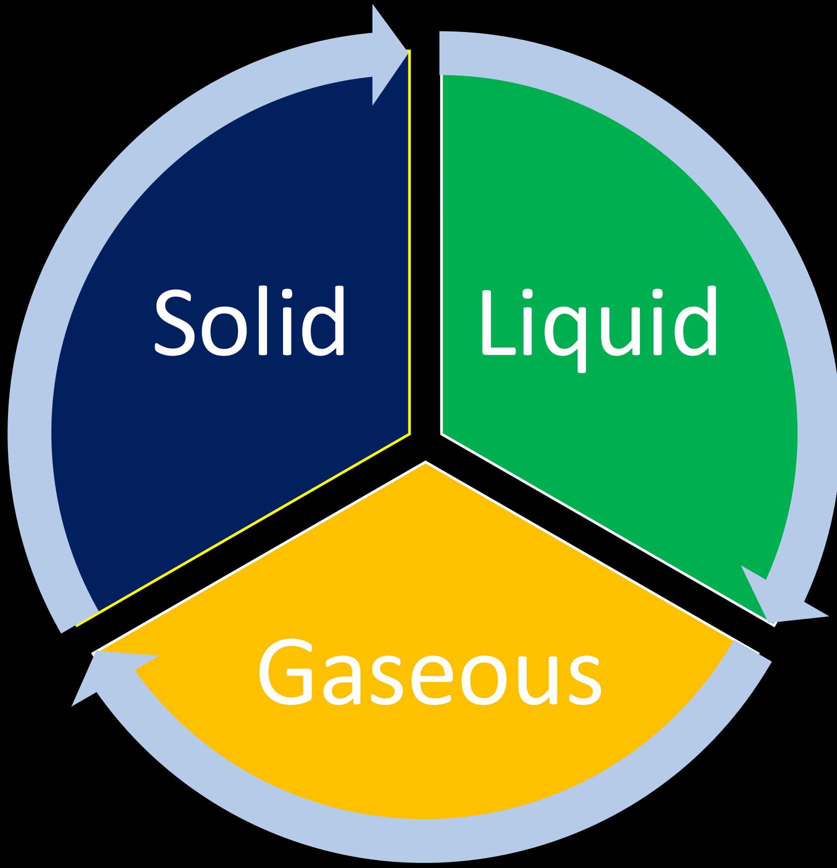
Fluid Mechanics by Modi & seth.

UNIT 1

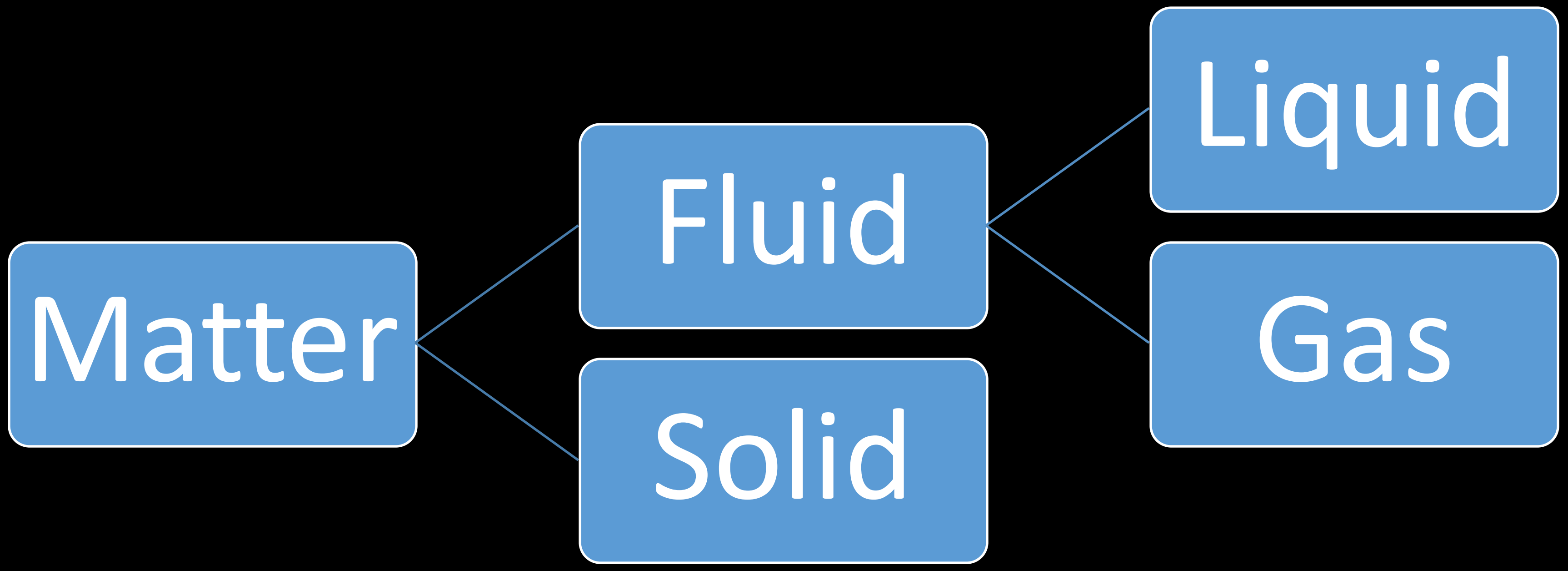
Chapter 1
Properties of fluids

States of matter:

Matter can exist in three distinct states



Water → Ice
Water → Water
Water → Vapour


Matter
Fluid
Solid
Liquid
Gas

- Matter consists of vast number of molecules because of their molecular structure.
- These molecules are separated by the empty spaces.

Solids:

- ❑ Molecules are closely spaced
- ❑ Solids possess compact or rigid form

Liquids:

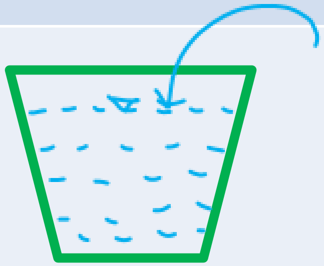
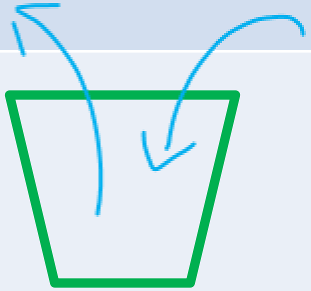
- ❑ Spacing between molecules is relatively large
- ❑ Liquids has definite volume

Gases:

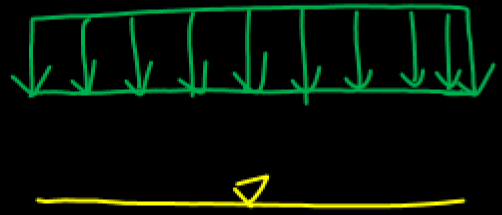
- ❑ Spacing between molecules is still larger than liquids
- ❑ Gases fill the entire volume of vessel in which it is contained.

Fluid:

A fluid is a substance, which is capable of flowing, even a small amount of shear force(tangential force)can cause continuous deformation.

Property	Liquid	Gas
Cohesive forces	predominant	negligible
Free surface		

uniform pressure



"Free Surface"

Shear stress, $\tau = 0$

Q. Statement I: Liquids have defined surfaces, whereas gases do not have.

Statement II: liquids have predominant cohesion compared to gases.

A

A. Both statements are correct and Correct explanation

B. " " " " and Incorrect explanation

C. 1 correct 2 wronge

D. 1 wronge 2 correct

Cohesion: Attraction between the molecules of same substance.

Adhesion: Attraction between the molecules of different substances.

Q. Statement I: If mercury is placed in hand it will not wet the hand. ✓

Statement II: Cohesion of mercury is more than the adhesion ✓ (A)

between mercury and hand.

Cohesion of mercury is more than adhesion with hand.

Q. Statements I: If cohesion is more, fluids will not wet ✓

Statements II: Mercury has more cohesion compare to adhesion with a ✓ hand.

A

Vapour:

It is a gas very nearer to the liquid phase.

Ex: steam

Mass density (or) specific mass: (ρ)

It is the ratio of mass per unit volume.

$$\rho = \frac{Mass}{Volume} = \frac{m}{V} = \frac{Kg}{m^3}$$

Dimensional formula:

$$\rho = \frac{M}{L^3}$$

$$\rho = M L^{-3} T^{0}$$

- Unit of mass in M.K.S system is **Slug**
- Unit of mass in S.I system is **Kg**

Note : mass is the quantity of matter contained in a body, therefore mass is constant everywhere it does not depend upon location and gravitational force

Units : M.K.S $\Rightarrow \dfrac{m.slug}{m^3}$

S.I $\Rightarrow \dfrac{kg}{m^3}$

Density of Water, $\rho_w = 1000\,kg/m^3$

Density of mercury, $\rho_m = 13600\,kg/m^3$

Density of kerosine, $\rho_k = 810\,kg/m^3$

Factors:

1. Temperature: $\rho = \frac{m}{V}$ $T\uparrow \Rightarrow \rho\downarrow$

If $T \to 0°C$ to $4°C$, $\rho\uparrow$

If $T > 4°C$, $\rho\downarrow$

2. Pressure: $\rho = \frac{m}{V}$ $P\uparrow \Rightarrow \rho\uparrow$

3. Location: [at constant T & P]

ρ does not change with location.

$\left\{\rho_{air} = 1.2\,kg/m^3\right\}$

Weight density (or) Specific weight [γ (or) w]

$$\gamma = \frac{\text{Weight}}{\text{Volume}}$$

Weight: It is a force with which a body attracted towards the center of a clelestial body (either earth or Moon)

$$F = ma$$

$$W = mg$$

$$\gamma = \frac{W}{V} = \frac{mg}{V} = \left[\frac{m}{V}\right] \cdot g = \rho g$$

$$\boxed{\gamma = \rho g}$$

Units: weight in M.K.S : kg.f

S.I : Newton (N)

Dimensional formula: $\gamma = \frac{W}{V} = \frac{mg}{V} = \frac{ML}{T^2 L^3} = ML^{-2}T^{-2}$

g = acc. due to gravity.

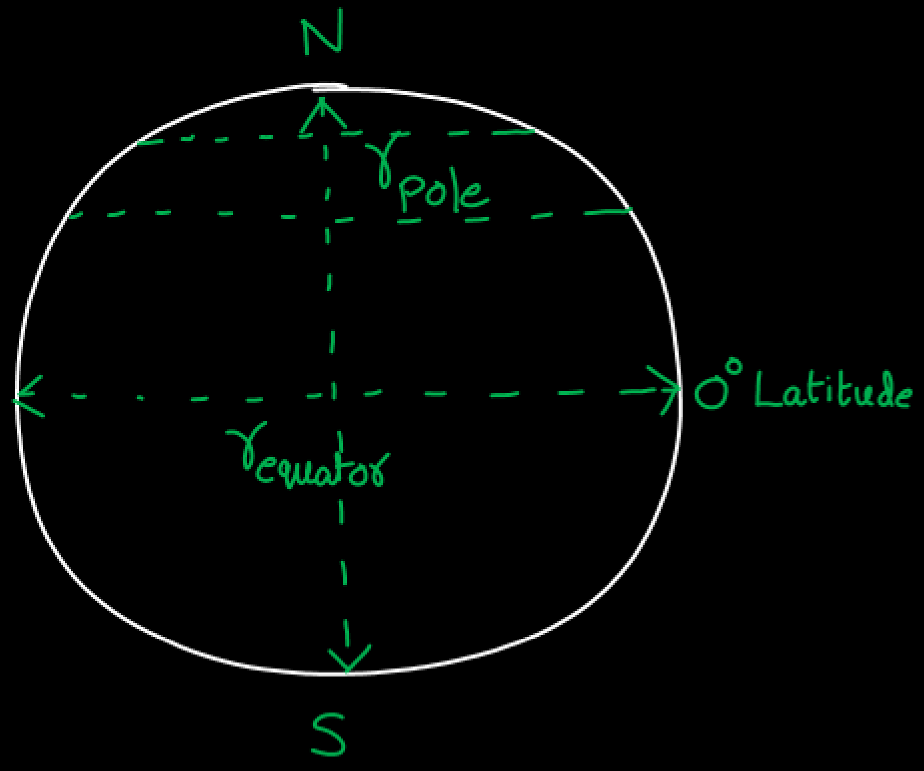
$$r_{equator} > r_{pole}$$

$$F = G\frac{m_1 m_2}{r^2}$$

$$F \propto \frac{1}{r^2} \Rightarrow r\uparrow, F\downarrow$$

$$g_{pole} > g_{equitorial}$$

$$\left.\begin{aligned} g_{pole} &= 9.83\ m/s^2 \\ g_{equivator} &= 9.73\ m/s^2 \end{aligned}\right\} \quad g = 9.81\ m/s^2$$



"Oblate spheriod"

Specific Volume: (v)

It is the reciprocal of density

$$v = \frac{1}{\rho} = \frac{\text{Volume}}{\text{mass}} \left\{ m^3/Kg \right\}$$

Specific Volume is Important in the case of gases.

$$v = \frac{1}{\rho}$$

$$\rho_{gas} \downarrow \Rightarrow v \uparrow$$

Ideal gas equation, $\boxed{PV = nRT}$

$$PV = nRT$$

$$P = \frac{nRT}{V}$$

$$\boxed{P = \rho RT}$$

$P \rightarrow$ Pressure (Atm)

$\rho \rightarrow$ density

$R \rightarrow$ universal gas constant $\left\{ \frac{Atm \cdot m^3}{mol \cdot {}^\circ K} \right\}$

$T \rightarrow$ Absolute temperature $({}^\circ K)$

Specific gravity (or) relative density: (S)

$$S = \frac{\rho_{body}}{\rho_{stnd.\,fluid}} = \frac{\gamma_b}{\gamma_{stnd.\,fluid}}$$

Standard fluid is water at 4°c

$$S_{water} = \frac{\rho_{water}}{\rho_{water}} = 1$$

$$S_{mercury} = \frac{\rho_m}{\rho_w} = \frac{13600\ kg/m^3}{1000\ kg/m^3} = 13.6$$

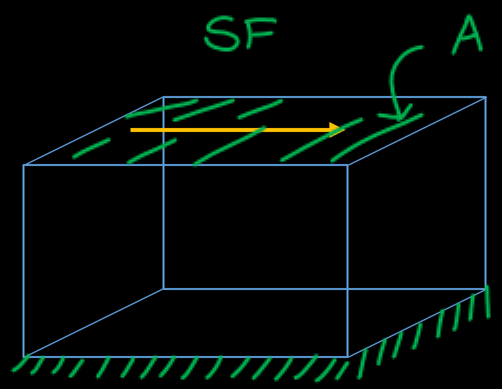
$$S_{oil} = \frac{\rho_{oil}}{\rho_w} = \frac{810\,Kg/m^3}{1000\,Kg/m^3} \simeq 0.8$$

Viscosity (or) dynamic viscosity (or) coefficient of dynamic viscosity (μ)

- ✓ If a layer of fluid tries to move over another layer, the resistance offered is called viscosity
- ✓ Viscosity is the internal resistance hence called molecular viscosity
- ✓ A fluid exhibits viscosity in motion hence called dynamic viscosity

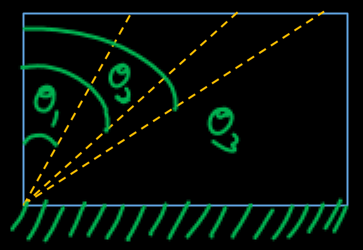
Viscosity Concept is developed by "Newton".

Prove newton's law of viscosity:



Shear stress, $\tau = \frac{SF}{A}$

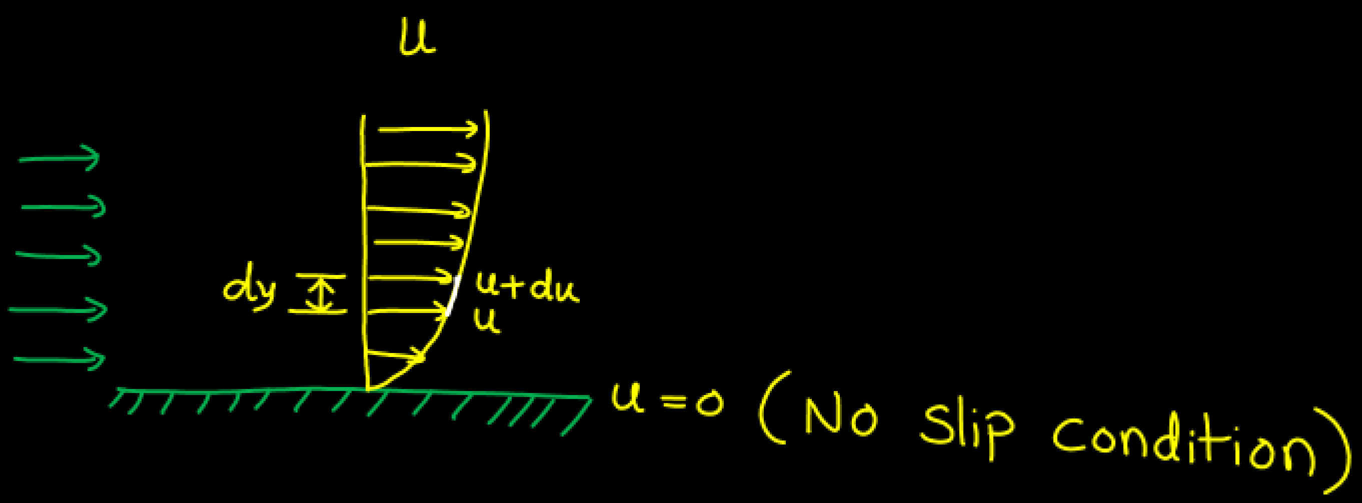
θ = shear strain (or) Angular deformation

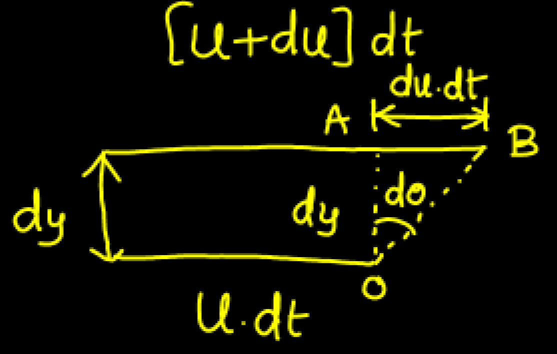


Newton observed τ is directly proportional to rate of shear strain $\left[\frac{d\theta}{dt}\right]$

$$\tau \propto \frac{d\theta}{dt}$$

he further observed $\frac{d\theta}{dt} = \frac{du}{dy}$





From Δle OAB, $\tan d\theta = \frac{AB}{OA} = \frac{du.dt}{dy}$

$\tan d\theta \simeq d\theta$

$$d\theta = \frac{du.dt}{dy} \Rightarrow \frac{du}{dy} = \frac{d\theta}{dt}$$

$$\tau \propto \frac{du}{dy}$$

$$\boxed{\tau = \mu \frac{du}{dy}}$$

Unit : $\dfrac{du}{dy} = \dfrac{m}{s.m} = \dfrac{1}{s} = s^{-1}$ [Velocity gradient]

μ = Coefficient of dynamic Viscosity

(or)

Viscosity

→ The fluid which follows Newton's law of viscosity is known as "Newtonian fluid"

Ex: Air, Water, Kerosine, Diesel, Petrol, -- etc

→ Which does not follow Newton's Law of Viscosity is known as "Non-Newtonian fluid"

Ex: Blood

Units: $\mu = \dfrac{\tau}{\left[\dfrac{du}{dy}\right]} = \dfrac{F}{A\ \bar{s}^{1}} = \left\{\dfrac{N.S}{m^2} = Pa.Sec\right\}$

$$1\,N = 10^5\ \text{dynes}$$

$$\frac{N\text{-}S}{m^2} = \frac{10^5\ \text{dyne-Sec}}{100^2\ cm^2} = 10\ \frac{\text{dyne-s}}{cm^2} \qquad \left\{\frac{\text{dyne-s}}{cm^2} = \text{Poise}\right\}$$

$$= 10\ \text{Poise}$$

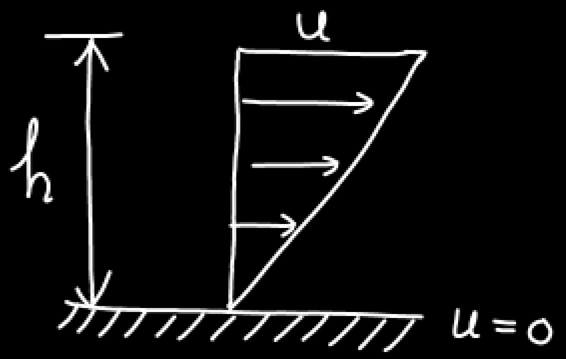
$$\boxed{\frac{1\,N\text{-}S}{m^2} = 10\,\text{Poise}}$$

$$\tau = \mu \frac{du}{dy} \Rightarrow \mu = \frac{\tau}{\left[\frac{du}{dy}\right]} = \frac{F}{A\, s^{-1}} = \frac{mass \cdot a}{m^2 \cdot s^{-1}}$$

$$= \frac{kg \cdot m \cdot s}{s^2 \cdot m^2}$$

$$\boxed{\mu = \frac{kg}{m\text{-}sec}}$$

Linearization of newtons law of viscosity:



$$\tau = \mu \frac{du}{dy}$$

$$= \mu \frac{|u-0|}{h}$$

$$\boxed{\tau = \mu \frac{u}{h}}$$

The gap is very small.

Factors affecting viscosity:

Viscosity is due to:

1) Cohesion
2) Molecular momentum exchange(MME)

MME: it is because of collision of different molecules. due to collision momentum transfer occurs in the transverse direction resisting the flow.

→ MME is important in the case of gases.

→ Cohesion is important in the case of Liquid.

Effect of temperature on fluid viscosity:

Liquid : $T \uparrow \Rightarrow$ cohesion $\downarrow \Rightarrow \mu \downarrow \Rightarrow \uparrow T \propto \dfrac{1}{\mu \downarrow}$

Gas : $T \uparrow \Rightarrow$ collisions $\uparrow \Rightarrow$ MME $\uparrow \Rightarrow \mu \uparrow \Rightarrow \uparrow T \propto \mu \uparrow$

Kinematic viscosity: $[\nu]$

$$\nu = \frac{\mu}{\rho}$$

fluids of less density having considerable kinematic viscosity.

$$\left\{ \nu_{air} > \nu_{water} \right\} \qquad \left\{ \mu_w > \mu_{air} \right\}$$

$$\rho_{air} < \rho_w$$

Unit : $\nu = \frac{\mu}{\rho} = \frac{M}{LT}\frac{L^3}{M} = \frac{L^2}{T} = \frac{m^2}{ses}$

$$\frac{cm^2}{sec} = \text{Stoke}$$

$$1 \text{ centistoke} = \frac{1}{100} \text{ Stokes}$$

$$1\frac{m^2}{s} = 10^4 \frac{cm^2}{s} = 10^4 \text{ stokes}$$

Q. A fluid is one which can be defined as a substance that

(GATE - 96)

(a) Has same shear stress at all points

(b) Can deform indefinitely under the action of the smallest shear force

(c) Has the small shear stress in all directions

(d) Is practically incompressible

Q. With increase of temperature, viscosity of a fluid

(GATE - 97)

(a) Does not change

(b) Always increases

(c) Always decreases

(d) Increases, if the fluid is a gas and decreases, if it is a liquid

Q. The unit of dynamic viscosity of a fluid is

(GATE - 97)

(a) m^2/s

(b) $\frac{N-s}{m^2}$

(c) $\frac{Pa-s}{m^2}$

(d) $\frac{Kg-s^2}{m^2}$

Q. The dimension for kinematic viscosity is

(GATE –14–Set 1)

(a) $\frac{L}{MT}$

(b) $\frac{L}{T^2}$

(c) $\frac{L^2}{T}$

(d) $\frac{ML}{T}$

Q. The viscosity of a fluid is 0.5poise, specific gravity is 0.5,then the kinematic viscosity of a fluid is ____1________ stokes.(answer up to nearest integer)

$\mu = 0.5$ Poise

$1 \frac{N-S}{m^2} = 10$ Poise

$\mu = 0.5 \times 0.1 = 0.05 \frac{N-S}{m^2}$

$S = 0.5$

$S = \frac{\rho}{\rho_w}$

$\rho = S \times \rho_w = 0.5 \times 1000$

$\nu = \frac{\mu}{\rho}$

$= \frac{0.05}{0.5 \times 1000} = 10^{-4}\ m^2/s$

$= 10^{-4} \times 10^{4}$ stokes

$\nu = 1$ stoke

Q. A liquid of density ρ and dynamic viscosity μ flows steadily down an inclined plane in a thin sheet of constant thickness t. Neglecting air friction the shear stress on the bottom surface due to the liquid flow is (where θ is the angle, the plane makes with horizontal).

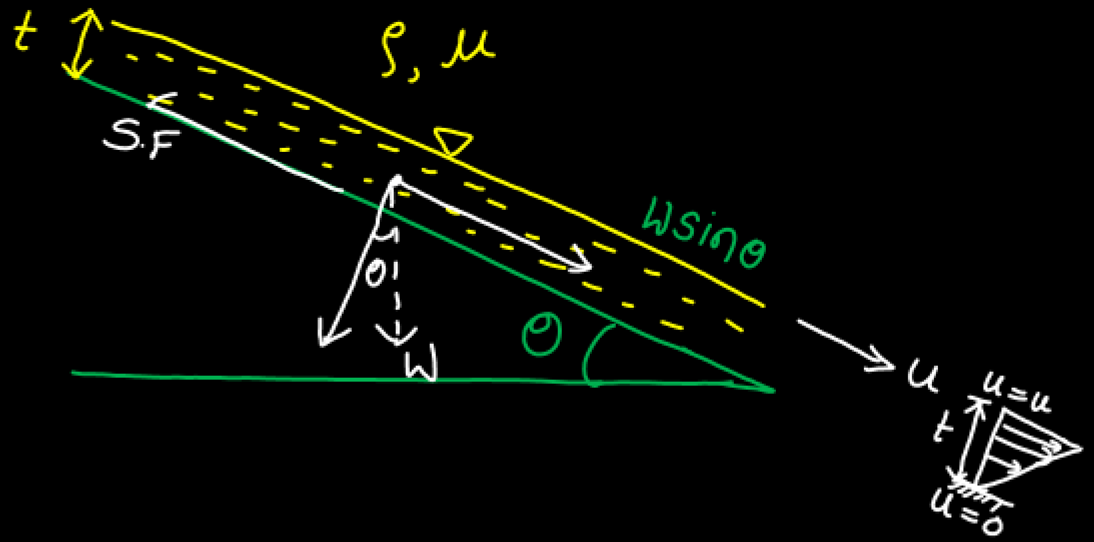
(GATE - 96)

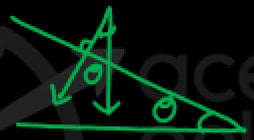
(a) $\rho\, g\, t \sin\theta$

(b) $\rho\, g\, t \cos\theta$

(c) $\mu \sqrt{\frac{g}{t}}$

(d) $\rho\, g$





At equilibrium, $SF = W\sin\theta$

$$\tau \cdot A = [\gamma \cdot \text{Volume}]\sin\theta$$

$$\tau \cdot A = \rho g [A \times t]\sin\theta$$

$$\boxed{\tau = \rho g t \sin\theta}$$

Q. What is the terminal velocity of fluid sliding down?

$$\tau = \mu \frac{du}{dy}$$

$$= \mu \frac{|u-0|}{t} = \frac{\mu u}{t}$$

$$\rho g t \sin\theta = \frac{\mu u}{t}$$

$$\left\{ u = \frac{\rho g t^2 \sin\theta}{\mu} \right\}$$

Power law: ① $A = \mu;\ n = 1;\ B = 0$

$$\tau = A\left[\frac{du}{dy}\right]^n + B$$

τ = Shear stress

A = Multiplying constant

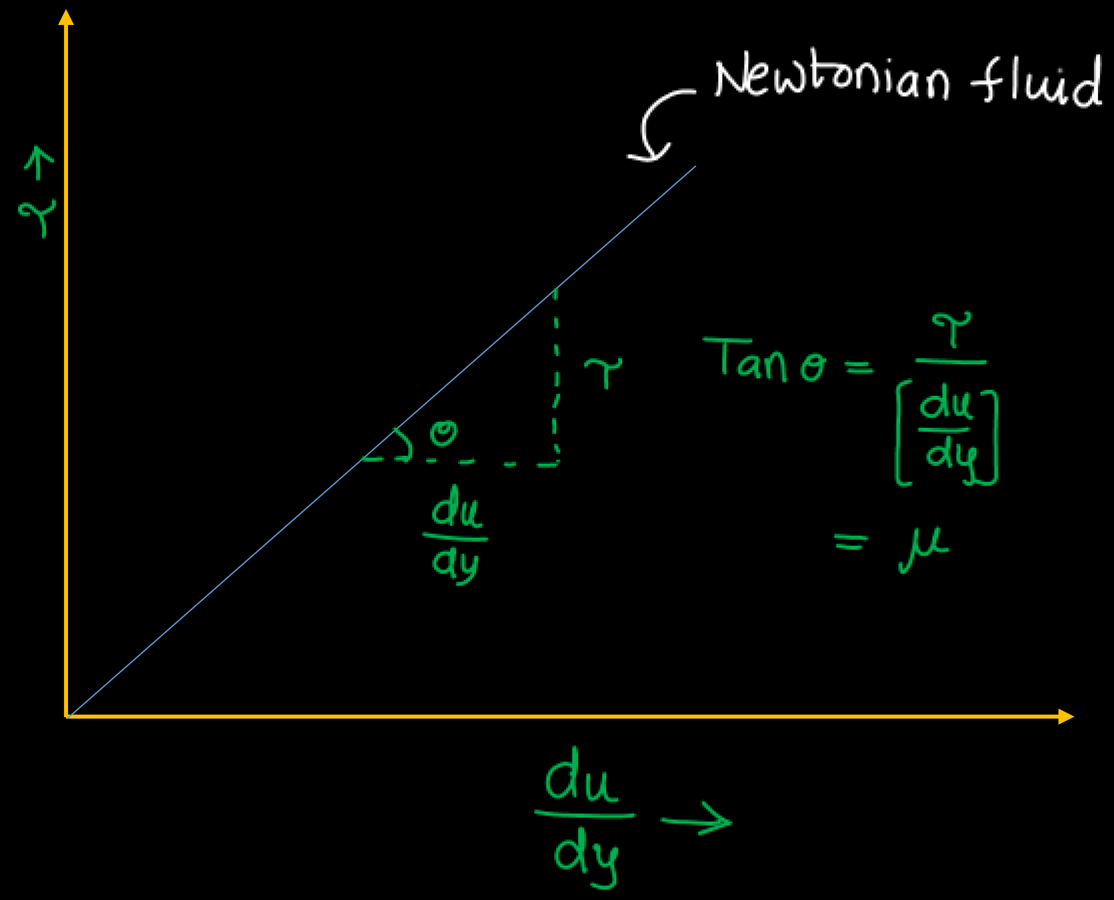
n = Power index

B = Addition constant

B is y intercept

(or)

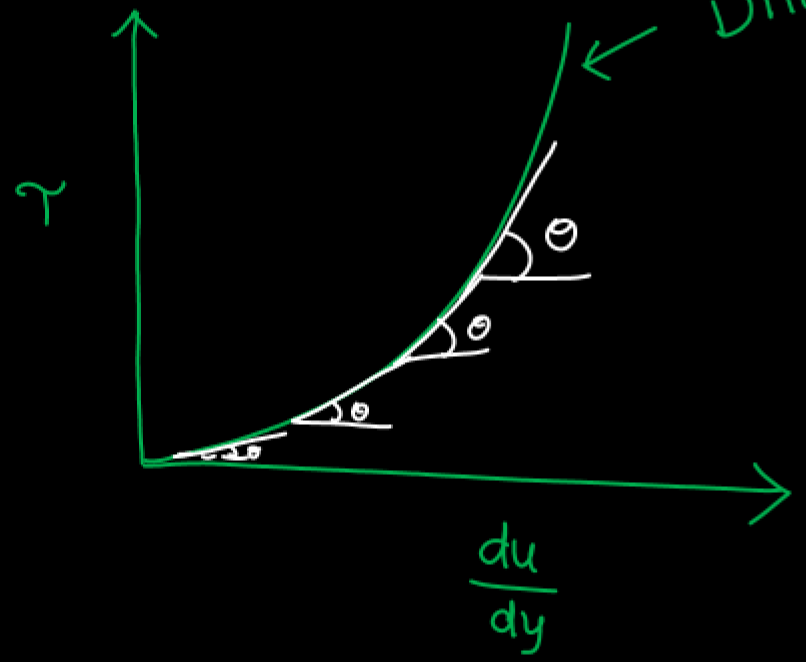
Minimum yield stress



$$\tan\theta = \frac{\tau}{\left[\frac{du}{dy}\right]} = \mu$$

$$\tau = \mu \frac{du}{dy}$$

② $A = \mu$; $n > 1$; $B = 0$



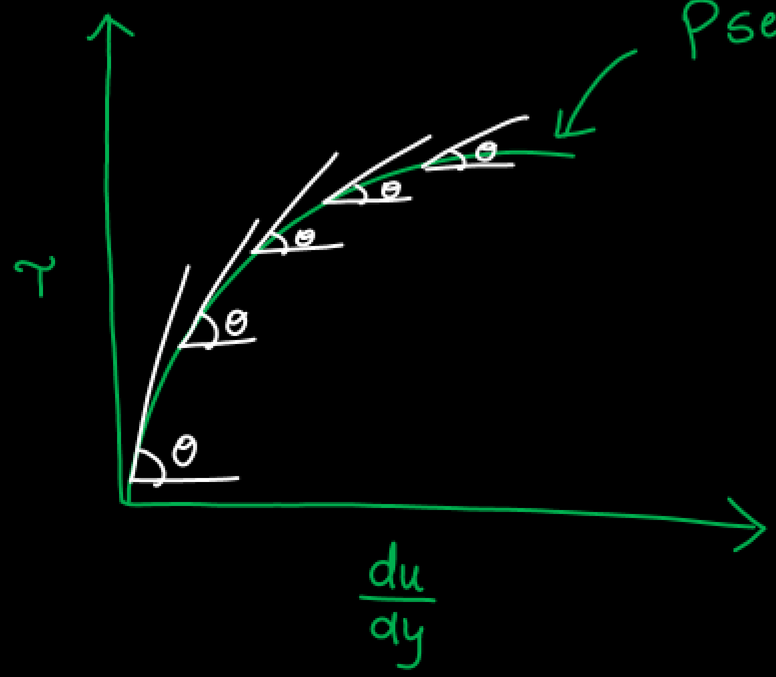
→ slope is increasing from point to point

→ μ is increasing

Ex: Sugar solution, Butter milk, Rice starch, Quick sand.

③ $A=\mu$; $n<1$; $B=0$

Pseudoplastic fluid / Shear thinning fluid

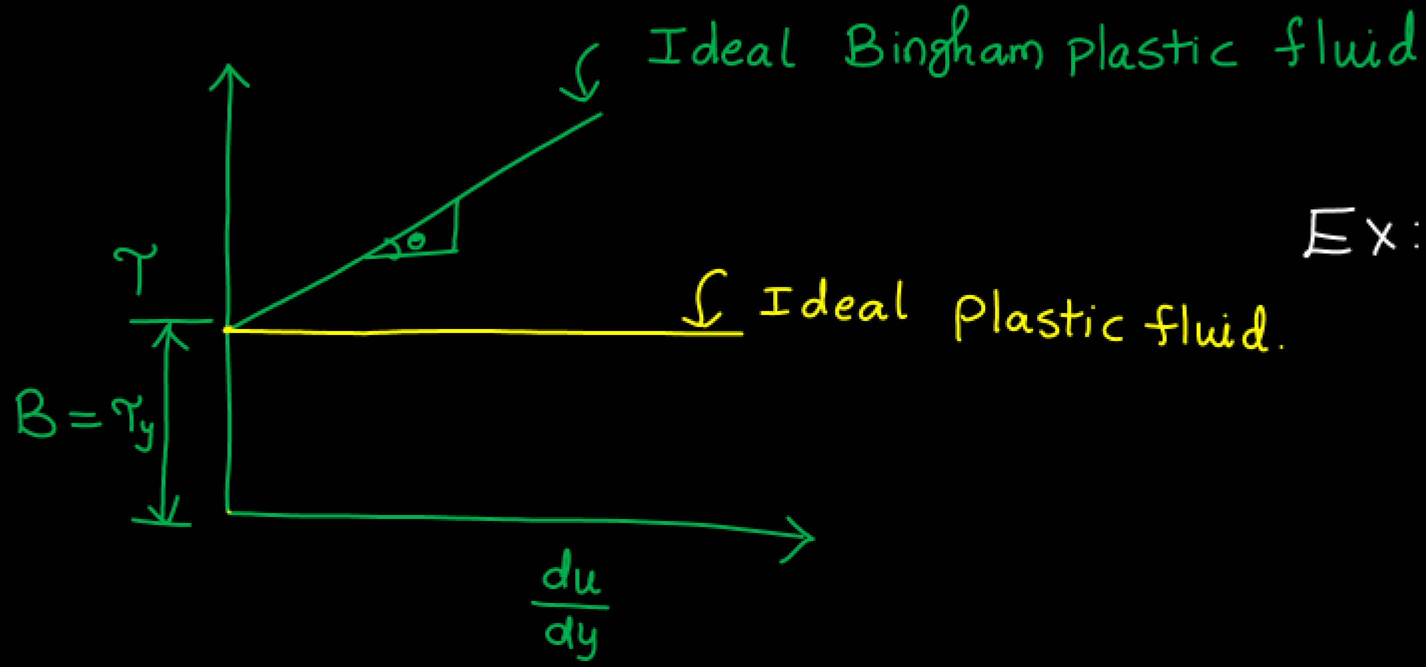


→ Slope is decreasing from point to point

→ μ is decreasing

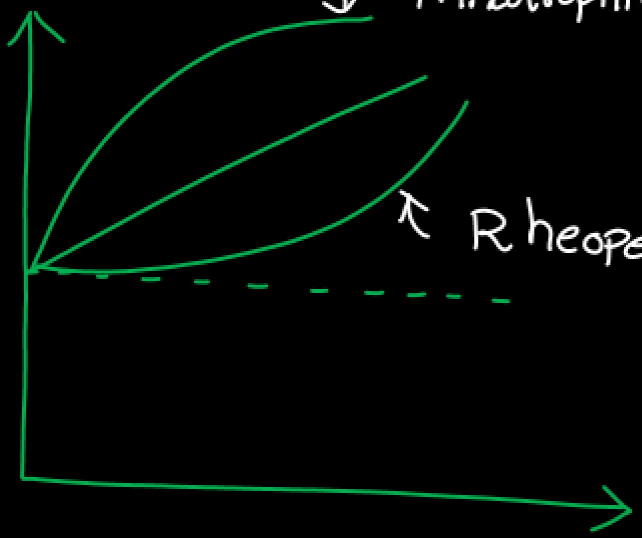
Ex: Blood, Paper pulp, Milk

④ $A = \mu$; $n = 1$; $B = \tau_y$



Ex: Tooth Paste, Sewage sludge, Drilling mud.

(5) Thixotrophic & Rheopectic:



Thixotrophic [μ decreases with time]

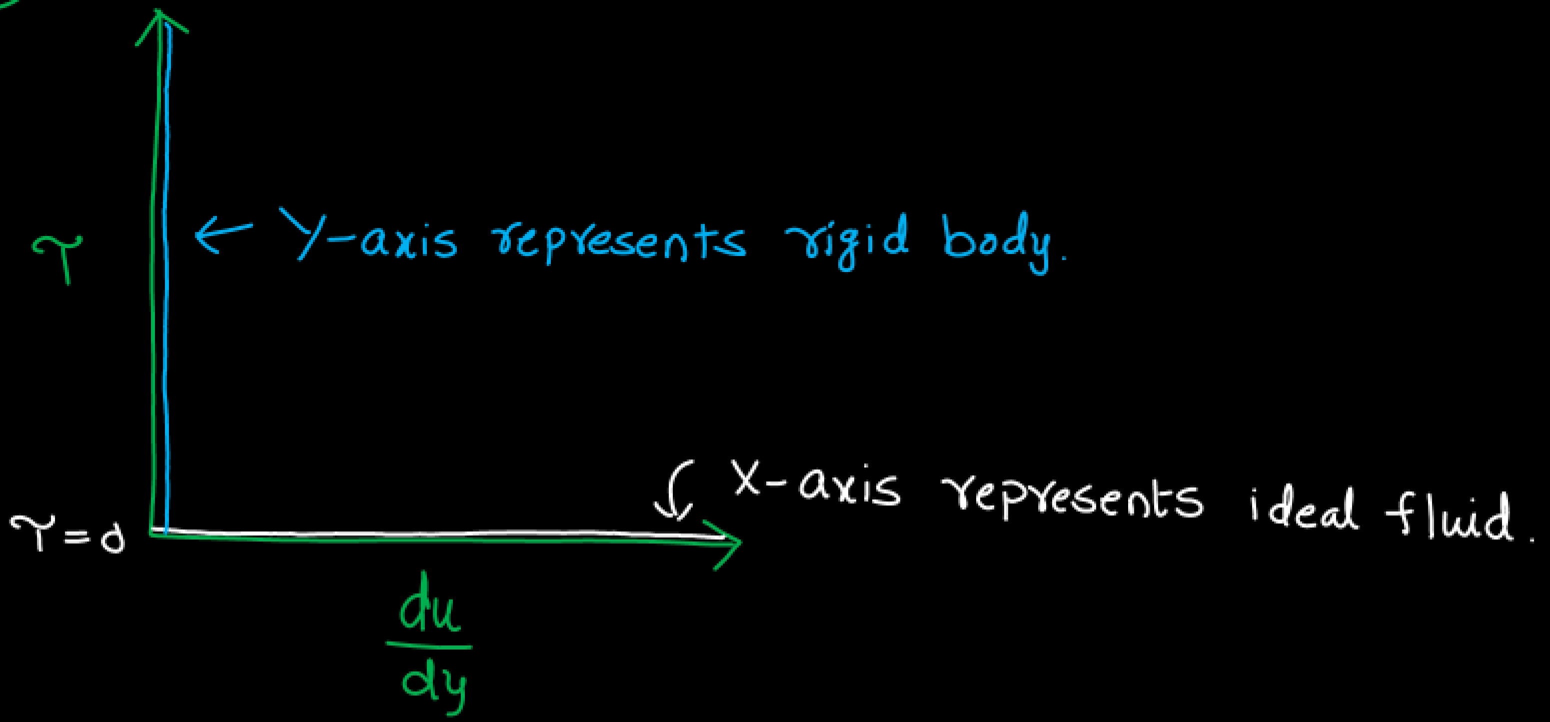
Ex: Some Paints,

Rheopectic [μ increases with time]

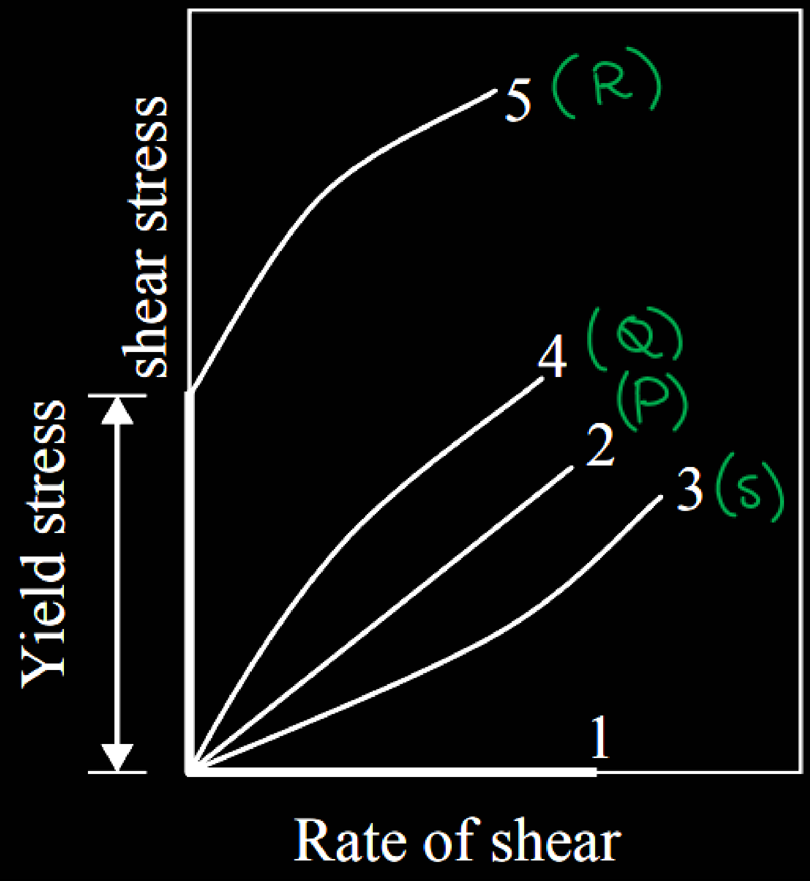
Ex: Printer's ink

Regain of Lost Strength : Thixotrophic

6


τ
$\tau = 0$
$\frac{du}{dy}$
Y-axis represents rigid body.
X-axis represents ideal fluid.

Q. Group I contains the types of fluids while Group II contains the shear stress-rate of shear relationship of different types of fluids, as shown in the figure



Group-I	Group-II
P. Newtonian fluid	1. Curve 1
Q. Pseudo plastic fluid	2. Curve 2
R. Plastic fluid	3. Curve 3
S. Dilatant fluid	4. Curve 4
	5. Curve 5

The correct match between Group I and Group II is

(GATE – 16 – Set 2)

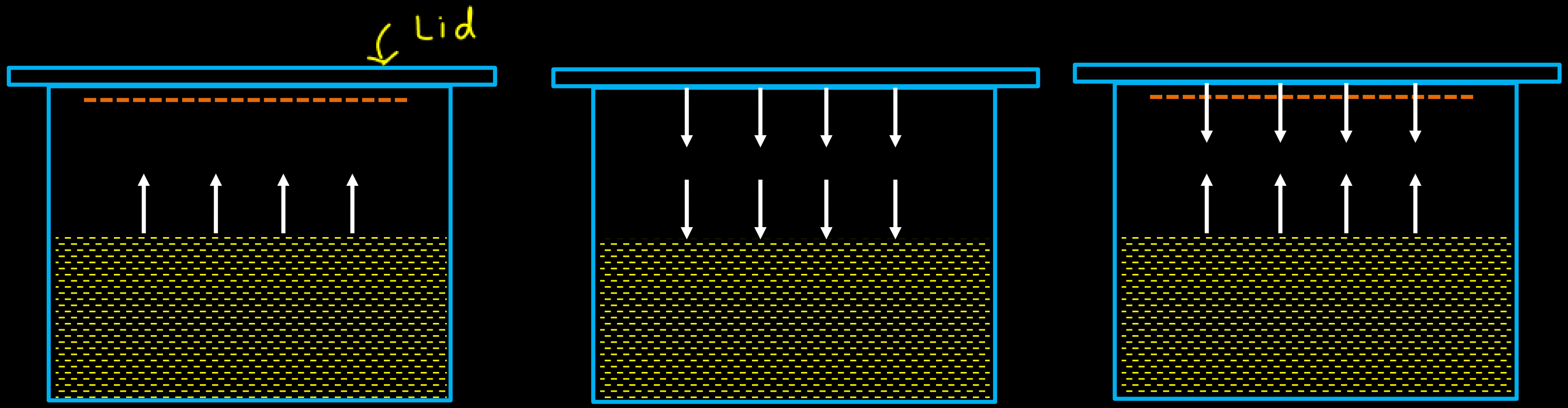
(a) P-2, Q-4, R-1, S-5

(b) P-2, Q-5, R-4, S-1

(c) P-2, Q-4, R-5, S-3

(d) P-2, Q-1, R-3, S-4

Vapour pressure:



Vapour pressure is the pressure exerted by an accumulated vapour on the parent liquid

If the vapour leaving parent liquid is in equilibrium with the vapour coming back is called saturation vapour pressure

Factors :

- Temperature   T ↑   Vapour pressure ↑

→ Vapour Pressure of Water is 2.5m of water head [Absolute Pressure]

Classification of Pressures:

1. Local atmospheric pressure:

The atmospheric pressure at a given location is called local atmospheric pressure.

it changes from place to place.

2. Standard atmospheric pressure:

the atmospheric pressure at average sea level

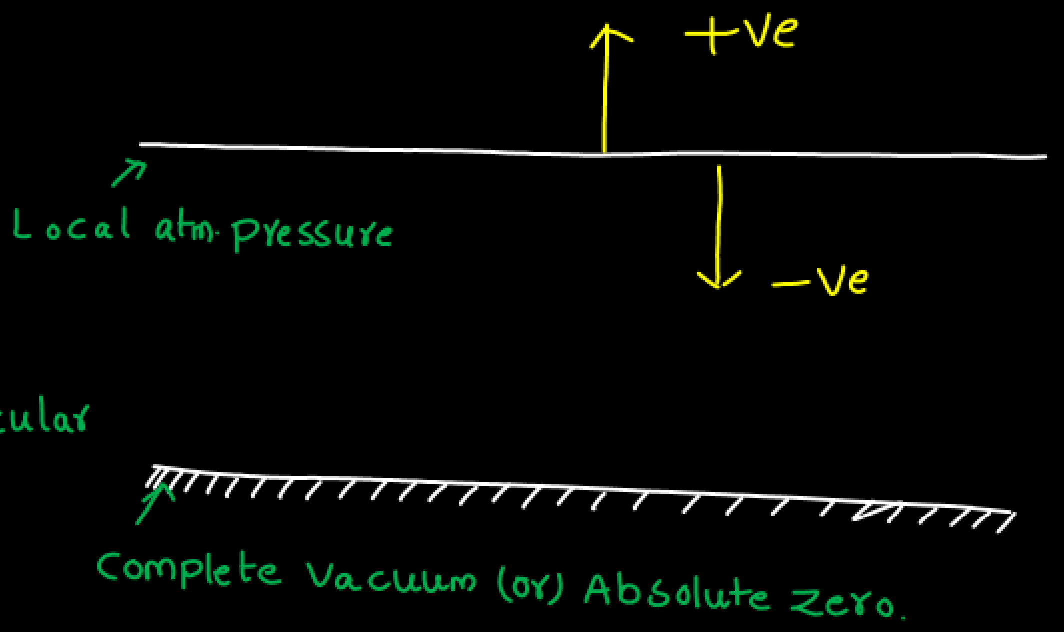
M.S.L [Mean sea level] : The average height of tides over a period of 19 yrs.

Stnd. Atmospheric Pressure : 76 cm of mercury. [Abs]

Absolute pressure:

Pressure measured w.r.t to Absolute zero (or) Complete vacuum.

Absolute zero: It is the point below which no molecular activity.



→ Absolute pressure is always positive.

$$\left\{ P_{abs} = P_{L.atm\,pressure} \pm P_{gauge} \right\}$$

Gauge Pressure:

Pressure measured w.r.t Local atm. Pressure

→ Local atm. pressure can be Positive or negative

Note: Local atm. pressure is zero on gauge scale.

Q. The standard atmospheric pressure at a location is 101.3 Kpa(abs), vacuum pressure is 30Kpa. Local atmospheric pressure is 100Kpa(abs). What is the absolute pressure at the given point?

Sol:
$$P_{abs} = P_{L.atm} \pm P_{gauge}$$
$$= 100 - 30$$
$$= 70 \, kPa \, (abs)$$

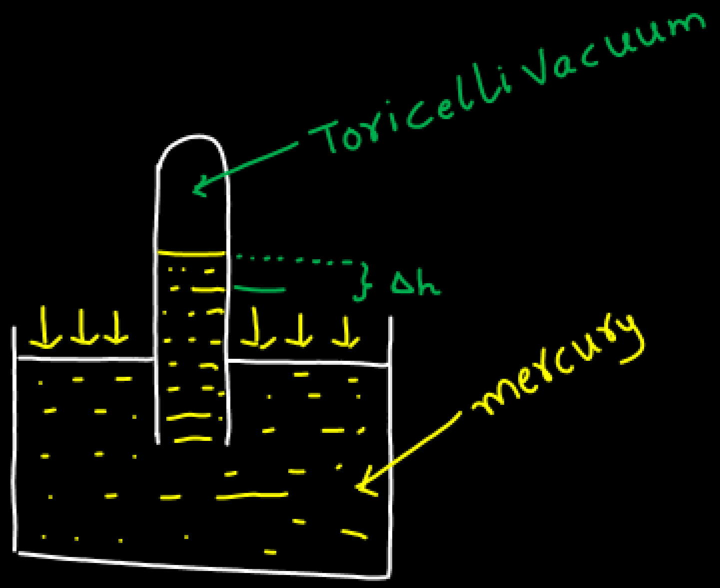
If Pressure inside a machine is 8.7 kPa, Find absolute pressure?

$$P_{abs} = 100 + 8.7$$
$$= 108.7 \, kPa$$

Practical applications of vapour pressure:

- Mercury is chosen as barometric fluid

Barometer is used to measure local atm. pressure.



Why Mercury is chosen as barometric fluid?

✓ Mercury has least vapour pressure and high density

✓ If Vapour Pressur is more the liquid column in the tube Compressed a little. It leads to error in local atm. pressure.

If a hole is made in the top of barometer(Torricelli vacuum)?

Result :

The level in the tube and reservoir are balanced.

$$P = \rho g h$$

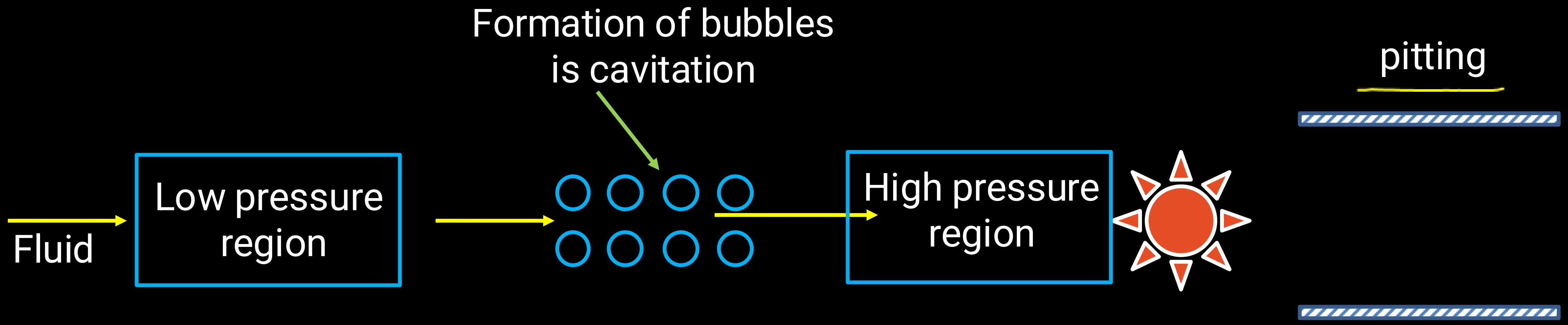
$$13600 \times 9.81 \times 76 = \rho_w g h_w$$

$$= 1000 \times 9.81 \times h_w$$

$$h_w = 1033.6\ cm$$

$$= 10.3\ m$$

Cavitation:



Pressure less than vapour pressure of flowing fluid

When a fluid is subjected to low pressure region bubbles formation occurs.

The phenomena of formation of bubbles is known as "Cavitation".

Those bubbles travels along with water and subjected to high pressure region, they colloapses and releases huge amount of energy. This energy damages adjoining machine parts known as "Pitting".

- Cavitation is due to low pressure
- High speed flows may cause cavitation
- Liquids shifted to higher altitudes will give low pressure and hence causes cavitation

$$H = \frac{P}{\gamma} + \frac{V^2}{2g} + z = \text{Constant}$$

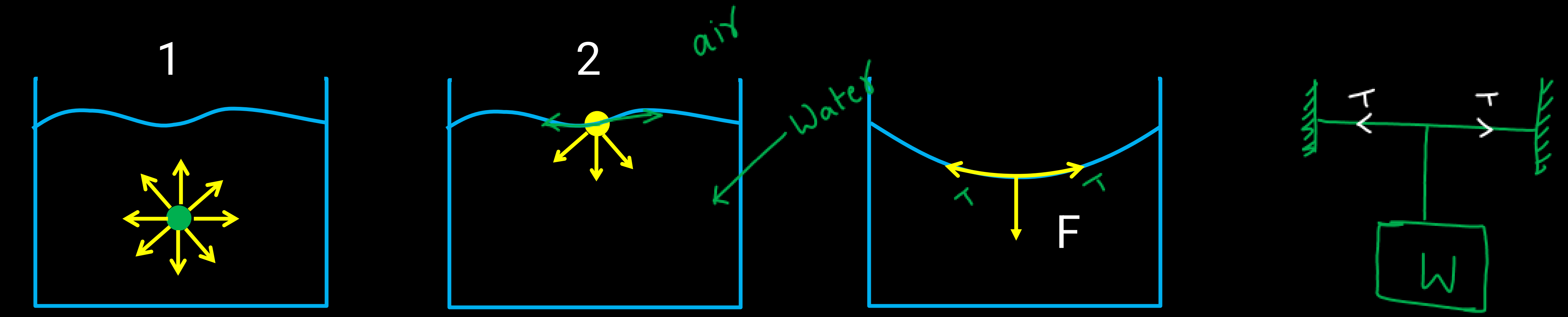
Effects of cavitation:

- Cavitation causes a lot of noise in the machine
- Damage to machine parts

Increasing order of Vapour Pressures:

mercury < Water < Petrol

Surface tension:



Internal molecule
Balanced state

Surface molecules
Un balanced state

Resultant
Downward normal

- Surface tension is due to cohesion only, if influence of air is neglected.
- Surface tension is due to unbalanced normal force at the interface of two different fluids.
- It is the tangential force on the surface

Applications :

Insects crawling on water, dust accumulated on the bubble and a small pin floating on water because of surface tension.

Note :

The floating of the above objects is not due to density criteria. It is because of the magnitude of force less than that of surface tension.

Surface tension (σ):

$$\sigma = \frac{\text{Surface energy}}{\text{Area}} = \frac{\text{Work done}}{\text{Area}} = \frac{F \cdot L}{L^2}$$

$$= \frac{F}{L}$$

$$\sigma = \frac{\text{Force}}{\text{Length}}$$

Units: SI $\Rightarrow \frac{N}{m}$

M.K.S $\Rightarrow \frac{Kg(f)}{m}$

Joule/m^2 (or) $\frac{N\text{-}m}{m^2}$

Factors effecting surface tension:

1. Cohesion
2. Temperature
3. Pressure
4. Impurities

σ_{water} @ $20^\circ c$ = 0.073 N/m

σ_{Hg} @ $20^\circ c$ = 0.446 N/m

1. Cohesion ↑ : Surface tension ↑
2. Temperature ↑ : Cohesion ↓ : surface tension ↓
3. Pressure is negligible
4. Impurities (Detergent Powder) ↑ : Cohesion ↓ : Surface tension ↓

Practical applications of surface tension:

Formation of spherical droplets

- A body will come to a stable state if it has least energy
- Sphere is the geometrical shape having least surface energy.

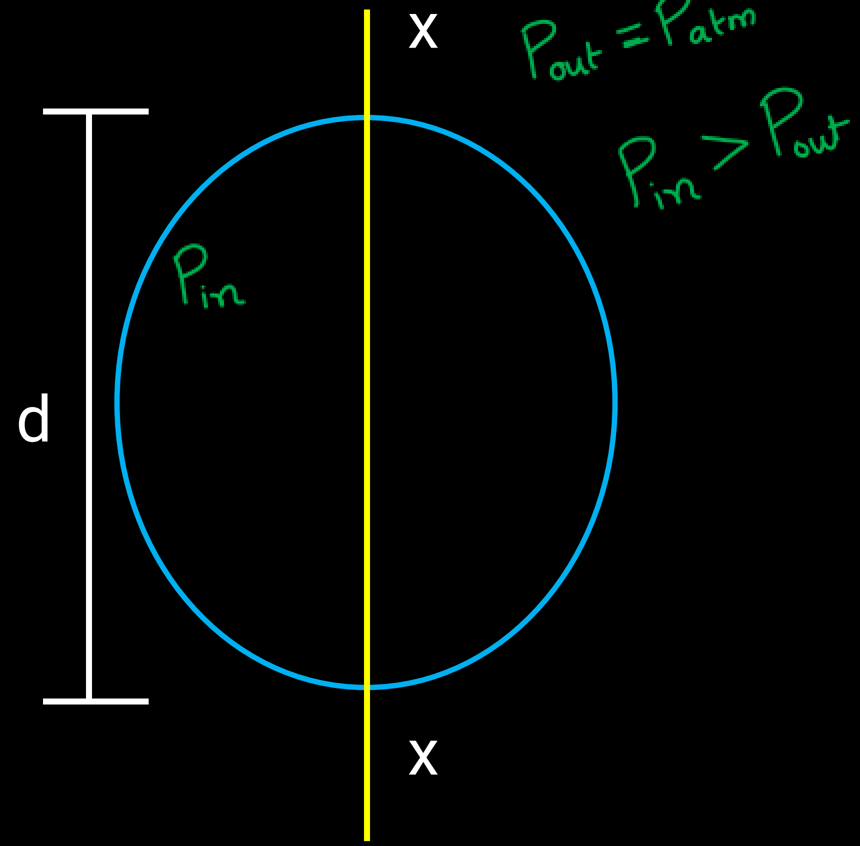
Q. Statement I: mercury has more surface tension than water

Statement II: surface tension is primarily depends upon cohesion. Mercury has more cohesion compared to water.

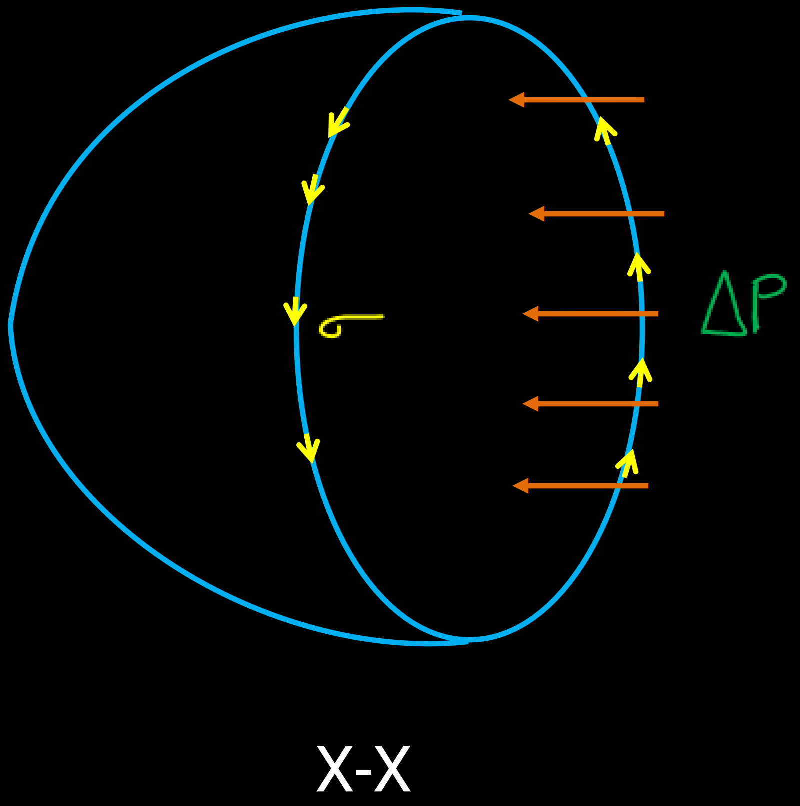
A

Excess hydrostatic pressure:

Droplet (or) Air Bubble:



$$P_{in} = P_{atm} + \underbrace{\Delta P}_{}$$ excess pressure.



Pressure force, $F = \Delta P \times$ Projected area

[Bursting force] $= \Delta P \times \frac{\pi d^2}{4}$

Surface tension force, $F_t = \sigma \times$ Length

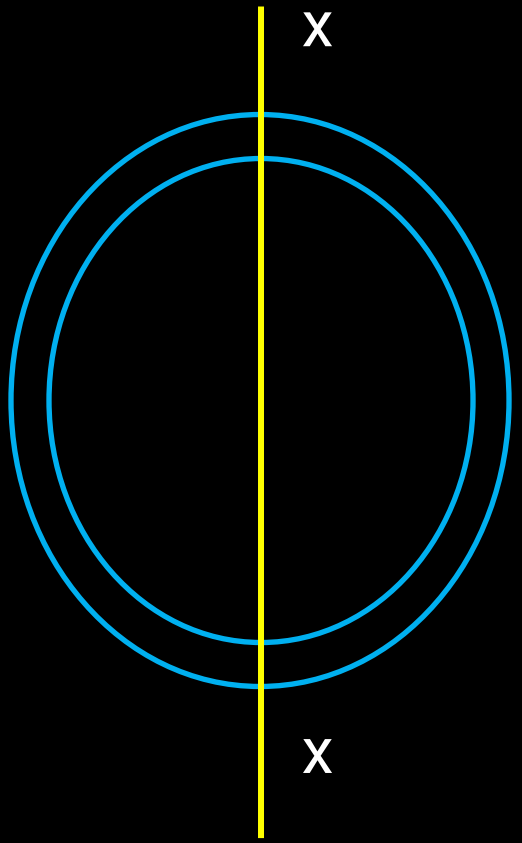
$= \sigma \times \pi d$

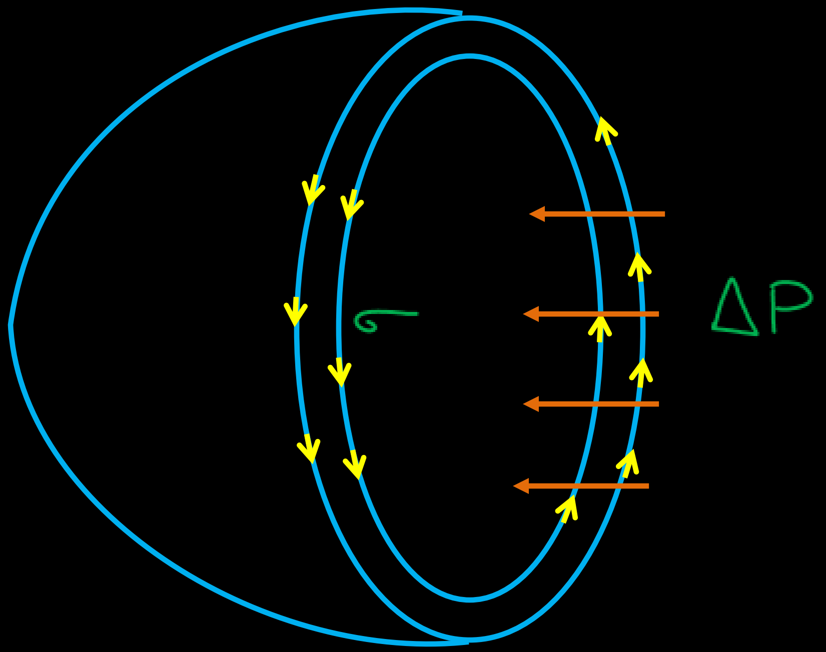
At equilibrium, $F = F_t$ $\left\{ \sum F_H = 0 \right\}$

$$\Delta P \times \frac{\cancel{\pi} d^{\cancel{2}}}{4} = \sigma \times \cancel{\pi d}$$

$$\left\{ \Delta P = \frac{4\sigma}{d} \right\}$$

Soap bubble:





X-X section

Pressure force, $F = \Delta P \times \frac{\pi d^2}{4}$

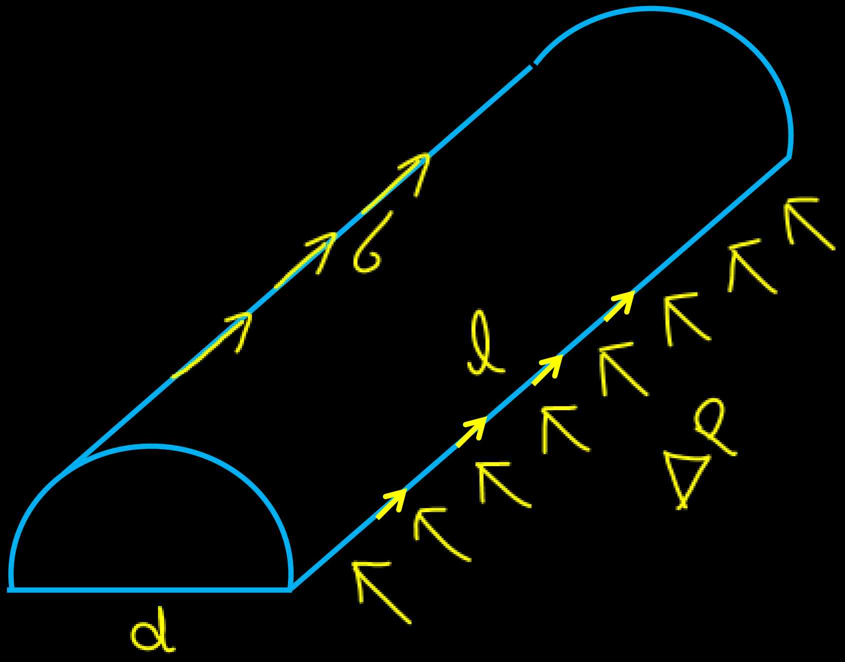
Surface tension force, $F_T = \{\sigma (\pi d)\} 2$

At Equilibrium, $F = F_T$

$$\Delta P \times \frac{\pi d^{\cancel{2}}}{4} = 2\sigma \cancel{\pi d}$$

$$\left\{ \Delta P = \frac{8\sigma}{d} \right\}$$

Liquid Jet:



Pressure force, $F = \Delta P \times (l d)$

Surface tension force, $F_T = \sigma \times 2 l$

At Equilibrium, $F = F_T$

$$\Delta P \times (l d) = \sigma (2 l)$$

$$\left\{ \Delta P = \frac{2\sigma}{d} \right\}$$

ESE PYQS

Q. Assertion (A): The movement of two blocks of wood welded with hot glue requires greater and greater effort as the glue is drying up.

Reason (R): Viscosity of liquids varies inversely with temperature.

(a) both A and R are true and R is the correct explanation of A

(b) both A and R are true but R is not a correct explanation of A

(c) A is true but R is false

(d) A is false but R is true

A

Q. The velocity distribution for flow over a plate is given by $u = 0.5y - y^2$ where 'u' is the velocity in m/s at a distance 'y' meter above the plate. If the dynamic viscosity of the fluid is 0.9 N-s/m², then what is the shear stress at 0.20 m from the boundary?

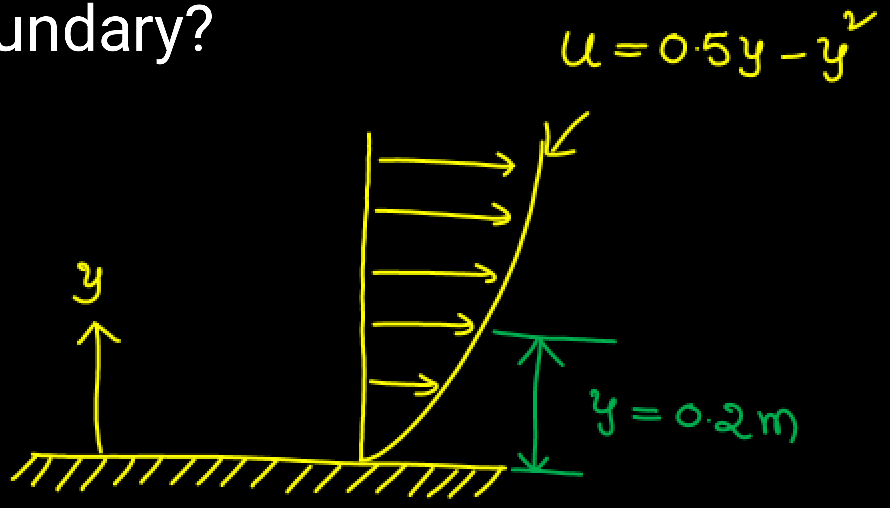
(a) 0.9 N/m²

(b) 1.8 N/m²

(c) 2.25 N/m²

(d) 0.09 N/m²

$\mu = 0.9\ N\text{-}s/m^2$



$$\tau = \mu \frac{du}{dy}$$

$$\frac{du}{dy} = \frac{d}{dy}\left[0.5y - y^2\right] = 0.5 - 2y$$

$$\left.\frac{du}{dy}\right|_{@\,y=0.2m} = 0.5 - 2(0.2) = 0.1\,s^{-1}$$

$$\tau_{@\,y=0.2m} = \mu \left.\frac{du}{dy}\right|_{@\,y=0.2m} = 0.9 \times 0.1 = 0.09\ N/m^2$$

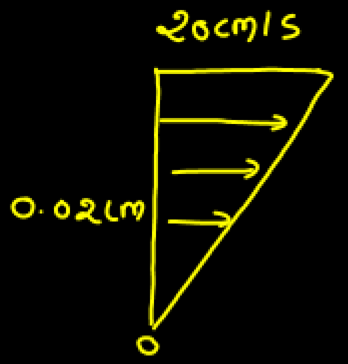
Q. A flat plate of 0.15 m² is pulled at 20 cm/s relative to another plate, fixed at a distance of 0.02 cm from it with a fluid having μ = 0.0014 N-s/m² separating them. What is the power required to maintain the motion?

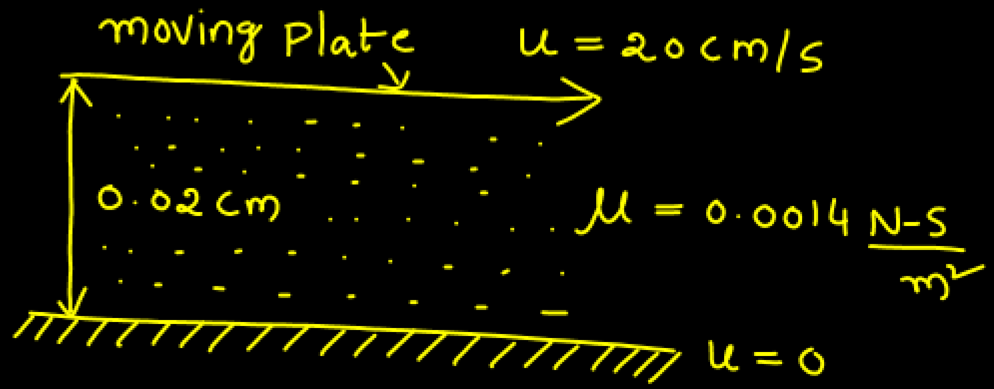
(a) 0.014 W

(b) 0.021 W

(c) 0.035 W

(d) 0.042 W





$$\text{Power, } P = F \times V$$

$$= \text{Shear force} \times u$$

$$= \tau \times \text{Area} \times u$$

$$= \left[\mu \frac{du}{dy}\right] \times A \times u$$

$$= 0.0014 \times \frac{20}{0.02} \times 0.15 \times \frac{20}{100}$$

$$= \frac{14}{10^4} \times \frac{\cancel{20}\,(\cancel{100})}{\cancel{2}} \times \frac{15}{\cancel{100}} \times \frac{2\cancel{0}}{10\cancel{0}}$$

$$= \frac{420}{10^4} = 0.042 \ \frac{N\text{-}m}{s} = 0.042 \text{ Watt}$$

Q. A jet of water has a diameter of 0.3 cm. The absolute surface tension of water is 0.072 N/m and atmospheric pressure is 101.2 kN/m². The absolute pressure within the jet of water will be

(a) 101.104 kN/m²

(b) 101.152 kN/m²

(c) 101.248 kN/m²

(d) 101.296 kN/m²

$d = 0.3\,cm$

$\sigma = 0.072\ N/m$

$P_{atm} = 101.2\ kN/m^2$

Excess Pressure, $\Delta P = \dfrac{2\sigma}{d}$

$= \dfrac{2 \times \cancel{0.072}}{0.3} \times 1\cancel{00} = 0.048\,kN/m^2$

$\Delta P = P_{in} - P_{atm}$

$$P_{in} = \Delta P + P_{atm}$$

$$= 0.048 + 101.2$$

$$= 101.248 \ KN/m^2$$

Q. The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm² (atmospheric pressure). What is the pressure within the droplet if surface tension is 0.0725 N/m of water?

(a) 11.045 N/cm²

(b) 10.32 N/cm²

(c) 9.45 N/cm²

(d) 8.595 N/cm²

$d = 0.04\ mm$

$P_{atm} = 10.32\ N/cm^2$

$\sigma = 0.0725\ N/m$

$$\Delta P = \frac{4\sigma}{d} = \frac{\cancel{4} \times 0.0725}{\cancel{4}} \times 10^5$$

$$= 7250\ N/m^2 = 0.7250\ N/cm^2$$

$$P_{in} = P_{atm} + \Delta P$$

$$= 10.32 + 0.7250$$

$$= 11.045\ N/cm^2$$

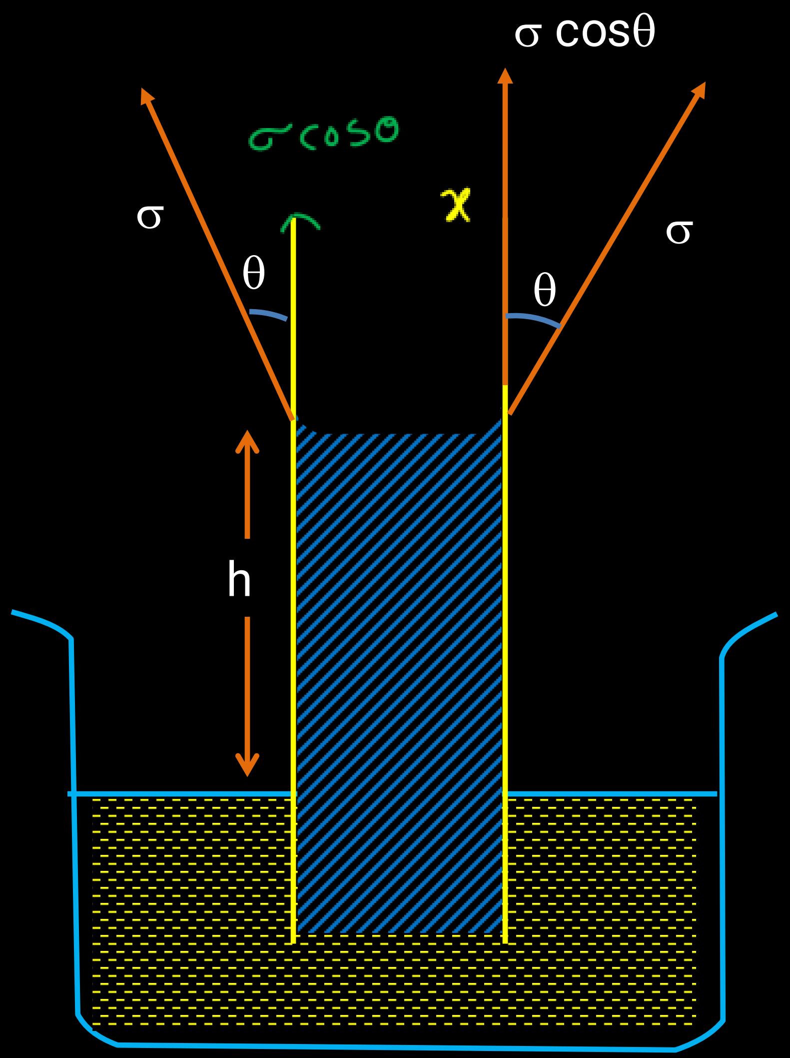
Capillarity :

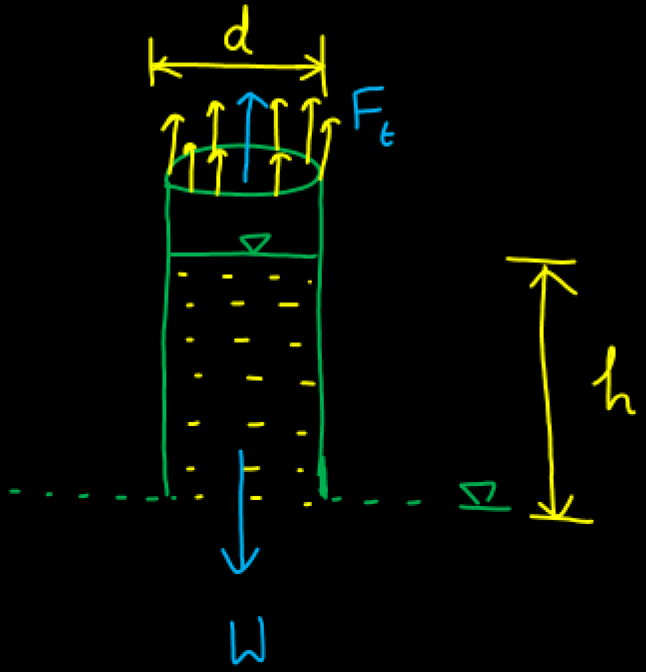
The rise or fall of liquid through small tube is known as capillarity.

- If fluid rises, it is capillary rise
- If fluid falls, it is capillary depression

Note : capillarity is due to cohesion of liquid and adhesion between liquid and the walls of the tube.

Capillary rise:





At Equilibrium

$$\sum F_v = 0$$

$$W - F_t = 0$$

$$W = F_t$$

Weight of liquid = Surface tension force.

Weight of liquid, $W = \gamma \cdot \text{Volume}$

$$= \rho_w g \left[\frac{\pi d^2}{4} \cdot h\right]$$

Surface tension force, $F_t = \sigma \cos\theta \, (\pi d)$

$$W = F_t$$

$$\rho_w g \left[\frac{\cancel{\pi} d^{\cancel{2}}}{4}\right] h = \sigma \cos\theta \, (\cancel{\pi d})$$

$$\boxed{h = \frac{4\sigma \cos\theta}{\rho_w g d}}$$

where, h = capillary rise (m)

σ = surface tension (N/m)

θ = Meniscus angle [Acute angle]

θ = Angle made by the tangent drawn to the menicus curve and the wall of tube.

$\theta = 0^\circ$ {for Pure Water and glass} for Pure water, $h = \dfrac{4\sigma}{\rho_w g d}$

d = dia of tube

$$\uparrow h \propto \frac{1}{d} \downarrow$$

→ Wetting fluids will rise

→ If $\theta < 90^\circ$, fluid will rise

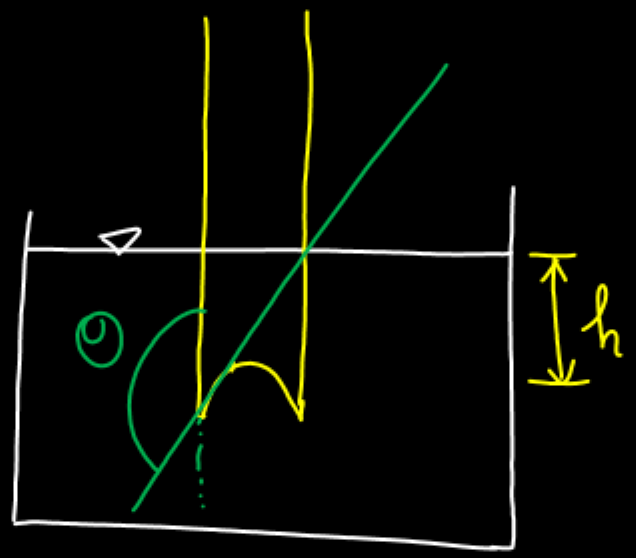
Capillary depression:

→ It is observed in case of mercury

→ θ is Approximately equal to 130°

→ $\theta > 90^\circ$ fluid will fall

→ Non-Wetting fluid will fall.



Q. The surface tension of water at 20°C is 75×10^{-3} N/m. The difference in water surfaces within and outside an open-ended capillary tube of 1 mm internal bore, inserted at the water surface, would nearly be

(a) 7 mm

(b) 11 mm

(c) 15 mm

(d) 19 mm

$$h = \frac{4\sigma}{S_w g d} = \frac{\cancel{4}^{2} \times 75}{1000 \times \cancel{1000} \times 10 \times \frac{\cancel{2}}{\cancel{1000}}}$$

$$= \frac{150}{10^4} \times 1000 \text{ mm}$$

$$= 15 \text{ mm}$$

bore = radius

$r = 1$ mm

$d = 2$ mm

Practical Applications of Capillarity:

→ Sap (or) Water rises through roots of a tree to the stem is due to capillarity

→ rise of oil through wicked lamp.

→ rise of blood in veins.

Bulk modulus of elasticity: (k)

It is the ratio of change in pressure to volumetric strain

$$k = -\frac{dP}{\left[\frac{dV}{V}\right]} \quad (\text{or}) \quad \frac{dP}{\left[\frac{d\rho}{\rho}\right]}$$

$$E = \frac{\sigma}{\varepsilon}$$

$$G = \frac{\tau}{\phi}$$

$$k = \frac{\sigma_v}{\varepsilon_v}$$

$$\rho = \frac{m}{V}$$

$$\rho V = m$$

$$\rho V = C$$

$$d[\rho V] = d[C]$$

$$\rho \, dV + V \, d\rho = 0$$

$$\left\{ \frac{dV}{V} = -\frac{d\rho}{\rho} \right\}$$

Compressibility: (β)

It is the inverse of bulk modulus of elasticity (k)

$$\beta = \frac{1}{k}$$

$$k = -\frac{dP}{\left[\frac{dV}{V}\right]}$$

$P\uparrow \Rightarrow k\uparrow \Rightarrow$ Difficult to compress $\Rightarrow \beta\downarrow$

$P\downarrow \Rightarrow k\downarrow \Rightarrow$ Easy to compress $\Rightarrow \beta\uparrow$

❑ Liquids are generally taken as incompressible.

Units:

$$k : S.I \Rightarrow \frac{N}{m^2} \text{ (or) } P_a$$

$$\beta : S.I \Rightarrow \frac{m^2}{N}$$

Bulk modulus & compressibility of important substances:

$k_{air} \approx 1.03 * 10^5$ Pa

$k_{water} \approx 2.06 * 10^9$ Pa

$k_{steel} \approx 2.06 * 10^{11}$ Pa

① $$\frac{\beta_{air}}{\beta_{water}} = \frac{k_{water}}{k_{air}} = \frac{2.06 \times 10^9}{1.03 \times 10^5} = 20000$$

② $$\frac{\beta_{water}}{\beta_{steel}} = \frac{k_{steel}}{k_{water}} = \frac{2.06 \times 10^{11}}{2.06 \times 10^9} = 100$$

Pressure wave velocity:

The velocity with which a small pressure disturbance will travel in the fluid medium.

$$C = \text{Pressure wave velocity} = \sqrt{\frac{k}{\rho}}$$

(or)

Velocity of sound

(or)

Sonic Velocity

Mach number : (M_a)

$$M_a = \frac{V}{C}$$

V = Velocity of the fluid (or) object

C = Pressure wave velocity

If $M_a < 0.3$, incompressible

If $M_a < 1$, sub sonic flow

If $M_a = 1$, sonic flow

If $1 < M_a < 3$, supersonic flow

If $M_a > 3$, hyper sonic flow

Q . A reservoir of capacity $0.01 m^3$ is completely filled with a fluid of coefficient of compressibility $0.75 * 10^{-9}\ m^2/N$. if pressure in the reservoir is reduced by $2 * 10^7\ N/m^2$.

the amount of fluid that spills over is ____1.5____($* 10^{-4}$in m^3).

Sol:

$$V = 0.01 m^3$$

$$\beta = 0.75 \times 10^{-9}\ m^2/N$$

$$dP = 2 \times 10^7\ N/m^2$$

$$\beta = \frac{1}{K} = \frac{\left[\frac{dv}{V}\right]}{dP}$$

$$0.75 \times 10^{-9} = \left[\frac{dv}{V}\right] / 2 \times 10^7$$

$$\frac{dv}{V} = 0.75 \times 10^{-9} \times 2 \times 10^7$$

$$dv = \frac{3}{4} \times 10^{-9} \times 2 \times 10^7 \times \frac{1}{100}$$

$$dv = 1.5 \times 10^{-4}\ m^3$$

Q. Determine the velocity of sound in air if the mass density of air at atmospheric pressure of 1bar is 0.92kg/m³

Sol:

$$P = 1 \text{ bar}$$
$$= 10^5 \, P_a$$
$$\rho_{air} = 0.92 \, kg/m^3$$

$\left\{ 1 \text{ bar} = 100 \, kN/m^2 = 10^5 \, N/m^2 \text{ (or) } P_a \right\}$

$$C = \sqrt{\frac{k}{\rho}}$$

$\left\{ \text{For Isothermal condition, } k = P \right\}$

$$= \sqrt{\frac{P}{\rho}} = \sqrt{\frac{10^5}{0.92}} = 330 \, m/s$$

jet velocity $V = 1800$ Kmph

$$= 1800 \times \frac{5}{18} = 500 \, m/s$$

$$\left[M_a\right]_{jet} = \frac{V}{C} = \frac{500}{330} = 1.5$$

$1 < M_a < 3$: Supersonic flow

For Isothermal Condition:

$$C = \sqrt{\frac{k}{\rho}} \qquad \{k = P\}$$

$$= \sqrt{\frac{P}{\rho}} \qquad \{P = \rho R T\}$$

$$= \sqrt{\frac{\rho R T}{\rho}}$$

$$\left\{ C = \sqrt{RT} \right\}$$

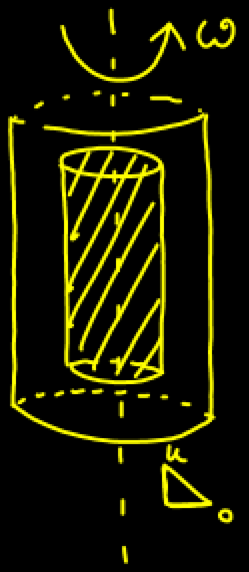
Q.No	Question	Marks
1. a)	Differentiate between: i) Liquids and Gases ii) Real and Ideal Fluids	7
b)	Determine the intensity of shear stress of an oil having viscosity = 1 poise. The oil is used for lubricating the clearance between a shaft of diameter 10 cm and its journal bearing. The clearance is 1.5 mm and the shaft rotates at 150 r.p.m.	7
2. a)	Define capillarity and derive the formula for finding the capillarity rise in a glass tube.	7
b)	The pressure outside the droplet of water of diameter 0.04 mm is 10.32 N/cm^2 (atmospheric pressure). Calculate the pressure within the droplet if surface tension is given as 0.0725 N/m of water.	7
3. a)	The pressure intensity at a point in a fluid is given ...	

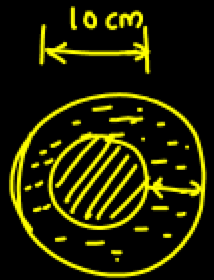
Sol: $\mu = 1 \text{ Poise} = 0.1 \text{ N-s/m}^2$

$N = 150 \text{ r.p.m}$

$$\tau = \mu \frac{du}{dy}$$

$$= \mu \frac{|V-0|}{y} = \mu \frac{V}{y}$$





dy = clearance = 1.5 mm

$$\omega = \frac{2\pi N}{60} \text{ rad/sec}$$

$$V = r\omega$$

$$V = r\frac{2\pi N}{60} = \frac{\pi D N}{60} \text{ m/s}$$

$$\left\{ V = \frac{\pi D N}{60} \right\}$$

$$V = \frac{\pi \times 0.1 \times 150}{60} = 0.7853 \text{ m/s}$$

$$\tau = \mu \frac{V}{y} = 0.1 \times \frac{0.7853}{1.5 \times 10^{-3}} = 52.353 \text{ N/m}^2$$

Chapter 2
Fluid statics

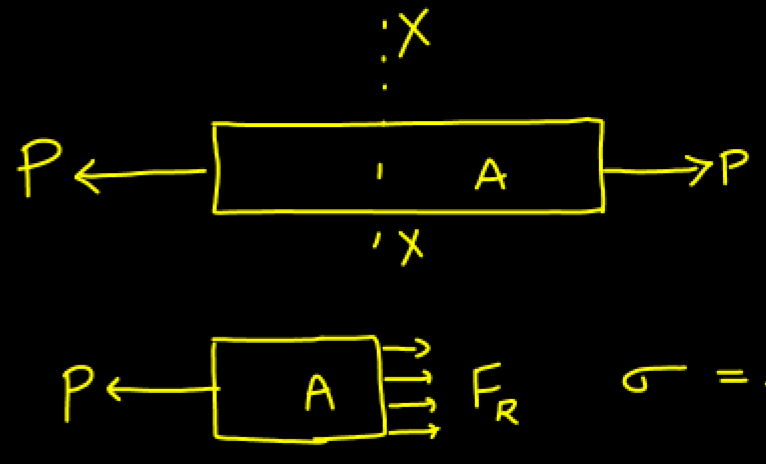
Pressure :

It is the intensity of force per unit area.

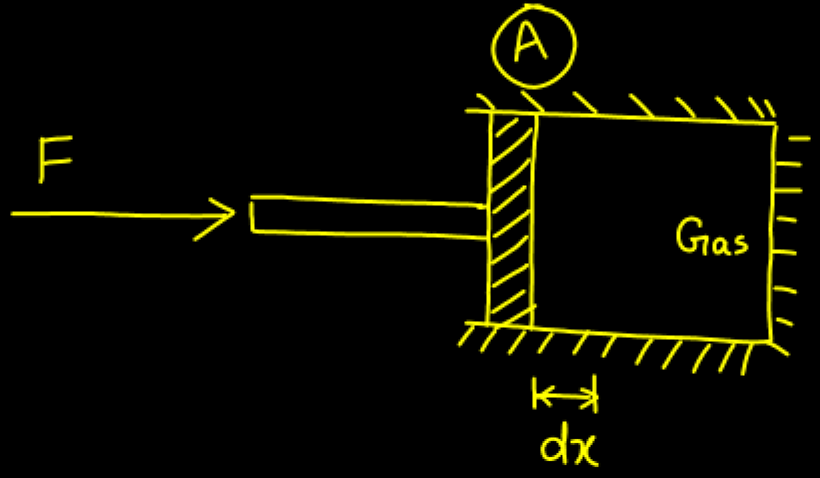
$$p = \frac{F}{A}$$

→ compressive in Nature.

→ Scalar quantity



$$\sigma = \frac{F_R \simeq P}{A}$$



$$dW = F \cdot dx$$

$\downarrow$

Compressive work

$$= \frac{F}{A} \cdot A \cdot dx$$

$$dW = P \cdot A \cdot dx$$

$$dW = P \cdot d\forall$$

$$P = \frac{dW}{d\forall} = \frac{\text{Compressive work}}{\text{unit volume}}$$

Units :

$$\frac{N}{m^2} = Pa$$

$$KPa = 10^3 Pa$$

$$MPa = 10^6 Pa$$

$$GPa = 10^9 Pa$$

$$1\ Bar = 10^5 Pa = 100 KPa$$

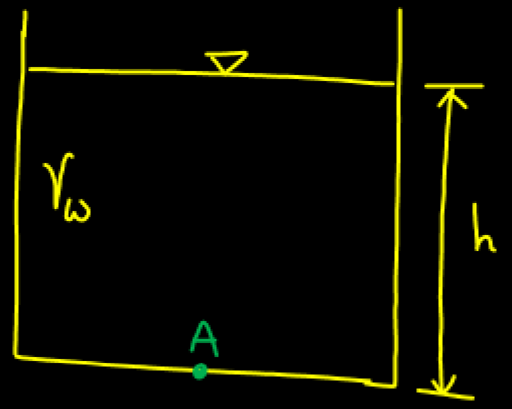
$$1\ atm = 101.3 KPa$$

$$= 1.013\ bar$$

$$\left\{ \begin{aligned} Kg &= 9.81 N \\ &\simeq 10 N \end{aligned} \right\}$$

Pressure in terms of a head of a fluid:

- Height of a liquid column required to create a particular amount of pressure



$P_A = h$ m of Water

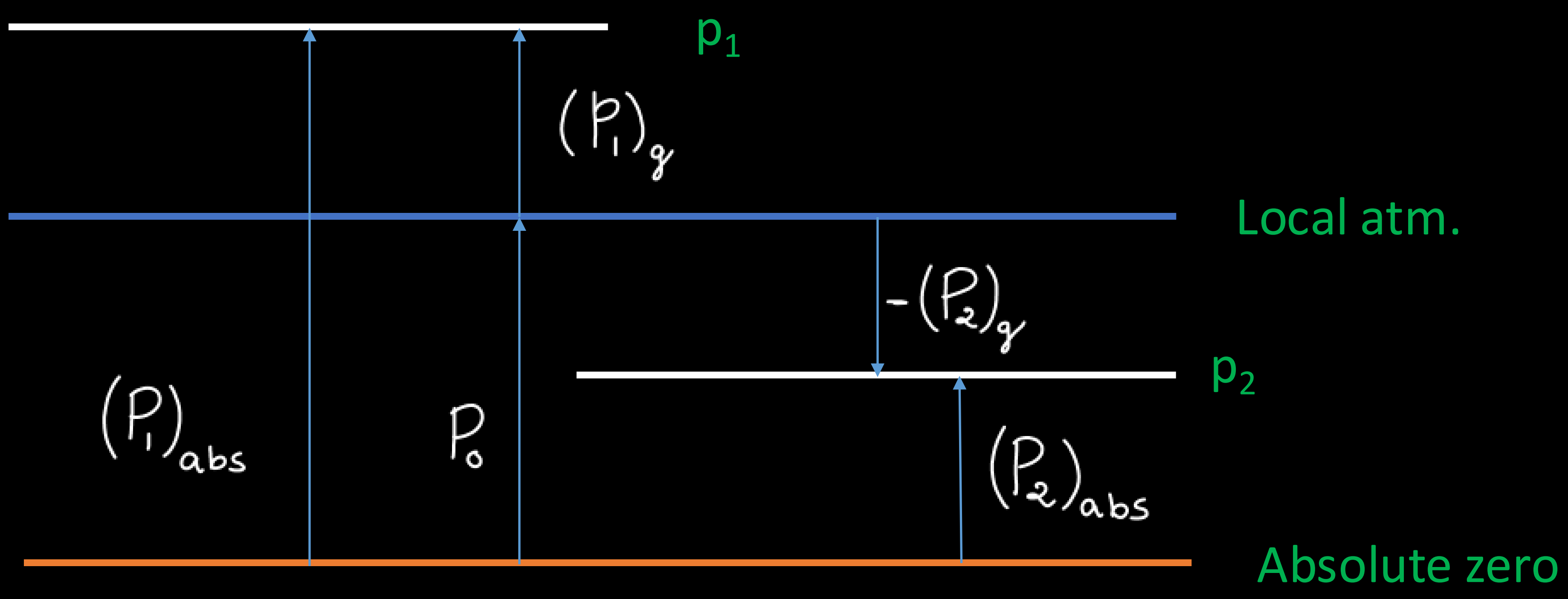
$P_A = \gamma h = \rho g h$

$h = \dfrac{P_A}{\gamma}$

1 atm = 76 cm of Hg = 10.3 m of H_2O

1 Torr = 1 mm of Hg = 133.4 Pa

Scale of measurement:

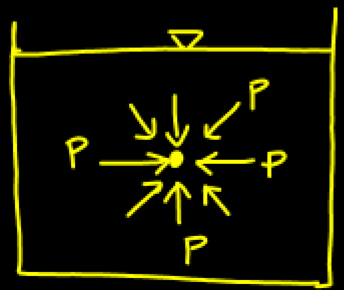


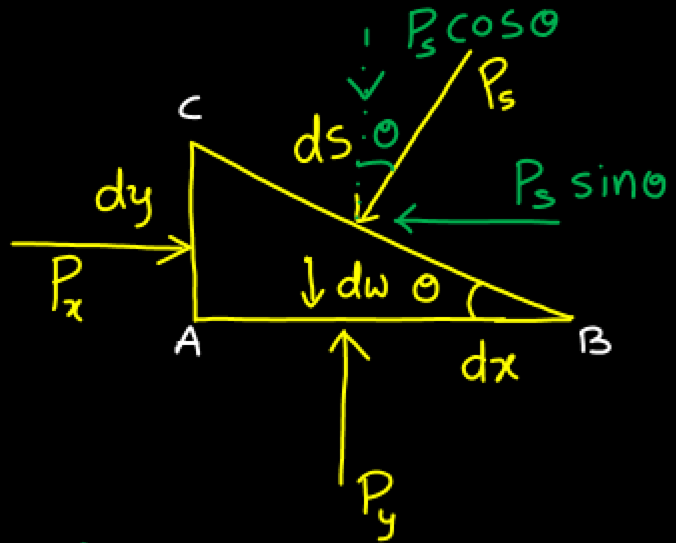
$$P_{abs} = P_o \pm P_{gauge}$$

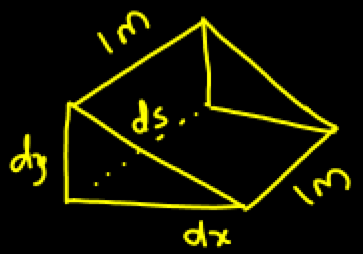
$$= \text{Local atm} \pm \text{gauge pressure.}$$

Pascal's law:

Pascal's law states that pressure is uniform in all directions at a point for a constrained fluid.







$\sum F_H = 0$;

$$P_x(dy \times 1) - P_s \sin\theta\,(ds \times 1) = 0$$

$$P_x\, dy = P_s(ds \sin\theta)$$

$$P_x\, \cancel{dy} = P_s\, \cancel{dy} \Rightarrow P_x = P_s \longrightarrow (1)$$

Δle ABC,

$$\sin\theta = \frac{dy}{ds} \Rightarrow dy = ds\sin\theta$$

$$\cos\theta = \frac{dx}{ds} \Rightarrow dx = ds\cos\theta$$

$$\Sigma F_v = 0$$

$$P_y(dx \times 1) - P_s \cos\theta (ds \times 1) - \cancel{dw}^{\,0} = 0$$

$$P_y dx = P_s ds \cos\theta$$

$$P_y dx = P_s dx$$

$$P_y = P_s \longrightarrow (2)$$

From (1) & (2) $\quad P_x = P_y = P_s$

Ideal fluid	Real fluid
It is incompressible, inviscid, and no surface tension	It is compressible, viscous and has a surface tension
In fact ideal fluids does not exist in nature Ex: Air, Water	All the fluids are real fluids

Inviscid = No viscosity [$\mu = 0$]

Validity of pascal's law:

- ✓ Pascal's law is valid only in the absence of shear stress $(\tau = 0)$
- ✓ It is always valid for an ideal fluid

Case 1 : Ideal fluid

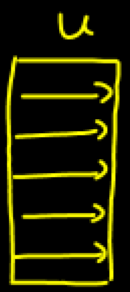
Rest : $u = 0$; $\mu = 0 \Rightarrow \tau = \mu \frac{du}{dy} = 0$ $(\tau = 0)$ ✓

Motion : $u \neq 0$; $\mu = 0 \Rightarrow \tau = 0$ ✓

Case 2 : Real fluid

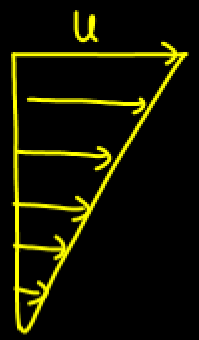
Rest : $u = 0$; $\mu \neq 0 \Rightarrow \frac{du}{dy} = 0 \Rightarrow \tau = 0$ ✓

Rigid body motion: $u \neq 0$; $\mu \neq 0 \Rightarrow \dfrac{du}{dy} = 0 \Rightarrow \tau = 0$

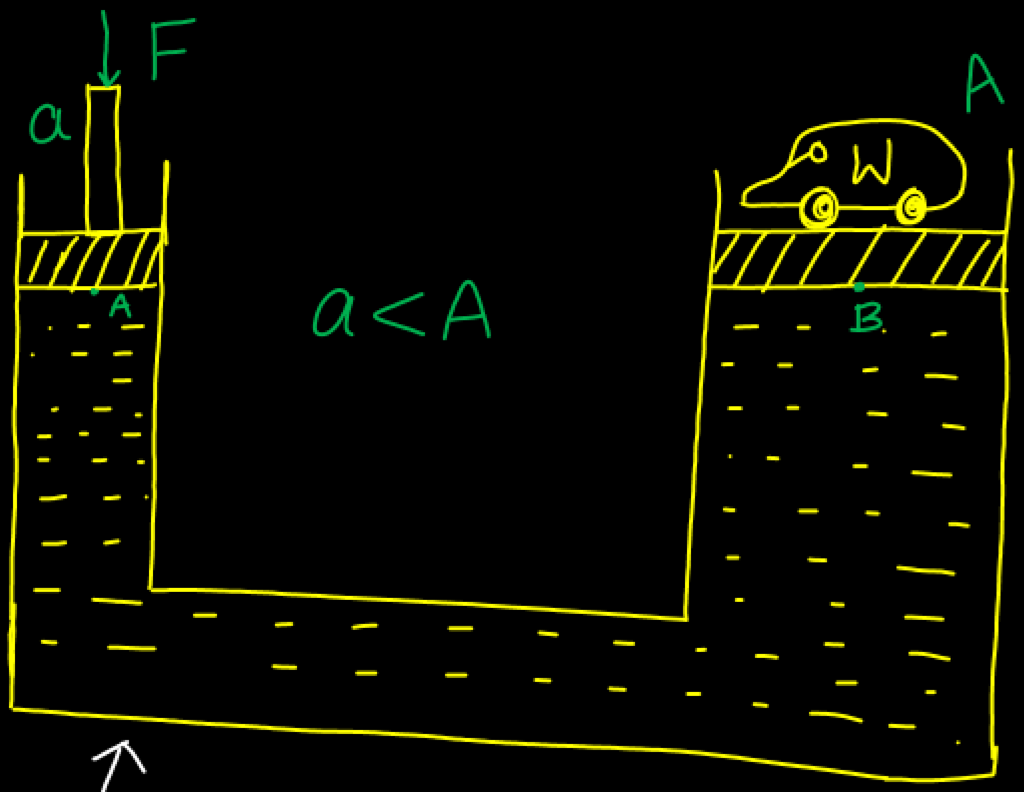


$$\frac{d}{dy}[C] = 0 \Rightarrow \frac{du}{dy} = 0$$

General body motion: $u \neq 0$; $\mu \neq 0$: $\dfrac{du}{dy} \neq 0 \Rightarrow \tau \neq 0$



Application of pascal's law:



Hydraulic lift (or) Hydraulic ram.

As per Pascal's law

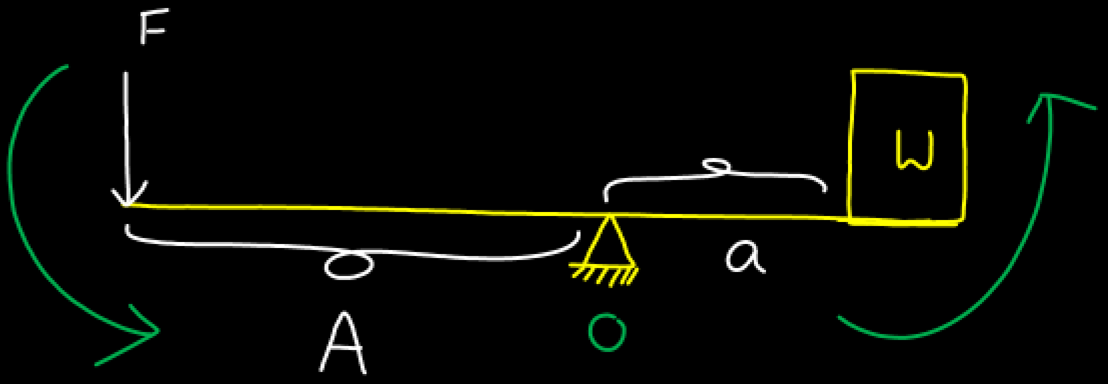
$$P_A = P_B$$

$$\frac{F}{a} = \frac{W}{A}$$

$$F = W\left[\frac{a}{A}\right]$$

$$F < W$$

$$\text{Machanical Advantage, MA} = \frac{\text{Load lifted}}{\text{force Applied}} = \frac{W}{F}$$



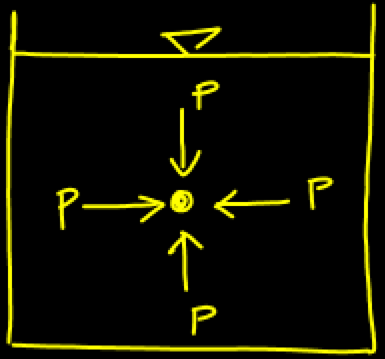
$F < W$

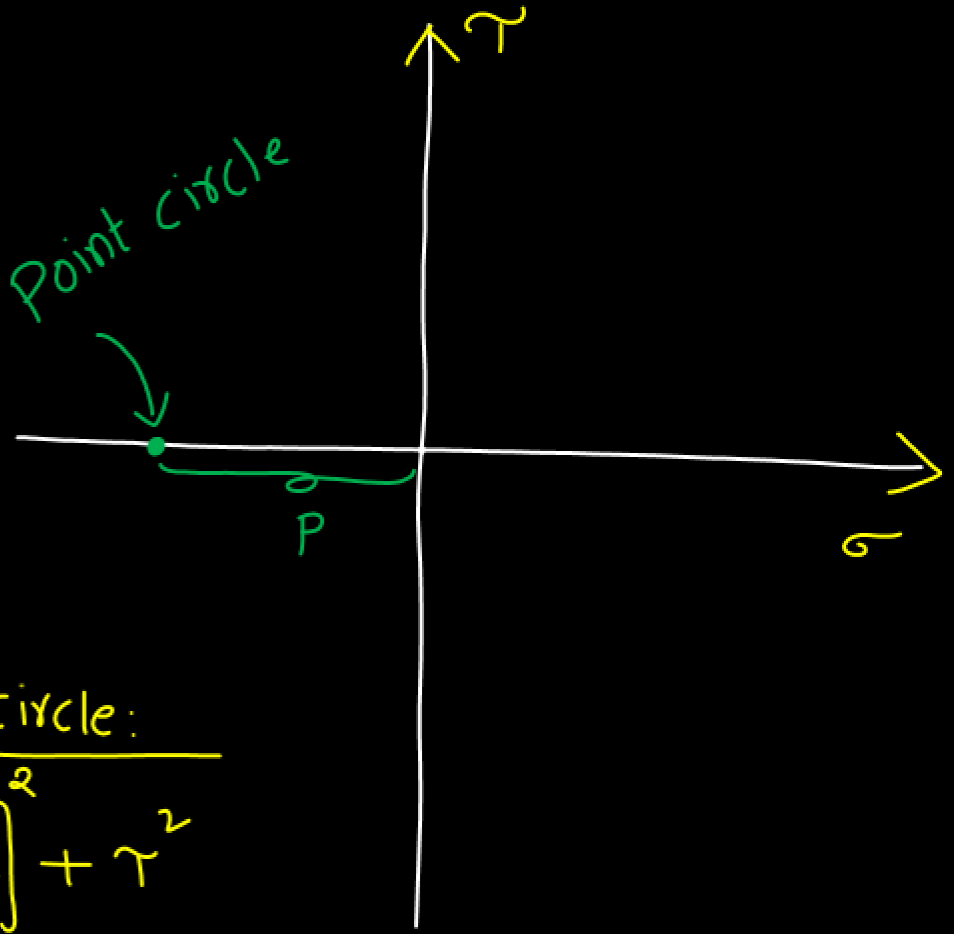
$$\sum M_o = 0$$

$$F[A] = W[a]$$

$$F = W\left[\frac{a}{A}\right]$$

Mohr circle for a point in a fluid at rest:





$$\left.\begin{aligned} \sigma_1 &= -P \\ \sigma_2 &= -P \\ \tau &= 0 \end{aligned}\right\}$$

Radius of Mohr circle:

$$r = \sqrt{\left[\frac{\sigma_1 - \sigma_2}{2}\right]^2 + \tau^2}$$

$$r = \sqrt{\left(\frac{-P+P}{2}\right)^2 + 0} = 0$$

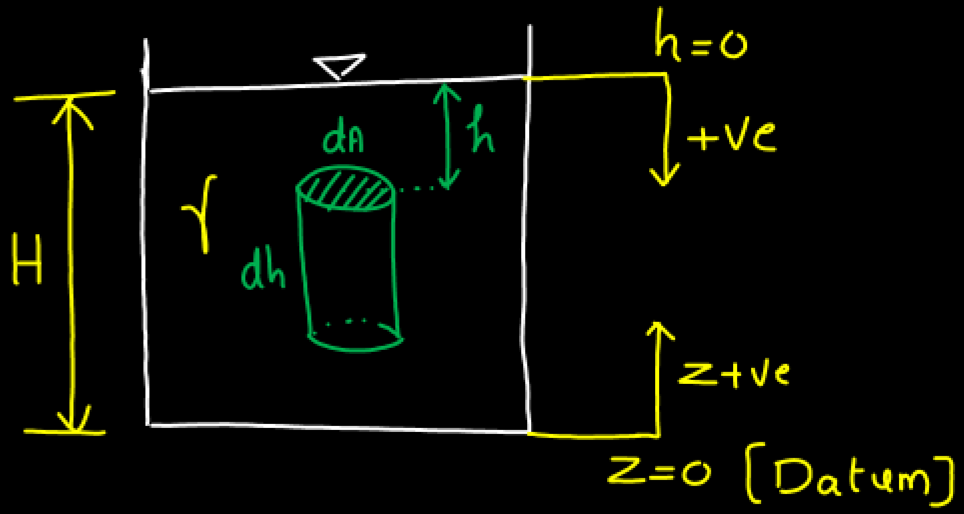
Distance b/w origin to Center of Mohr circle :

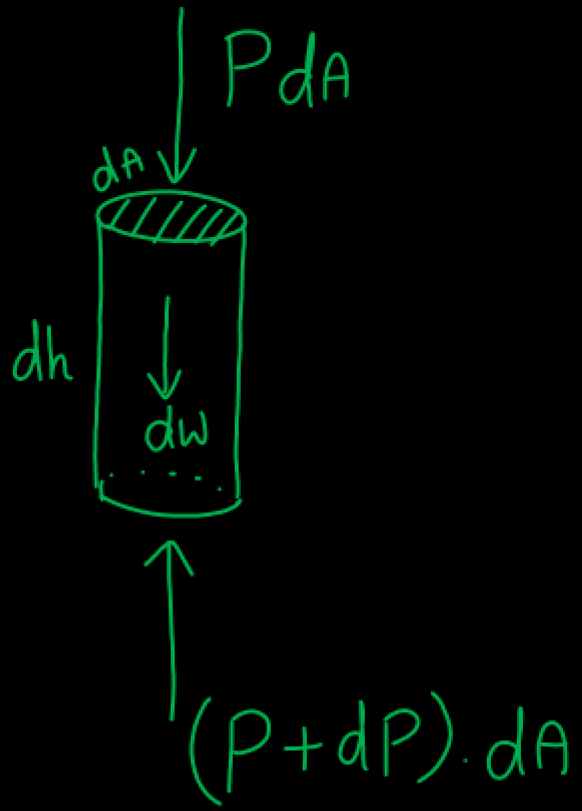
$$\sigma' = \left[\frac{\sigma_1 + \sigma_2}{2}\right]$$

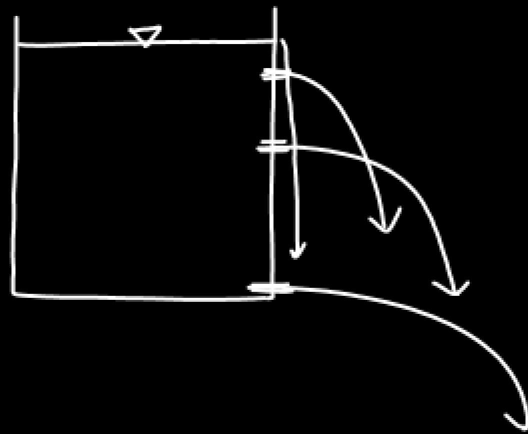
$$\sigma' = \frac{-P-P}{2} = -P.$$

Note: Mohr circle for hydrostatic Compression at a Point is the circle with zero radius [Point circle] on Normal stress axis.

Hydrostatic law:







$$\sum F_v = 0$$

$$P\,dA + dW - (P + dP)\,dA = 0$$

$$dW = dP\,dA$$

$$\gamma\,dV = dP\,dA$$

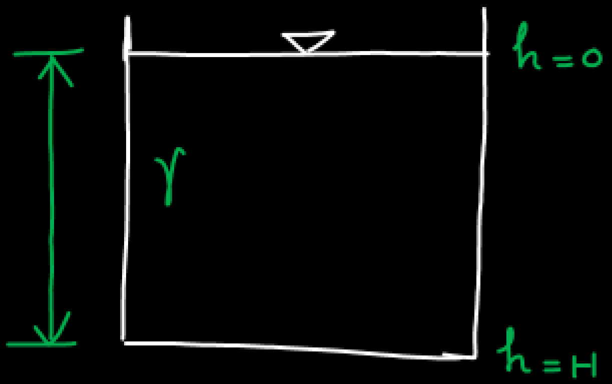
$$\gamma\,dA \cdot dh = dP \cdot dA$$

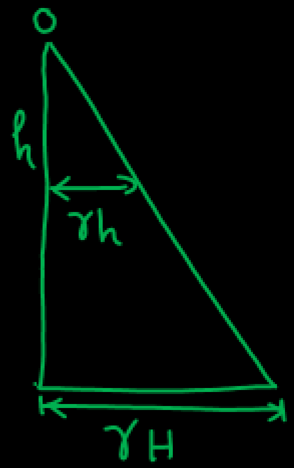
$$\boxed{\frac{dP}{dh} = \gamma} \quad \text{or} \quad \frac{dP}{dz} = -\gamma$$

Rate of change of Pressure w.r.t to vertical axis is given by hydrostatic law.

Application of Hydrostatic law:

Case 1: incompressible fluid $[\gamma = \text{Constant}]$





$$\frac{dP}{dh} = \gamma \Rightarrow \int_{P=0}^{P=P} dP = \gamma \int_{h=0}^{h=H} dh$$

$$\left\{ P = \gamma H = P = \rho g H \right\}$$

Case 2: Compressible fluid $[\gamma \neq C]$

$$\gamma = \gamma_0 + c\sqrt{h}$$

$$\frac{dP}{dh} = \gamma$$

$$dP = \left[\gamma_0 + c\sqrt{h}\right] dh$$

$$\int_0^P dP = \int_0^H (\gamma_0 + c\sqrt{h})\, dh$$

$$= \int_0^H \gamma_0\, dh + c\sqrt{h}\, dh$$

$$= \gamma_0 H + c \left. \frac{h^{1+\frac{1}{2}}}{1+\frac{1}{2}} \right|_0^H$$

$$= \gamma_0 H + \frac{2}{3} c \left. h^{3/2} \right|_0^H$$

$$P = \gamma_0 H + \underbrace{\frac{2}{3} c H^{3/2}}_{\text{excess pressure}}$$

Q. Shear stress develops on a fluid element, if the fluid

(GATE - 92)

(a) is at rest ✗ $u=0 \Rightarrow \tau=0$

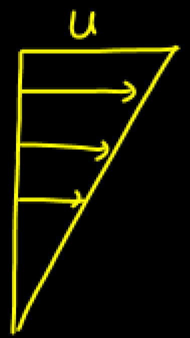
(b) if the container is subjected to uniform linear acceleration ✗ $u=\text{constant}$

$\frac{du}{dy}=0 \Rightarrow \tau=0$

(c) is inviscid ✗ $\mu=0 \Rightarrow \tau=0$

(d) is viscous and the flow is non-uniform. ✓

$\mu \neq 0$:



$\frac{du}{dy} \neq 0 \Rightarrow \tau \neq 0$

Q. If, for a fluid in motion, pressure at a point is same in all directions, then the fluid is

(GATE - 96)

(a) a real fluid

(b) a Newtonian fluid

(c) an ideal fluid

(d) a non-Newtonian fluid

Q. In a static fluid, the pressure at a point is

$u = 0$ At rest

(GATE - 96)

(a) Equal to the weight of the fluid above

(b) Equal in all directions

(c) Equal in all directions, only if, its viscosity is zero

(d) Always directed downwards

Q. If a small concrete cube is submerged deep in still water in such a way that the pressure exerted on all faces of the cube is p, then the maximum shear stress developed inside the cube is

(GATE - 12)

(a) 0

(b) $\frac{p}{2}$

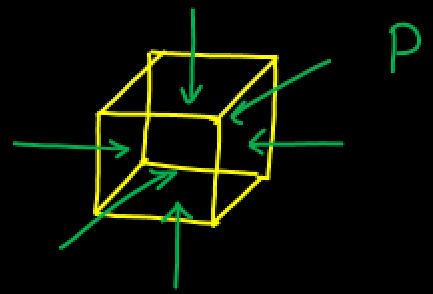
(c) p

(d) 2p

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2}$$

$$= \frac{-P-(-P)}{2}$$

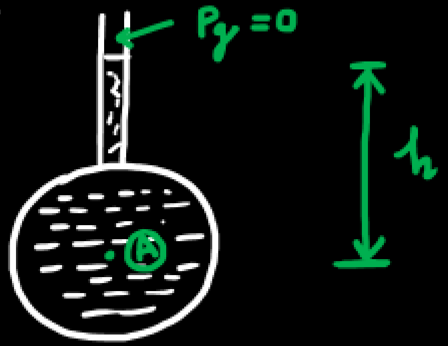
$$= 0$$



Pressure measurement:

1. Piezometer:

it is used to measure pressure at a point



$p_A = \gamma h$

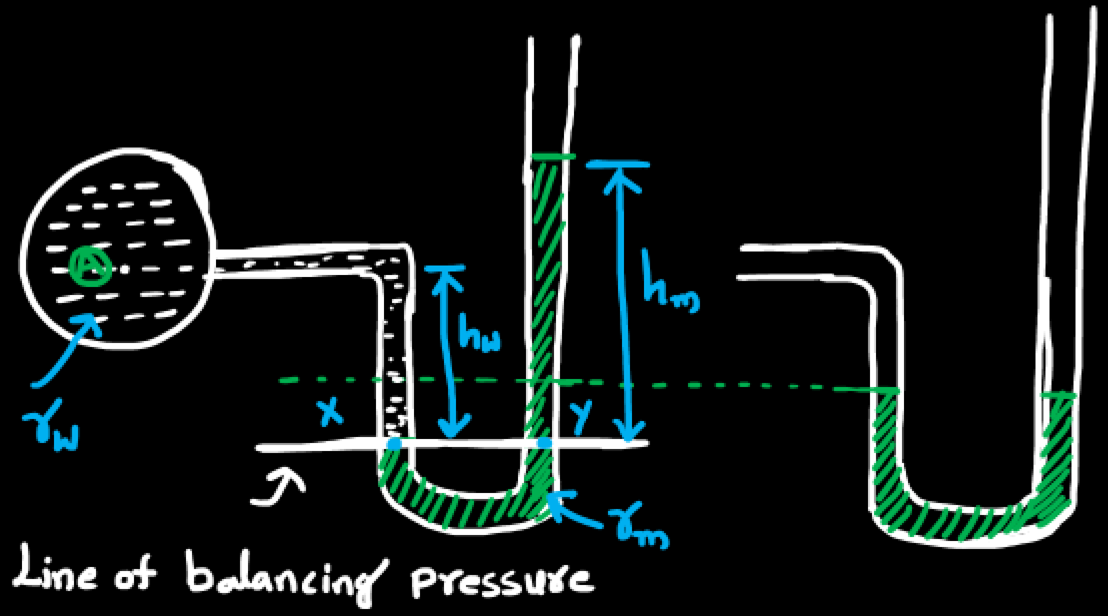
$= \rho g h$

Limitations :

1. It is not suitable to measure pressure of gases
2. It is not suitable to measure high pressures
3. It is not suitable to measure vacuum or suction pressure

Manometer: simple U–Tube manometer

It is a thin transparent glass tube filled with a manometric liquid

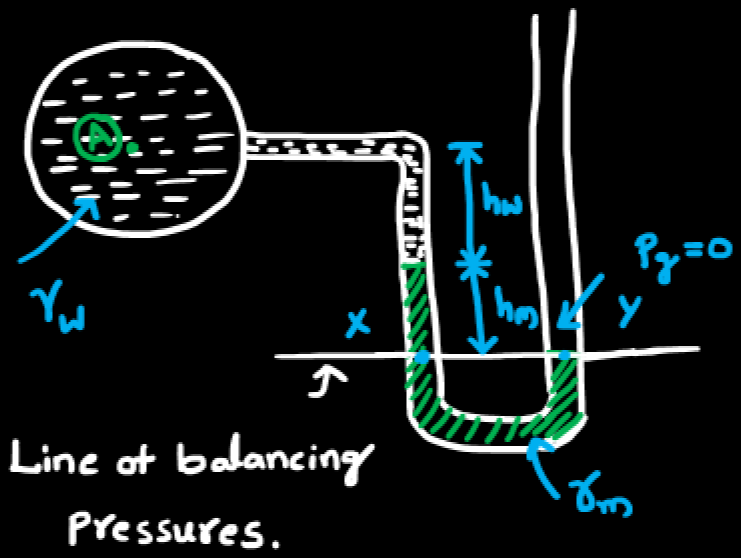


As per pascal's law,

$$P_x = P_y$$

$$P_A + \gamma_w h_w = \gamma_m h_m$$

$$\left\{ P_A = \gamma_m h_m - \gamma_w h_w \right\}$$



As per Pascal's law

$$P_x = P_y$$

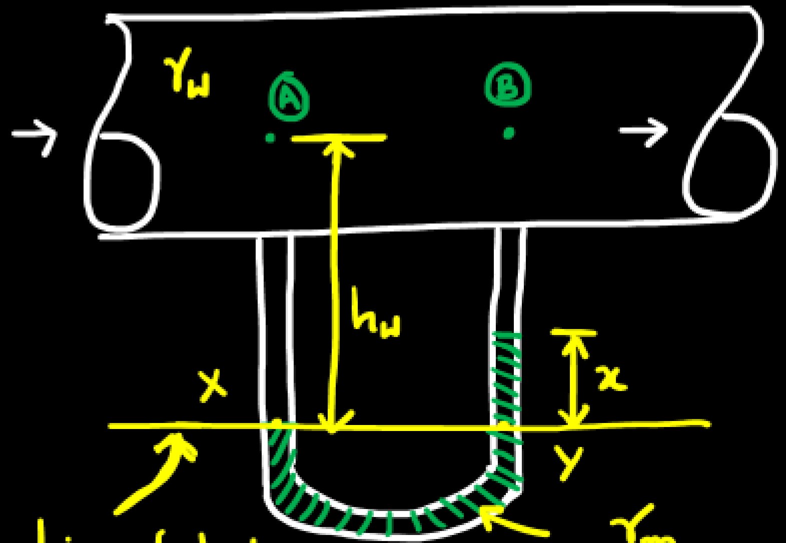
$$P_A + \gamma_w h_w + \gamma_m h_m = 0$$

$$P_A = -\left[\gamma_w h_w + \gamma_m h_m\right]$$

Differential manometers: U-Tube

It is used to measure difference in pressure between two points

$$\left\{ S_w = \frac{\gamma_w}{\gamma_w} = \frac{\rho_w}{\rho_w} \qquad S_m = \frac{\gamma_m}{\gamma_w} = \frac{\rho_m}{\rho_w} \right\}$$



$P_A - P_B = ?$

$$P_x = P_y$$

$$P_A + \gamma_w h_w = P_B + \gamma_w [h_w - x] + \gamma_m x$$

$$P_A - P_B = \gamma_m x - \gamma_w x$$

$$\Delta P = P_A - P_B = x[\gamma_m - \gamma_w]$$

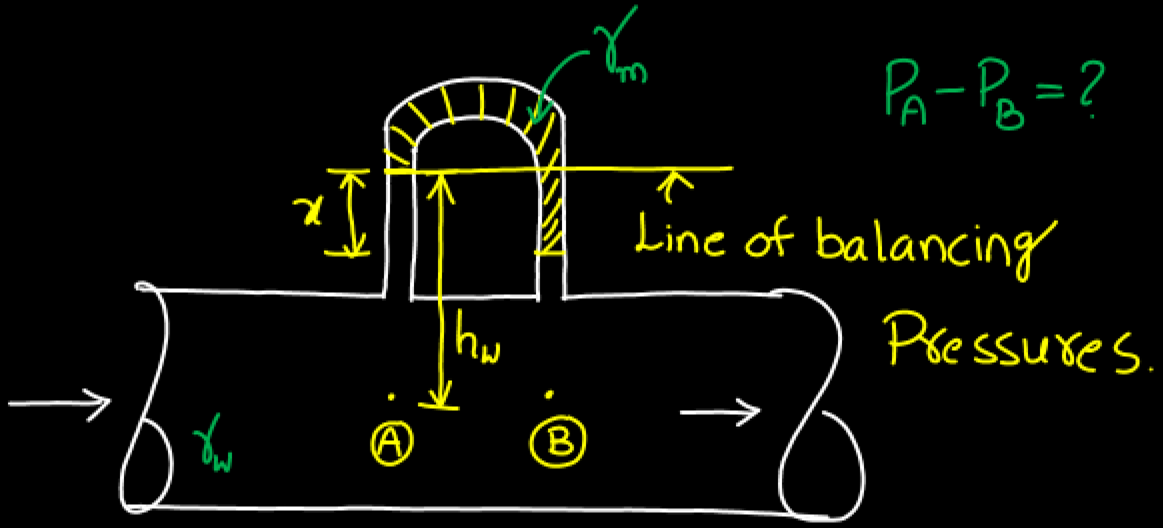
Pressure in terms of head of working fluid

$$\frac{\Delta P}{\gamma_w} = \frac{P_A - P_B}{\gamma_w} = H_w = x\left[\frac{\gamma_m}{\gamma_w} - 1\right]$$

$$\boxed{H_w = x\left[\frac{S_m}{S_w} - 1\right]}$$

x = differential level of Manometric fluid.

Inverted U-Tube:



$P_A - P_B = ?$

$\gamma_m < \gamma_w$

$$P_A - \gamma_w h_w = P_B - \gamma_w [h_w - x] - \gamma_m x$$

$$P_A - P_B = \gamma_w x - \gamma_m x$$

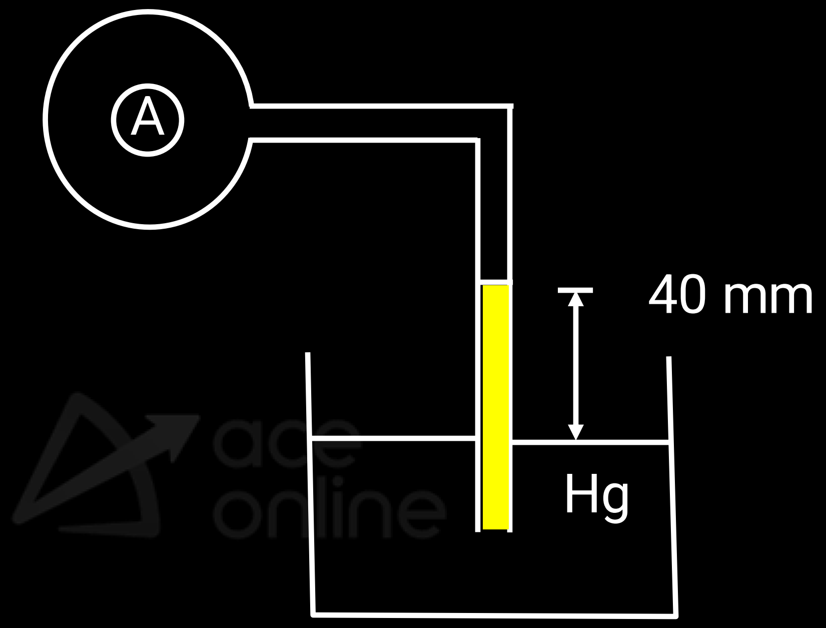
$$\Delta P = P_A - P_B = x[\gamma_w - \gamma_m]$$

Pressure in terms of head of working fluid.

$$H_w = \frac{P_A - P_B}{\gamma_w} = x\left[1 - \frac{\gamma_m}{\gamma_w}\right]$$

$$\left\{ H_w = x\left[1 - \frac{S_m}{S_w}\right] \right\}$$

Q. Determine the pressure at A ?



$$(P_A)_{abs} = P_{atm} - [P_A]_g$$

$$= 760 \text{ mm of Hg} - 40 \text{ mm of Hg}$$

$$= 720 \text{ mm of Hg}$$

(a) 40 mm of Hg

(b) 40 mm of Hg(vacuum)

(c) 720 mm of Hg (abs)

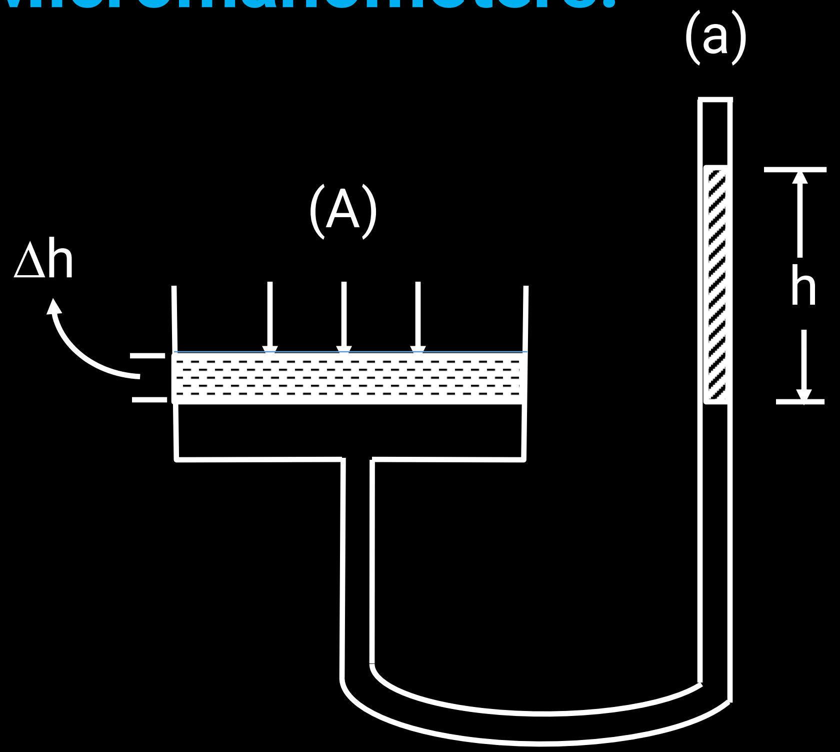
(d) 40 mm of gas (vacuum)

Inclined leg manometer:

The manometer with one limb inclined w.r.t vertical is known as inclined leg manometer.

- It increases the sensitivity of the pressure measurement
- Sensitivity : it is the ability to measure small pressure

Micromanometers:



Volume fall = Volume rise

$$A(\Delta h) = a \cdot h$$

$$\frac{\Delta h}{h} = \frac{a}{A}$$

$$\frac{\Delta h}{h} \times 100 = \frac{a}{A} \times 100$$

- The change in level of small limb is generally considered as the change in pressure.
- The error in the above measurement is given by:

$$e = \frac{a}{A} \times 100$$

Pressure gauges:

Bourdon pressure gauge:

It is the mechanical gauge used for measuring high pressures where high precision is not required.

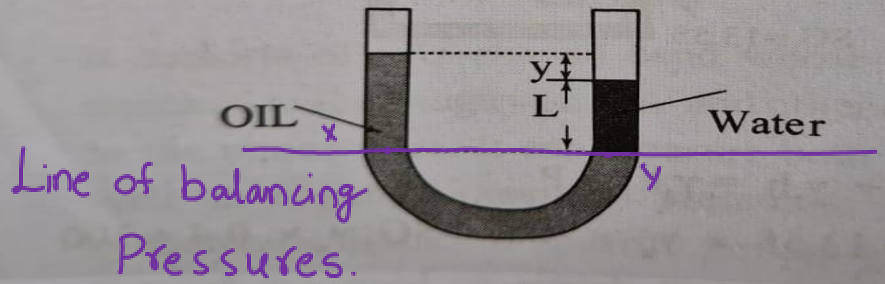
Ex: Automobile tyre pressure.

Aneroid barometer:

It is used to measure atmospheric pressure on gauge scale.

Example: 2.10

The U-tube in figure below contains two liquids in static equilibrium: water is in the right arm, and oil of unknown specific gravity (S_o) is in the left. Measurement gives L = 13.5cm and y = ~~1.55 mm~~ 1.5cm. The specific gravity of the oil is 0.9.



Sol: $\rho_{oil} g (y + L) = \rho_w g L$

below. T

meters of

C

Sol: $P_B - 1$

$$S_o = \frac{\rho_o}{\rho_w}$$

$$P_x = P_y$$

$$\gamma_o (L+y) = \gamma_w L$$

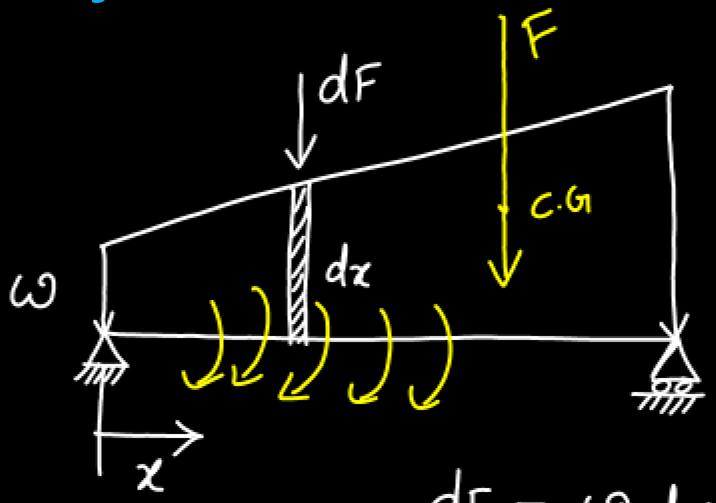
$$\rho_o g (L+y) = \rho_w g L$$

$$\rho_o = \left[\frac{L}{L+y}\right] \rho_w$$

$$S_o \rho_w = \left[\frac{L}{L+y}\right] \rho_w \Rightarrow S_o = \frac{L}{L+y} = \frac{13.5}{13.5+1.5} = \frac{135}{150}$$

$$= 0.9$$

Hydrostatics :



$$dF = \omega \cdot dx$$

$$F = \int_0^L \omega \cdot dx$$

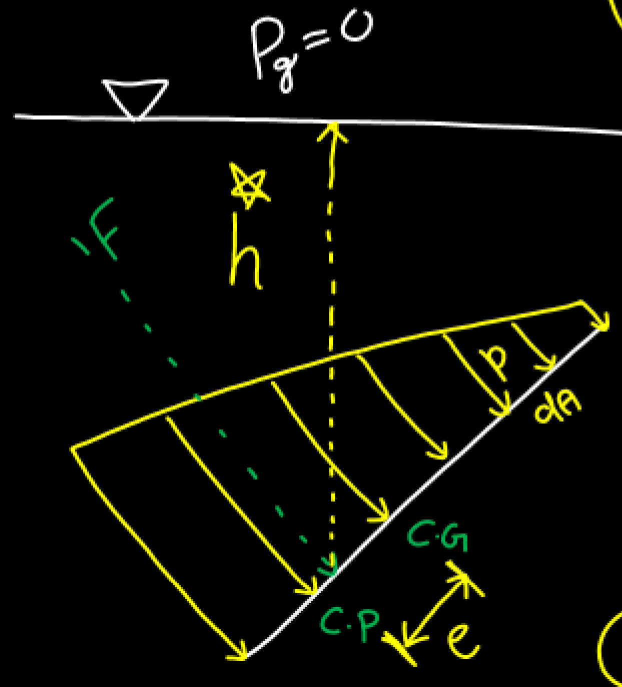
$$\Sigma\, dM = M_R$$

$$= F\bar{x} \qquad = \text{Area under loading curve.}$$

Beam : Length defined

$$\omega = \frac{W}{L}$$

"Varigonon's theorm"



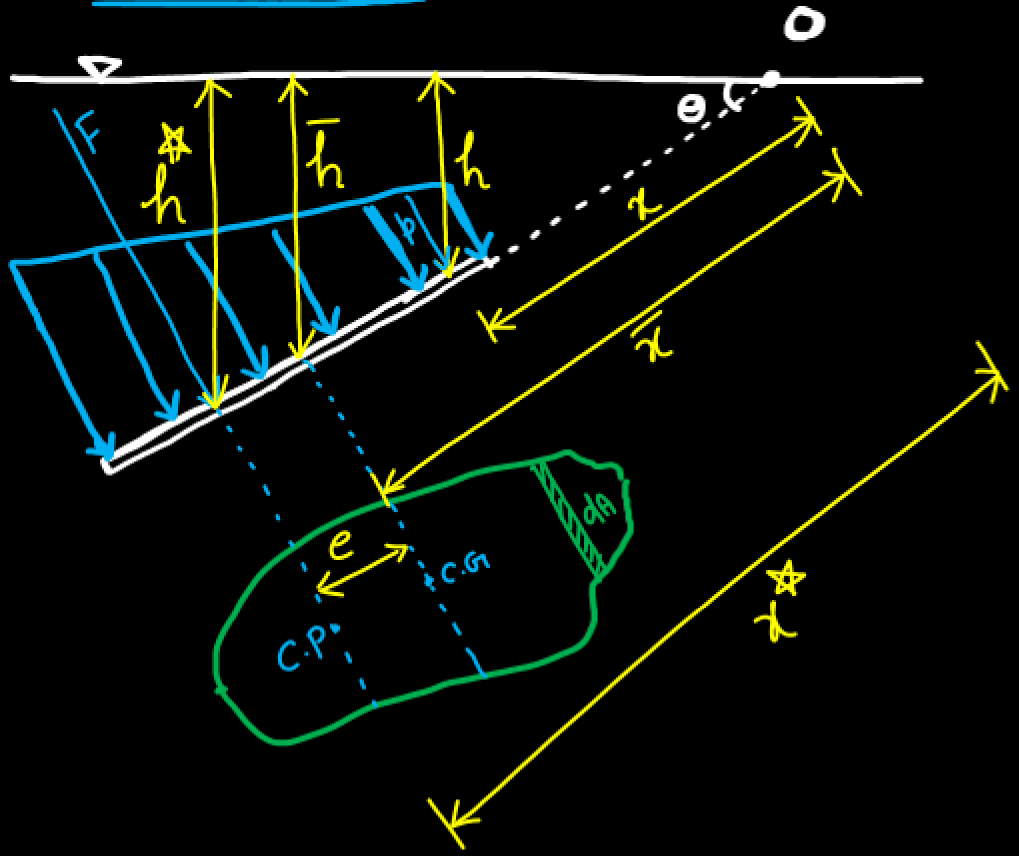
$$dF = P\,dA$$

1. $F = \int_A P\,dA$
2. h^*
3. e

Plate : Area defined

$$P : \frac{F}{A}$$

Flat plate:



① Force, F:

$$dF = P \cdot dA$$
$$= \gamma h \, dA$$
$$dF = \gamma x \sin\theta \, dA$$
$$F = \gamma \sin\theta \int_A x \, dA$$

Moments of area:

$$\int_A x \, dA = A \cdot \bar{x}$$

$$\therefore \; F = \gamma \sin\theta \, A \bar{x}$$

$$\boxed{F = \gamma A \bar{h}}$$

(2) Center of pressure h^*:

$$dM_o = dF \cdot x$$
$$= [\gamma x \sin\theta . dA]\, x$$
$$= \gamma \sin\theta\, x^2 dA$$

$$M_o = \gamma \sin\theta \int_A x^2 dA$$

Second Moments of area:

$$\int_A x^2 dA = I_{y-o}$$

As per Parallel axis theorm,

$$I_{y-o} = I_G + A\bar{x}^2$$

$$M_o = \gamma \sin\theta \left[I_G + A\bar{x}^2 \right] \longrightarrow (1)$$

$$M_o = F\, x^*$$
$$= \gamma A \bar{h}\, x^*$$
$$M_o = \gamma A \bar{x}\, x^* \sin\theta \longrightarrow (2)$$

$$(1) = (2)$$

$$\gamma \sin\theta \left[I_G + A\bar{x}^2 \right] = \gamma A \bar{x}\, x^* \sin\theta$$

$$x^* = \bar{x} + \frac{I_G}{A\bar{x}}$$

$$\frac{h^{*}}{\sin\theta} = \frac{\bar{h}}{\sin\theta} + \frac{I_G \sin\theta}{A\bar{h}}$$

$$\boxed{h^{*} = \bar{h} + \frac{I_G \sin^2\theta}{A\bar{h}}}$$

③ e:

$$e = x^{*} - \bar{x}$$

$$= \frac{h^{*} - \bar{h}}{\sin\theta}$$

$$= \frac{1}{\sin\theta}\left[\frac{I_G \sin^2\theta}{A\bar{h}}\right]$$

$$\boxed{e = \left[\frac{I_G \sin\theta}{A\bar{h}}\right]}$$

① $F = \gamma A\bar{h}$

② $h^{*} = \bar{h} + \dfrac{I_G \sin\theta}{A\bar{h}}$

③ $e = \dfrac{I_G \sin\theta}{A\bar{h}}$

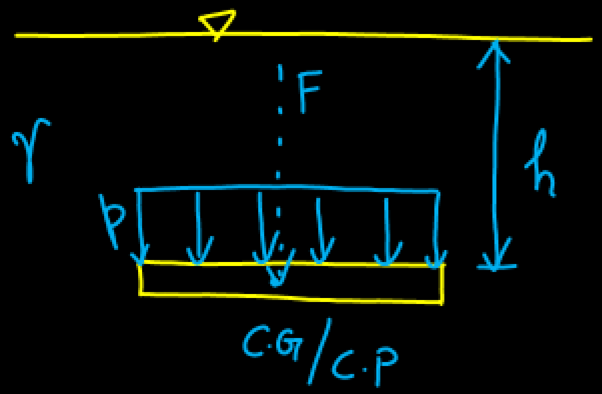
Where $e = 0$:

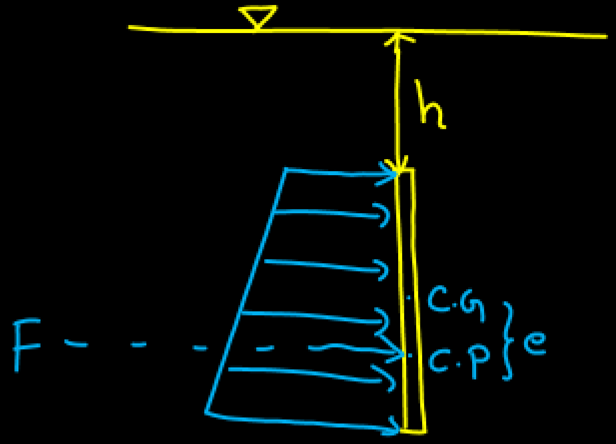
① If $\sin\theta = 0 \Rightarrow \theta = 0$

Horizontal Plate

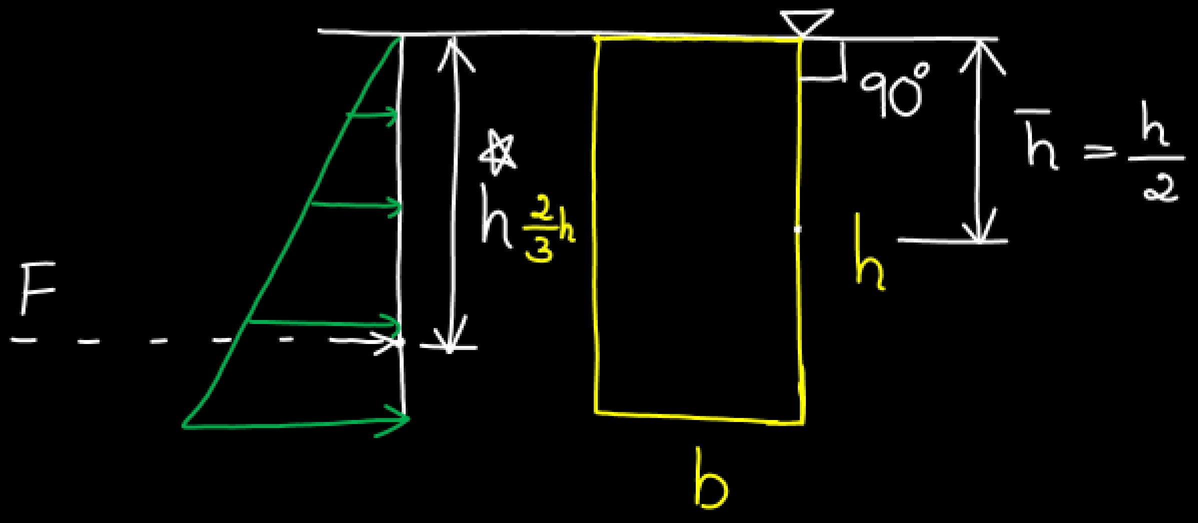
② If $A\bar{h} >> I_G \sin\theta$

$e \rightarrow 0$





Rectangular plate:

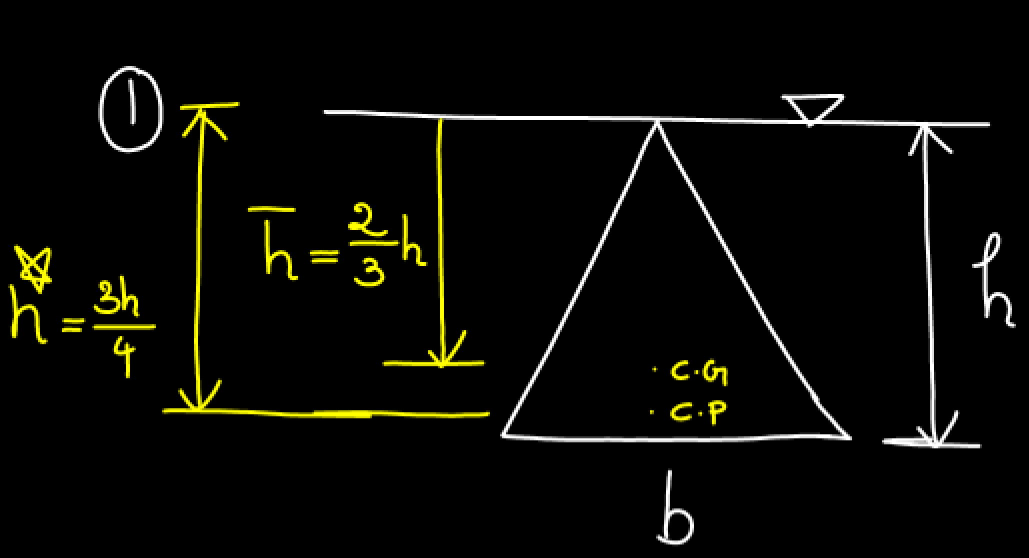


$$h^{*} = \bar{h} + \frac{I_G}{A\bar{h}} \quad [\sin 90^\circ = 1]$$

$$= \frac{h}{2} + \frac{bh^3[2]}{12[bh][h]}$$

$$= \frac{h}{2} + \frac{h}{6} = \frac{4h}{6} = \frac{2}{3}h$$

Triangular plate:



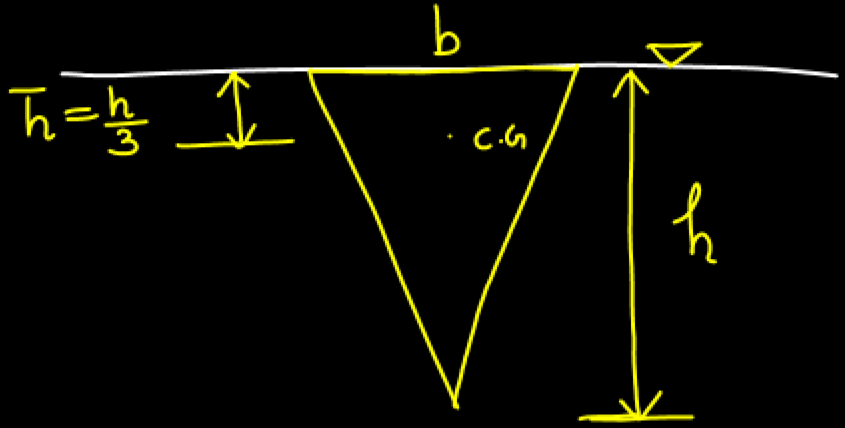
$$h^* = \bar{h} + \frac{I_G}{A\bar{h}}$$

$$= \frac{2}{3}h + \frac{bh^3}{36\left[\frac{1}{2}bh\right]\left[\frac{2}{3}h\right]}$$

$$= \frac{2h}{3} + \frac{h}{12}$$

$$= \frac{8h+h}{12} = \frac{9h}{12} = \frac{3h}{4}$$

2



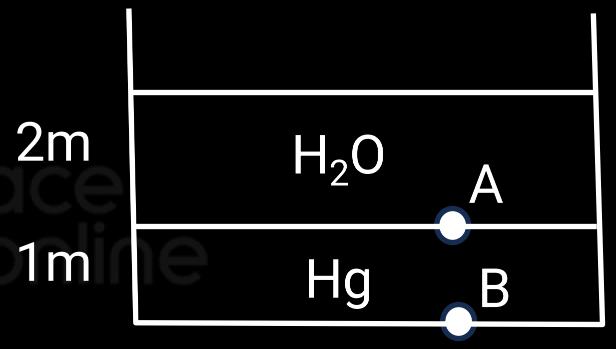
$$h = \frac{h}{3} + \frac{bh^3}{36\left[\frac{1}{2}bh\right]\frac{h}{3}}$$

$$= \frac{h}{3} + \frac{h}{6} = \frac{h}{2}$$

Q. Calculate the pressures at A and B In terms of

(i) KPa

(ii) In terms of m of water



(i) $P_A = \gamma h$

$= 9.81 \times 2 = 19.62 \, KN/m^2$

$P_B = \gamma_w h_w + \gamma_m h_m$

$= 9.81 \times 2 + S\gamma_w h_m$

$= 19.62 + 13.6 \times 9.81 \times 1$

$= 153.036 \, KN/m^2$

(ii) $P_A = 2m$ of Water

$$P_A = 19.62 = \gamma_m h_m$$

$$= S\gamma_w h_m$$

$$2 \ \cancel{19.62} = 13.6 \times \cancel{9.81} \times h_m$$

$$h_m = \frac{2}{13.6} = 0.1470\,m = 14.7\,cm \text{ of Hg}$$

$$\boxed{S_1 h_1 = S_2 h_2}$$

$$13.6 \times 1 = 1(h_w)$$

$$P_B = 2 \times 9.81 + 1 \times 13.6 \times 9.81 = 153.036\,kN/m^2$$

$$P_B = \gamma_w h_w$$

$$153.036 = 9.81 \times h_w \implies h_w = 15.6\,m \text{ of Water.}$$

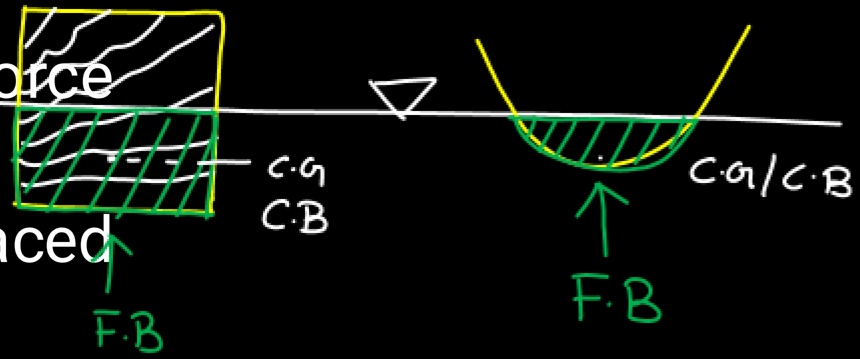
Buoyancy :

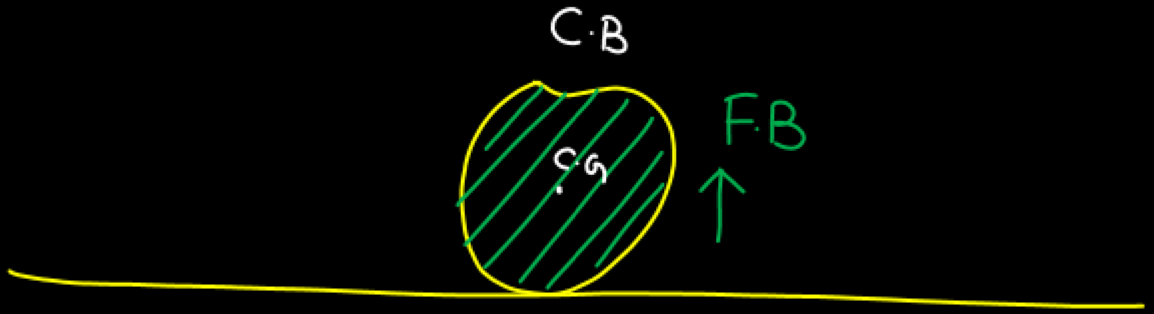
Definition:

If a body is submerged in a fluid either fully or partially it is subjected to an upward force which tends to lift it up. This tendency for a submerged body to be lifted up in the fluid, due to an upward force is known as Buoyancy.

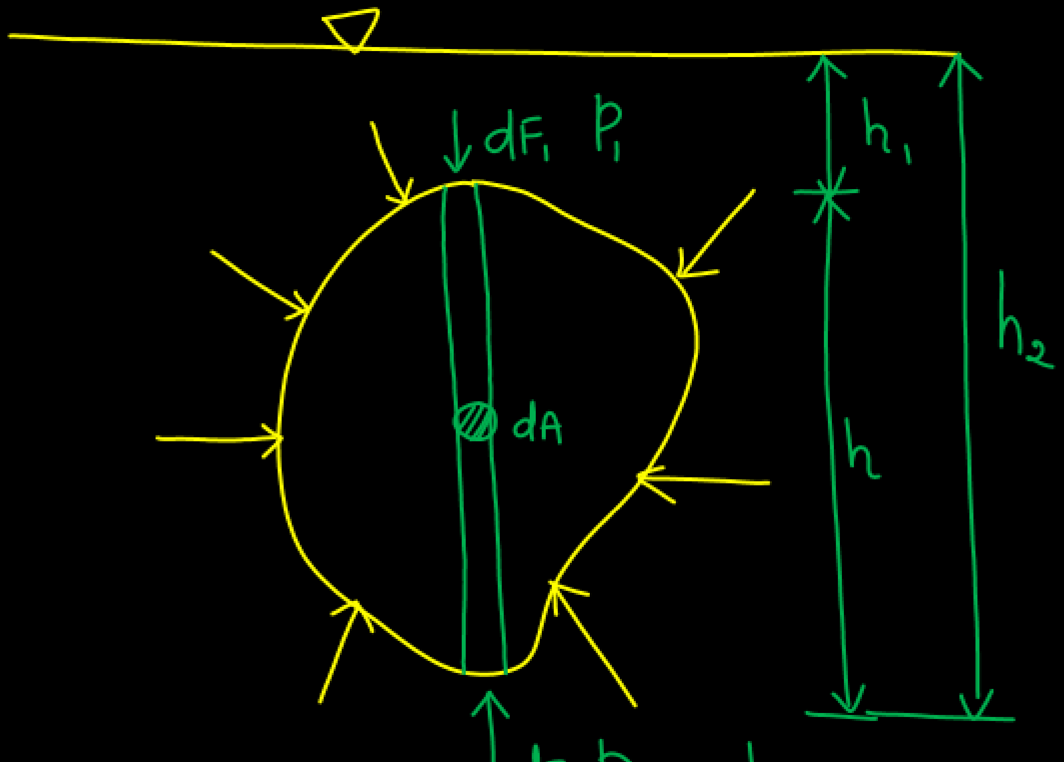
- The apparent loose of weight is called buoyancy.

- Buoyancy force is equal to the weight of the fluid displaced ← **Archimedes principle**
- It will act upwards
- The point of application of resultant buoyancy force is called center of buoyancy
- C.B will be at the C.G of the volume of displaced fluid.





Proof of Archimedes principle:



$$\sum F_H = 0$$

$$\sum F_V = ?$$

$$h_2 - h_1 = h$$

$$
\begin{aligned}
dF &= \overrightarrow{dF_1} + \overrightarrow{dF_2} \\
&= -dF_1 + dF_2 \\
&= -P_1\,dA + P_2\,dA \\
&= -\gamma h_1\,dA + \gamma h_2\,dA \\
&= \gamma\,dA\,[h_2 - h_1] \\
&= \gamma\,[dA\,h] \\
&= \gamma\,d\forall
\end{aligned}
$$

$$dF = dW$$

$$F = W$$

Chapter 4
Fluid dynamics

Energy Eqn. & Applications

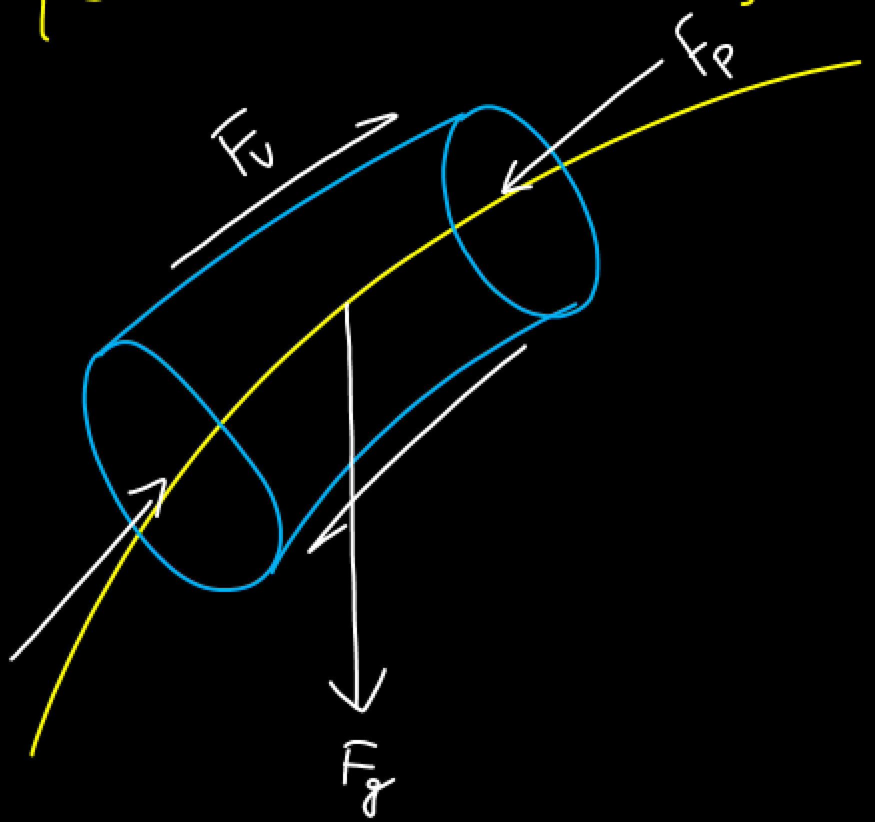
→ Bernoulli's Eqn.

Momentum Eqn. & Applications.

Bernoulli's Eqn:

It is based on Newton's 2^{nd} Law

$$\sum F_s = m a_s$$ [S: Streamline direction]



① F_p = Pressure force

② F_g = gravitational force.

③ F_v = viscous force

④ F_t = Turbulant force

⑤ F_s = Surface tension force.

⑥ F_c = Compressible force.

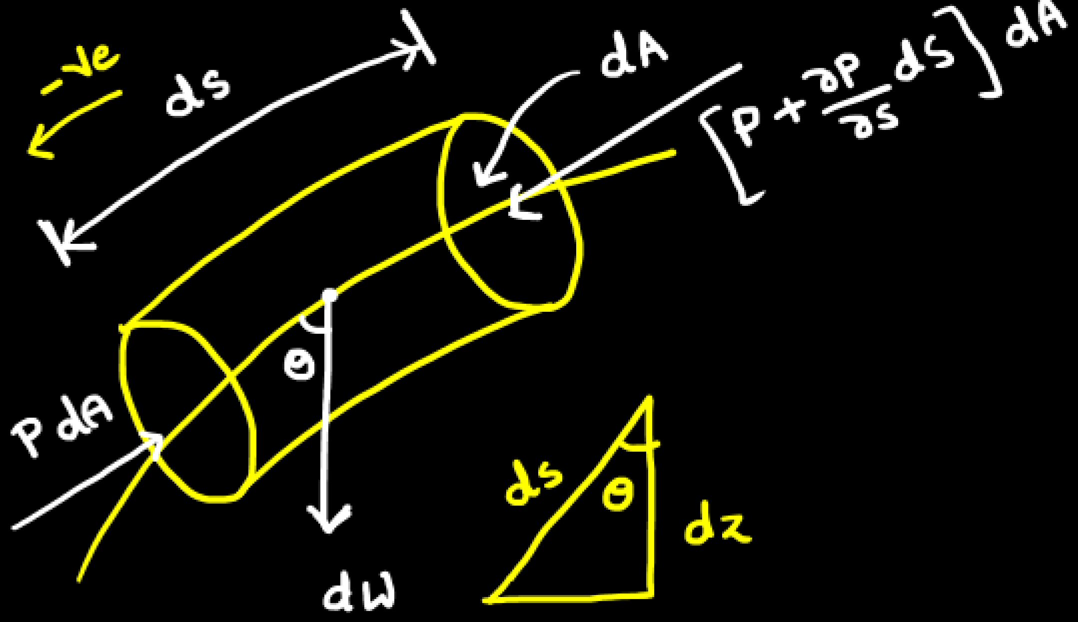
$$F_p + F_g + F_v + F_t + F_s + F_c = m a_s \longrightarrow \text{Newton's eqn}$$

$$F_p + F_g + F_v + F_t = m a_s \longrightarrow \text{Reynold's eqn}$$

$$F_p + F_g + F_v = m a_s \longrightarrow \text{Navier-Stokes eqn}$$

$$F_p + F_g = m a_s \longrightarrow \text{Euler's eqn}$$

Proof of Bernoulli's equation:



$$\cos\theta = \frac{dz}{ds}$$

$$\Sigma F = m . a_s$$

$$PdA - dW\cos\theta - \left(P+\frac{\partial p}{\partial s}ds\right)dA = dm . a_s$$

$$-dW\cos\theta - \frac{\partial p}{\partial s}ds . dA = dm . a_s$$

$$-dW\frac{dz}{ds} - \frac{\partial p}{\partial s} . d\forall = dm . a_s$$

$$\therefore a_s = \frac{\partial V_s}{\partial t} + V_s\frac{\partial V_s}{\partial s}$$

For a steady flow: $\frac{\partial V_s}{\partial t} = 0$

$$a_s = V_s\frac{dV_s}{ds}$$

$$-dW \frac{dz}{ds} - \frac{dp}{ds} d\forall = dm \cdot V_s \frac{dV_s}{ds}$$

$$-\gamma d\forall \frac{dz}{ds} - \frac{dp}{ds} d\forall = \rho \, d\forall \, V \frac{dV}{ds}$$

$$-\gamma dz - dp = \rho V dV$$

$$dp + \rho V dV + \gamma dz = 0$$

$$\left\{ \frac{dp}{\rho} + V dV + g dz = 0 \right\}$$ Euler's equation

$$\int \frac{dp}{\rho} + V dV + g dz = \text{constant}$$

$$\left\{ \frac{P}{\rho} + \frac{V^2}{2} + g z = H \right\}$$ Bernoulli's equation

$$\frac{P}{\rho g} + \frac{v^2}{2g} + z = H$$

Various forms of Bernoulli's equation:

1. Energy / unit mass
2. Energy / unit volume
3. Energy / unit weight

① $$\frac{P}{\rho} + \frac{V^2}{2} + gz = \text{Constant.}$$

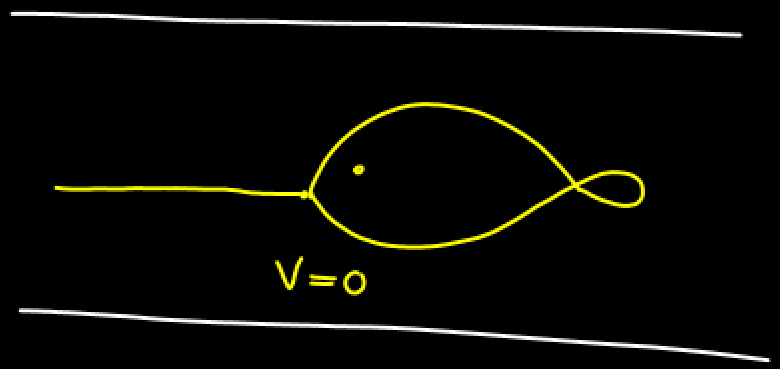
② $$P + \frac{\rho V^2}{2} + \rho g z = \text{constant}$$

P : Static Pressure

$\frac{\rho V^2}{2}$: Dynamic pressure.

Rise in pressure due to drop in K.E

$\left\{ P + \frac{\rho V^2}{2} \right.$: Stagnation pressure. $\left. \right\}$



$\rho g z$: Hydrostatic pressure.

Rise in pressure due to drop in P.E

$P + \rho g z$: Piezometric Pressure.

③ $$\frac{P}{\rho g} + \frac{V^2}{2g} + z = H$$

$\frac{P}{\rho g}$: Static / Pressure head

$\frac{V^2}{2g}$: Velocity head / Dynamic head

z : Datum head.

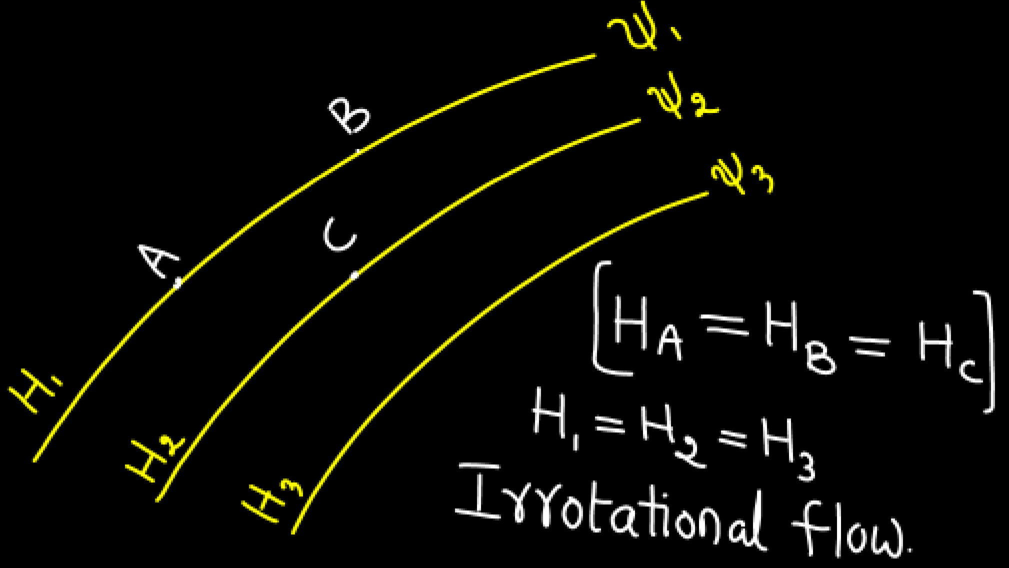
$$\frac{P}{\gamma} + \frac{V^2}{2g} \quad : \quad \text{stagnation head}$$

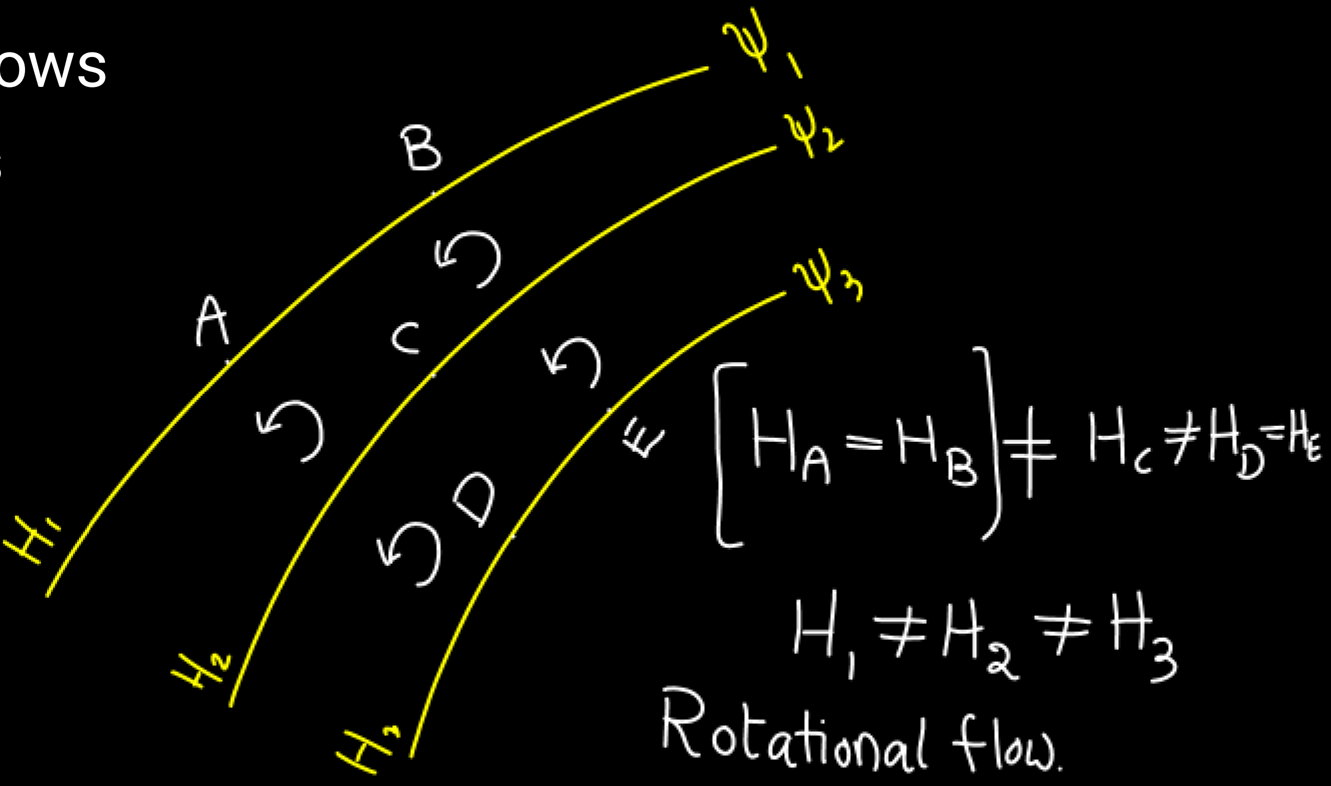
$$\frac{P}{\gamma} + z \quad : \quad \text{Piezometric head.}$$

Limitations & Assumptions in Bernoulli's equation:

1. Flow is steady
2. Flow is incompressible
3. Heat transfer effects are beyond the scope of Bernoulli's equation
4. Bernoulli's equation is valid:

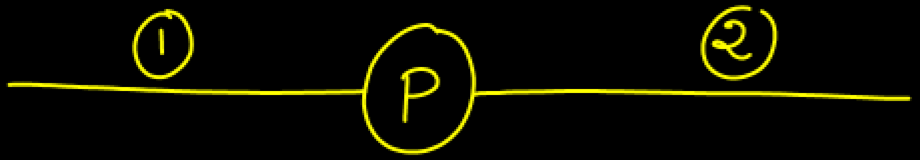
- Across a streamlines for irrotational flows
- Along a streamline for rotational flows





5. Involvement of hydraulic machines

Pump:



$$H_1 < H_2$$

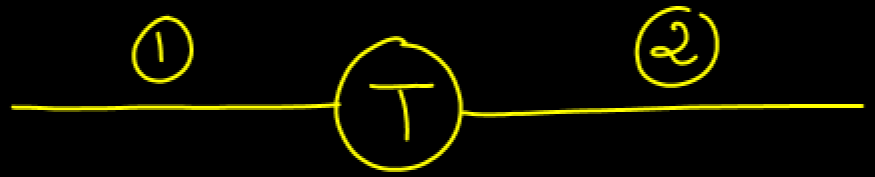
$$H_1 = H_2 - H_p$$

$$\left(\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1\right) = \left(\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2\right) - H_p$$

$$\Delta H = H_p$$

Energy gained by the fluid from Pump per unit wt.

Turbine:



$$H_1 > H_2$$

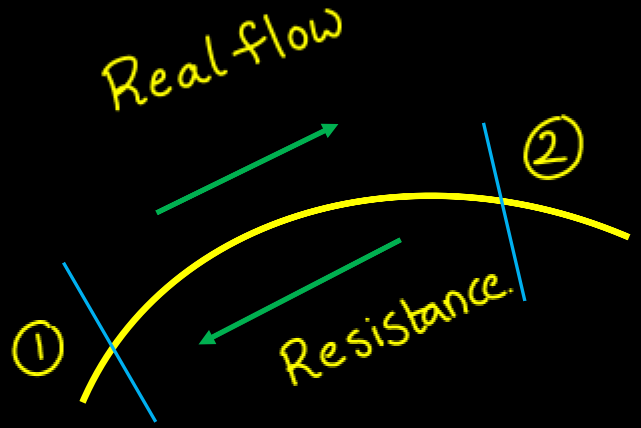
$$\Delta H = H_T$$

Energy given by the fluid to turbine.

$$H_1 - H_T = H_2$$

$$\left(\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1\right) - H_T = \left(\frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2\right)$$

6. Valid only for ideal flow



$$H_1 > H_2$$

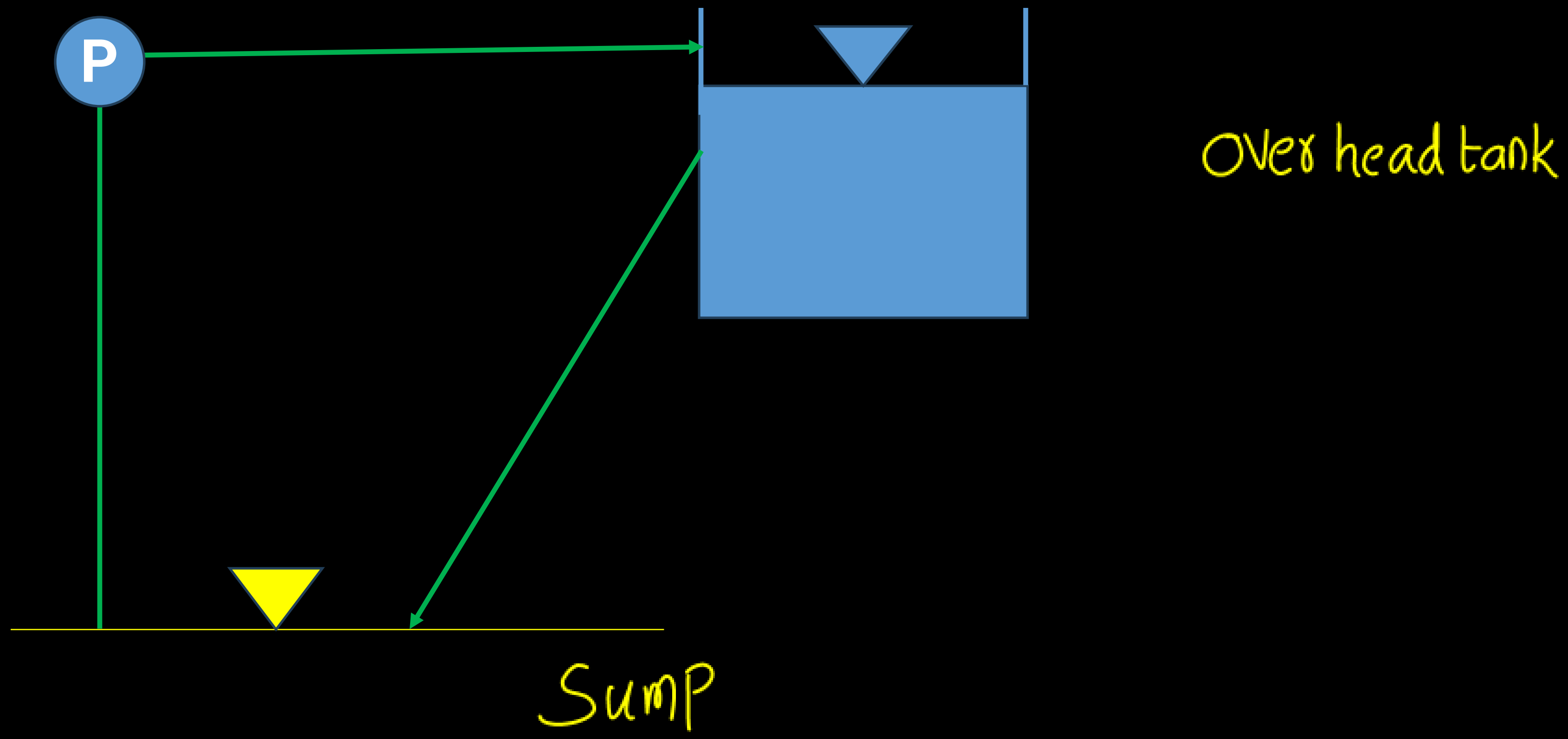
$$H_1 - h = H_2$$

$$\boxed{H_1 = H_2 + h}$$

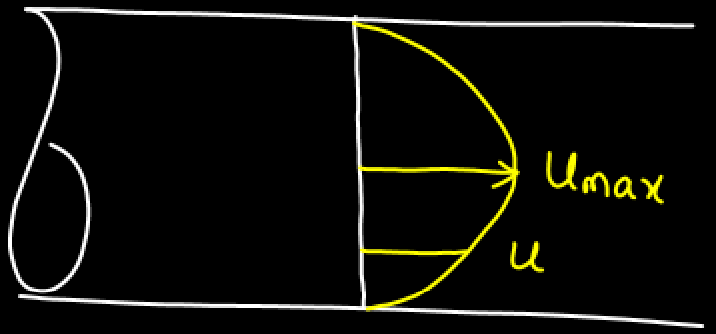
$$\Delta H = H_1 - H_2 = h$$

Energy given to the fluid to overcome resistance to flow.

Note : in the absence of a pump real flow takes place from higher total head to lower total head.

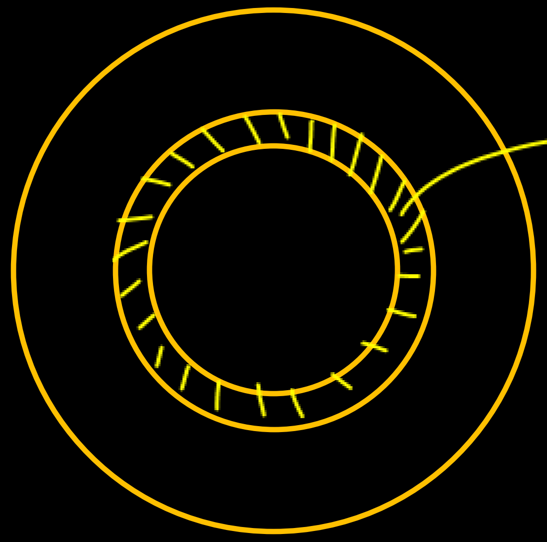


Kinetic energy correction factor: (α)



$$\alpha \frac{V^2}{2g}$$

V : Avg. velocity of flow.



$$d(\dot{K.E}) = \frac{1}{2}\dot{m}u^2$$

$$= \frac{1}{2}(\rho\, dA\, u)\cdot u^2$$

$$\int d(\dot{K.E}) = \frac{1}{2}\rho\int_A u^3\, dA$$

$$(\dot{K.E}) = \frac{1}{2}\rho\int_A u^3\, dA \longrightarrow (1)$$

$$(\dot{K.E}) = \alpha \frac{1}{2} \dot{m} V^2$$

$$= \alpha \frac{1}{2} (\rho A V) . V^2$$

$$(\dot{K.E}) = \alpha \frac{1}{2} \rho A V^3 \longrightarrow (2)$$

$$(1) = (2)$$

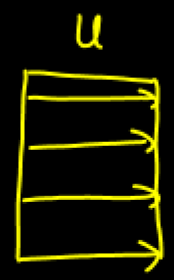
$$\cancel{\frac{1}{2}\rho} \int_A u^3 dA = \alpha \cancel{\frac{1}{2}\rho} A V^3$$

$$\left\{ \alpha = \frac{1}{AV^3} \int_A u^3 dA \right\}$$

$$\alpha = \frac{\frac{1}{A} \int_A u^3 dA}{V^3}$$

$$\{ \alpha \geq 1 \}$$

$\alpha = 1$ for uniform Velocity Profile



$\alpha = 2$ for Laminar flow through cir pipes.

$\alpha = 1.1$ to 1.2 for Turbulant flow through cir pipes.

Water power:

Rate of energy or work per unit time.

$$P = \frac{\text{Energy}}{\text{time}} = \frac{mgh}{t}$$

$$\left\{ W = mg \right\}$$

$$\left[\frac{m}{t} = \dot{m} \right]$$

$$\left[\frac{\text{Energy}}{\text{wt}} = \text{head} \right] \quad = \dot{m}gh$$

$$= (\rho Q)gh$$

$$\left\{ P = \rho g Q h \text{ (or) } \gamma Q h \right\}$$

Velocity measurement:

$$\frac{P}{\gamma} + \frac{V^2}{2g} + z = H$$

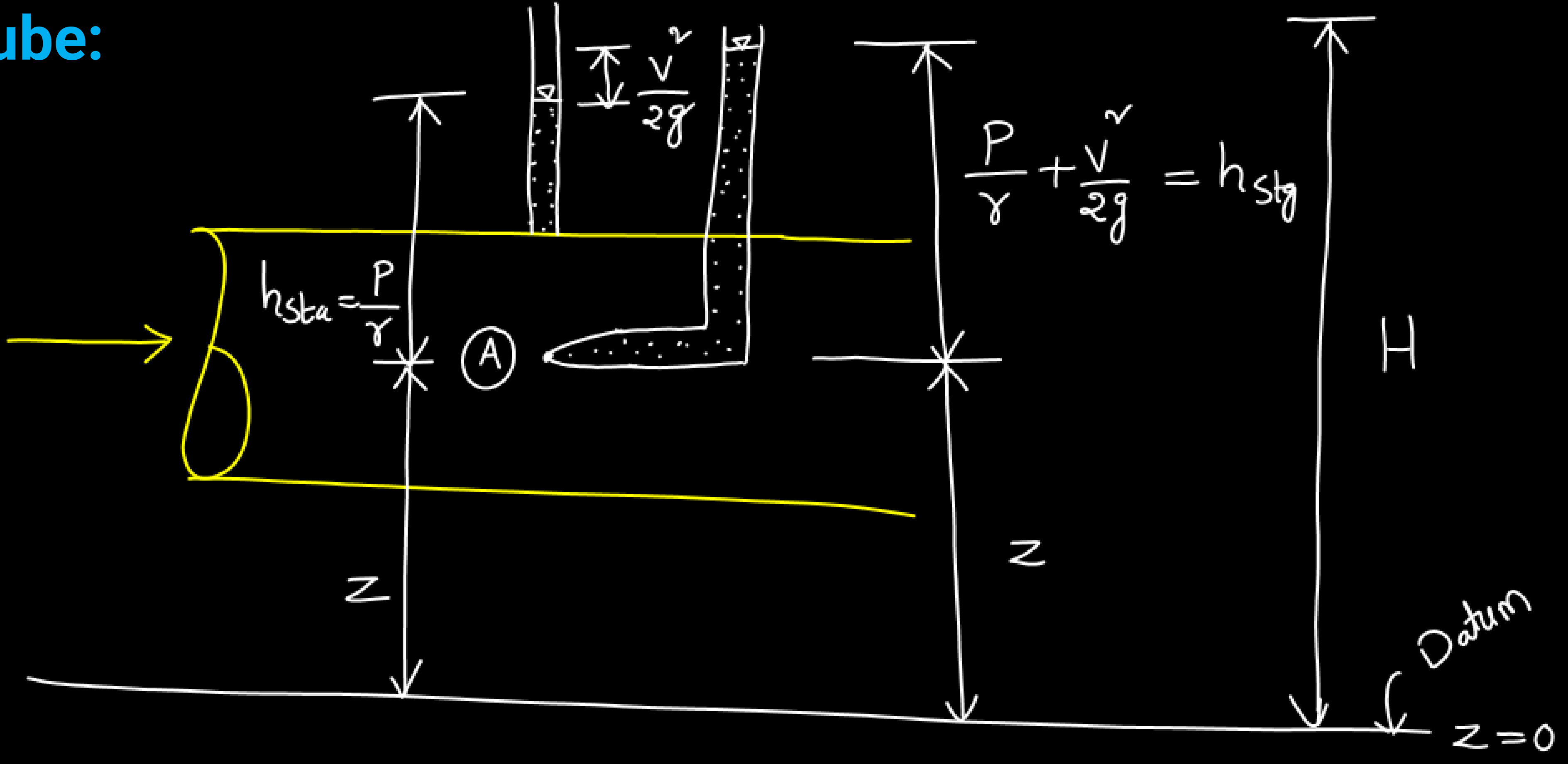
$$\frac{V^2}{2g} = H - \left(\frac{P}{\gamma} + z\right)$$

= Total Head − Piezometric head.

Total head : Pitot tube

Piezometric head : Piezometer.

Pitot tube:



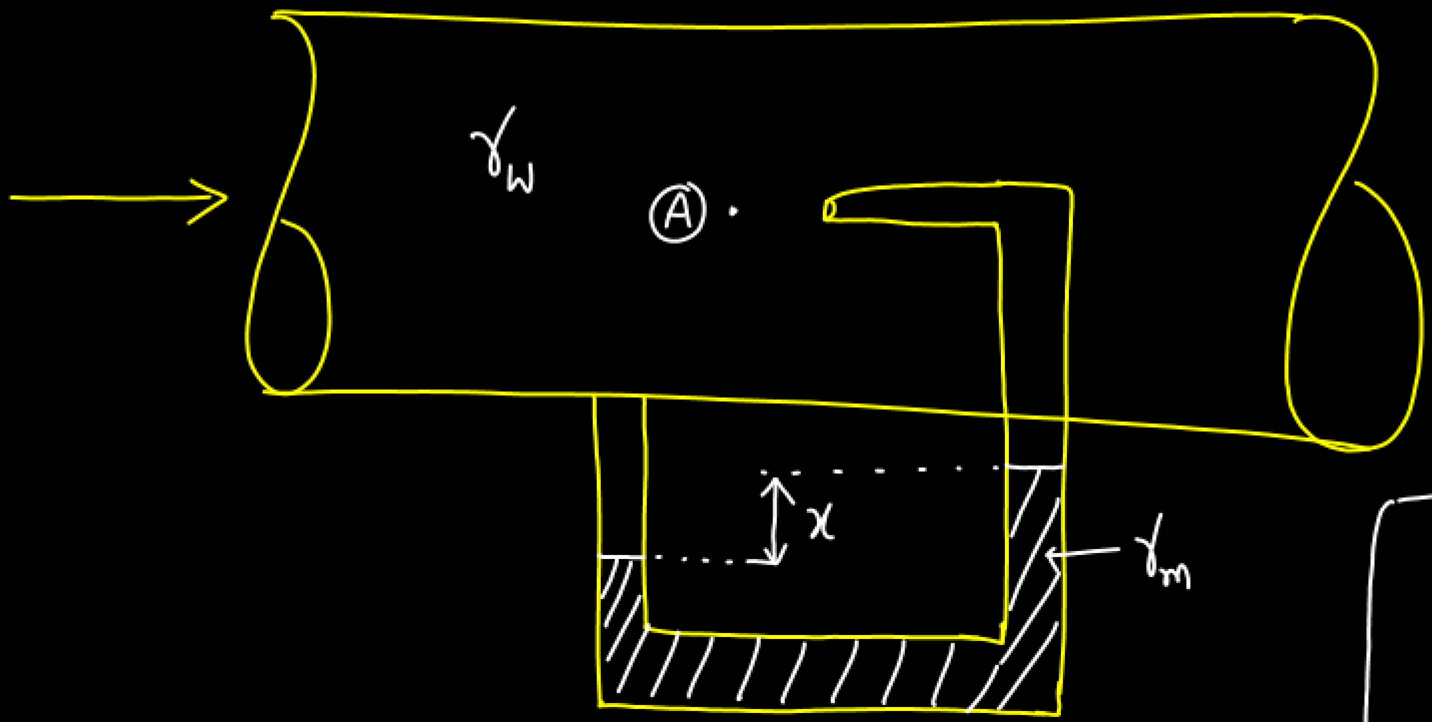
$$\frac{V^2}{2g} = \Delta h \implies V = \sqrt{2g\Delta h}$$

$$V_{th} = \sqrt{2g\,\Delta h}$$

$$V_a = C_v \sqrt{2g\Delta h}$$

$$C_v = \frac{V_a}{V_{th}}$$ Coefficient of velocity.

Pitot – static tube :



$$h = x\left[\frac{S_m}{S_w} - 1\right]$$

$$V = \sqrt{2gx\left[\frac{S_m}{S_w} - 1\right]}$$

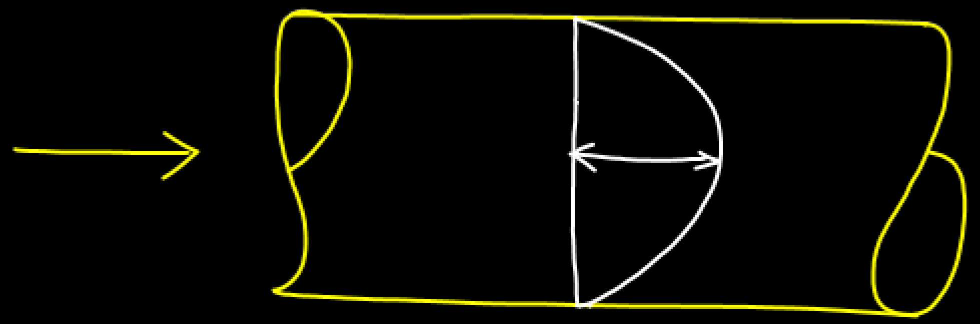
Gate CE

1. $V = \sqrt{2gh_m}$
2. $V = \sqrt{2gx\left(1 - \frac{S_m}{S_w}\right)}$
3. $V = \sqrt{2gx\left(\frac{S_m}{S_w}\right)}$
4. $V = \sqrt{2gx\left(\frac{S_w}{S_m} - 1\right)}$

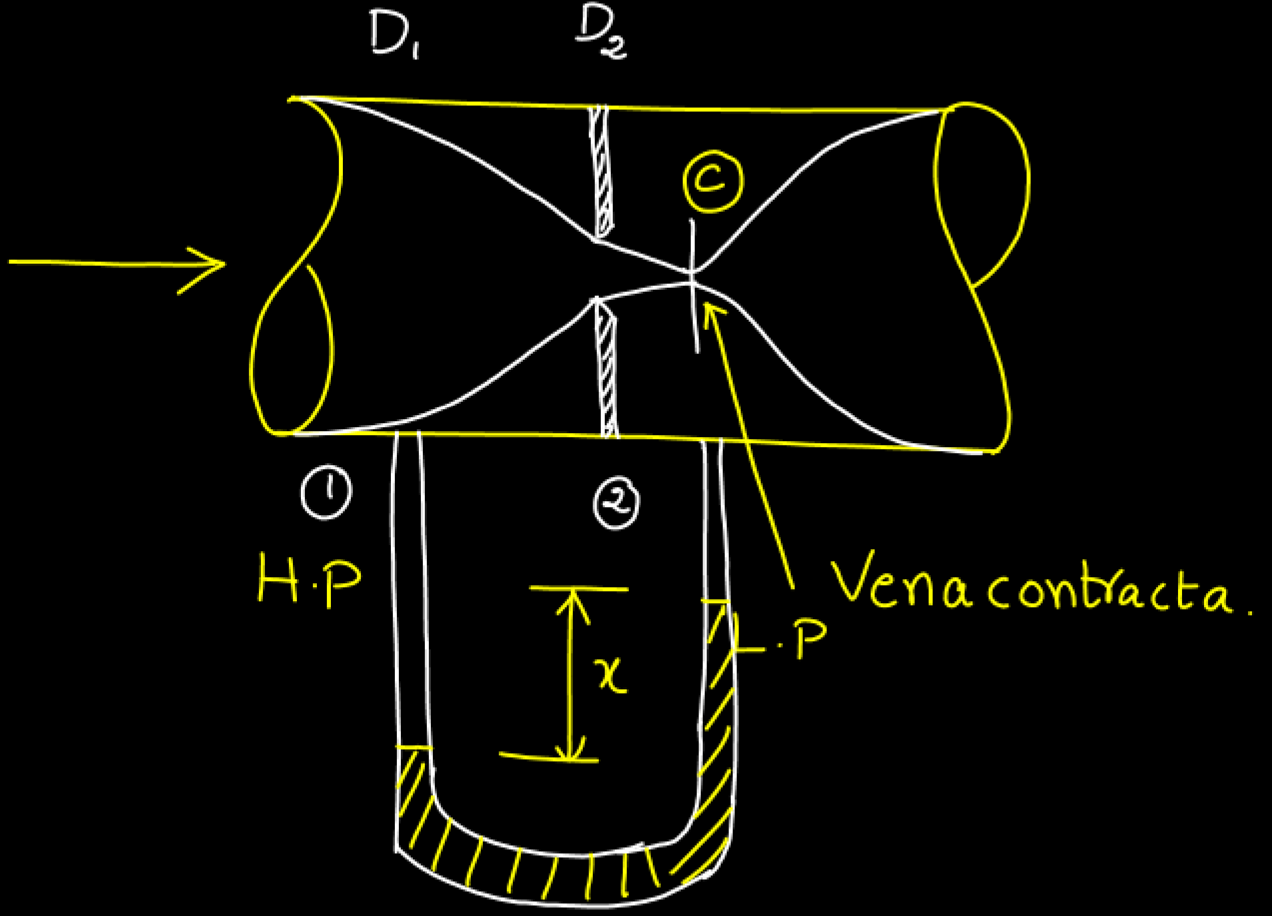
Discharge measurement:

$$Q = A \cdot V$$

$\downarrow$ Avg. velocity of flow.



Orifice meter:



$D_1 > D_2 > D_c$

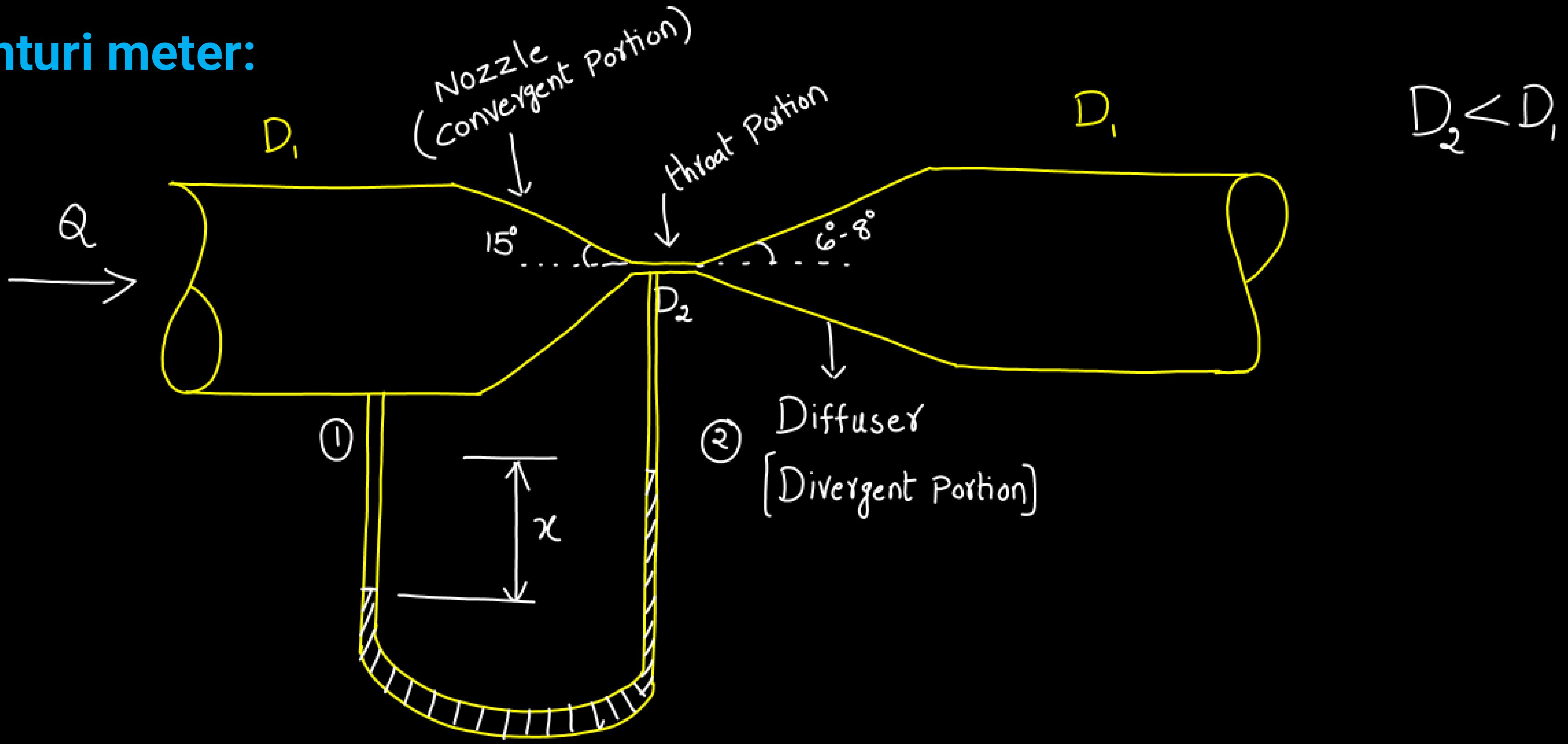
$A_1 > A_2 > A_c$

$\frac{P}{\gamma} + \frac{V^2}{2g} + Z = H$

$V_1 < V_2 < V_c$

$P_1 > P_2 > P_c$

Venturi meter:



$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + Z_2$$

$$\frac{V_2^2 - V_1^2}{2g} = \left(\frac{P_1}{\gamma} + Z_1\right) - \left(\frac{P_2}{\gamma} + Z_2\right)$$

$$= \Delta h$$

$$\left\{ Q = A \cdot V \right\}$$

$$V_2^2 - V_1^2 = 2g\Delta h$$

$$\frac{Q^2}{A_2^2} - \frac{Q^2}{A_1^2} = 2g\Delta h$$

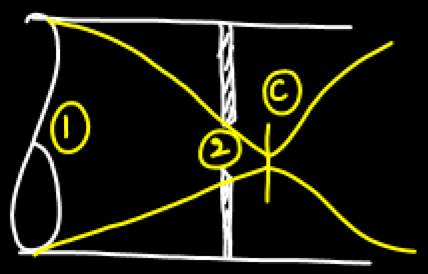
$$Q^2 \left\{ \frac{A_1^2 - A_2^2}{(A_1 A_1)^2} \right\} = 2g\Delta h$$

$$Q_{th} = \frac{A_1 A_2 \sqrt{2 g \Delta h}}{\sqrt{A_1^2 - A_2^2}}$$

$$\left\{ C_d = C_v \times C_c \right\}$$

C_d = coefficient of discharge

$$= \frac{Q_a}{Q_{th}}$$



$$\left\{ Q_a = C_d \frac{A_1 A_2 \sqrt{2 g \Delta h}}{\sqrt{A_1^2 - A_2^2}} \right\}$$

$$\left\{ C_c = \frac{A_c}{A_2} = \frac{V_2}{V_c} \right\}$$

$$Q_a = \frac{C_d A_1 A_2 \sqrt{2g\,\Delta h}}{\sqrt{A_1^2 - A_2^2}}$$

$$* \quad \sqrt{2g\,\Delta h} = \sqrt{2g\,x\left[\frac{S_m}{S_w} - 1\right]} = \sqrt{\frac{2\,\Delta P}{\rho}}$$

$$\Delta h = x\left[\frac{S_m}{S_w} - 1\right]$$

$$\Delta h = \frac{\Delta P}{\gamma}$$

$$\sqrt{2g\,\Delta h} = \sqrt{\frac{2\cancel{g}\,\Delta P}{\rho\cancel{g}}}$$

Key points about venturi meter:

d_1 = dia of main pipe

d_2 = throat dia.

- Designation of venturi meter: $d_1 \times d_2$
- Dia ratio, $\frac{d_2}{d_1}$ = 1/3 to 2/3

[Practically $\frac{d_2}{d_1} = \frac{1}{2}$]

- Length of divergent cone is more than convergent cone.

Head loss through Venturimeter:

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + z_2 + h_L$$

$$\left(\frac{P_1}{\gamma} + z_1\right) - \left(\frac{P_2}{\gamma} + z_2\right) - h_L = \frac{V_2^2 - V_1^2}{2g}$$

$[Q = A_1 V_1 = A_2 V_2]$

$$\Delta h - h_L = \frac{Q_a^2}{2g}\left[\frac{A_1^2 - A_2^2}{(A_1 A_2)^2}\right]$$

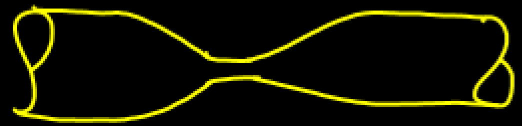
$$= \frac{C_d^2 \cancel{(A_1 A_2)^2}\, \cancel{2g}\, \Delta h}{\cancel{A_1^2 - A_2^2}} \frac{1}{\cancel{2g}} \frac{\cancel{(A_1^2 - A_2^2)}}{\cancel{(A_1 A_2)^2}}$$

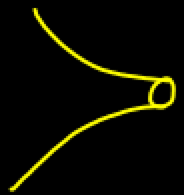
$$\Delta h - h_L = C_d^2 \Delta h$$

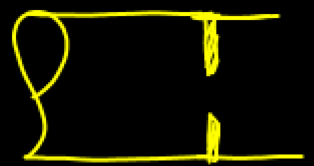
$$\boxed{h_L = \Delta h\left[1 - C_d^2\right]}$$

$$Q \propto C_d$$

Venturi meter   Nozzle meter   orifice meter







C_d : 0.95 - 0.98   0.85   0.65


When a physicist
plays basketball
GO!

VYAS EDIFICATION

Bro Again Breaking Physics Rules

Bernoulli Principal, Suction Pressure, Flow, Compression, Expansion etc

Chapter 5
Momentum equation

Momentum equation:

$$\vec{F} = m\vec{a}$$

$$= m\frac{d\vec{v}}{dt}$$

$$\vec{P} = mv$$

$$\vec{F} = \frac{d(\vec{mv})}{dt}$$

$\vec{F}$ = Rate of change of Linear momentum.

$$\vec{F}.dt = d(\vec{mv})$$

$\downarrow$

Impulse - momentum

$$F = ma$$

$$= m\frac{dV}{dt}$$

$$F = d(\dot{m}V)$$

$F =$ change in momentum flux

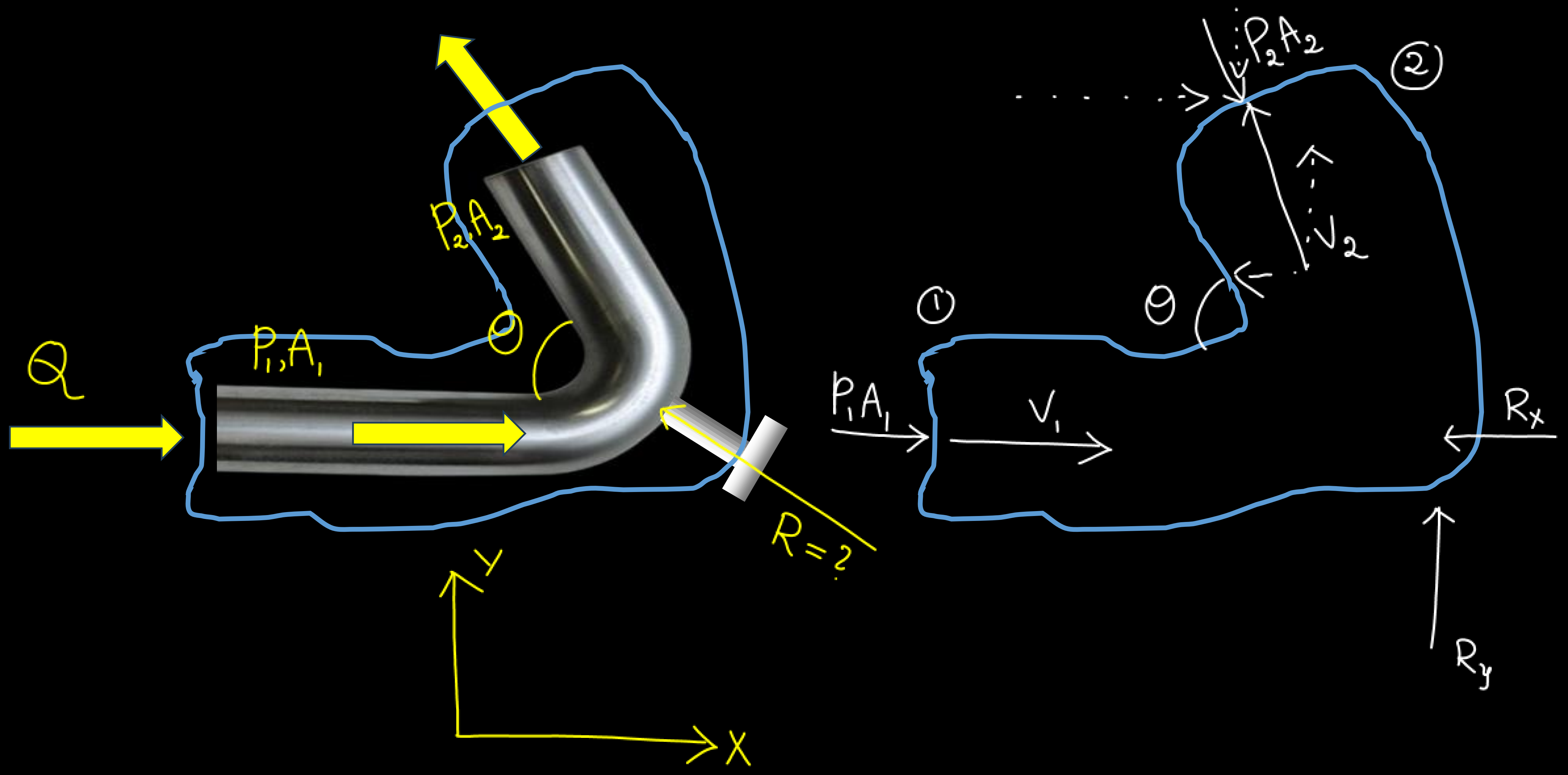
$$F = (\dot{m}V)_f - (\dot{m}V)_i$$

$$F_x = (\dot{m}V_x)_f - (\dot{m}V_x)_i$$

Control volume:

- ❑ Free body diagram kind of modelling used in fluid mechanics is known as control volume.
- ❑ To analyze the fluid flow certain volume is chosen
- ❑ There is a boundary of control volume known as control surface.

1. CV can be static or moving
2. CV can be of any shape and size
3. The velocities drawn are perpendicular to CS



Q
P_1, A_1
P_2, A_2
θ
$R = ?$
Y
X
$P_2 A_2$
②
①
θ
V_2
$P_1 A_1$
V_1
R_x
R_y

$$F = d(\dot{m}V)$$

$$F_x = d(\dot{m}V_x) = (\dot{m}V_x)_f - (\dot{m}V_x)_i$$

$$P_1A_1 - R_x + P_2A_2\cos\theta = \dot{m}(-V_2\cos\theta) - \dot{m}V_1$$

$$R_x = P_1A_1 + P_2A_2\cos\theta + \dot{m}\left(V_1 + V_2\cos\theta\right)$$

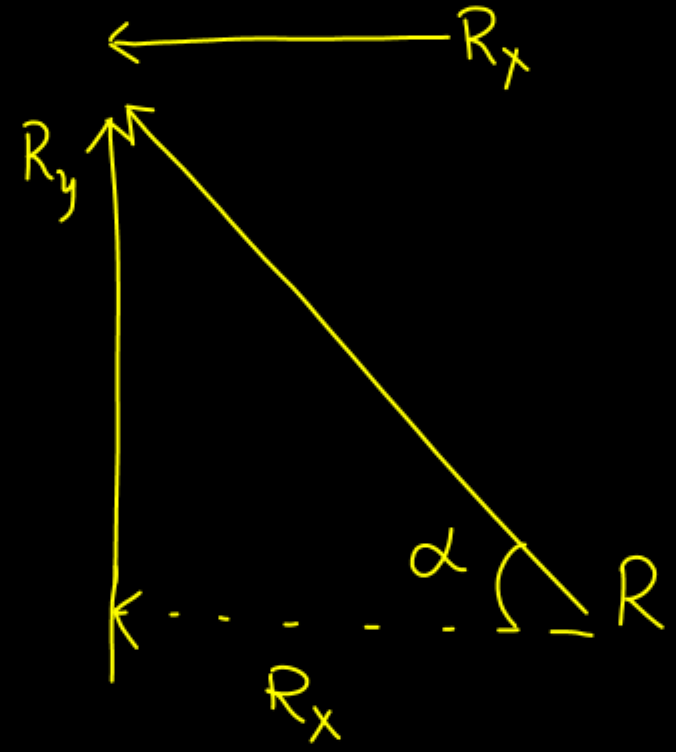
$$F_y = (\dot{m}V_y)_f - (\dot{m}V_y)_i$$

$$R_y - P_2A_2\sin\theta = \dot{m}V_2\sin\theta - \dot{m}\cancelto{0}{(0)}$$

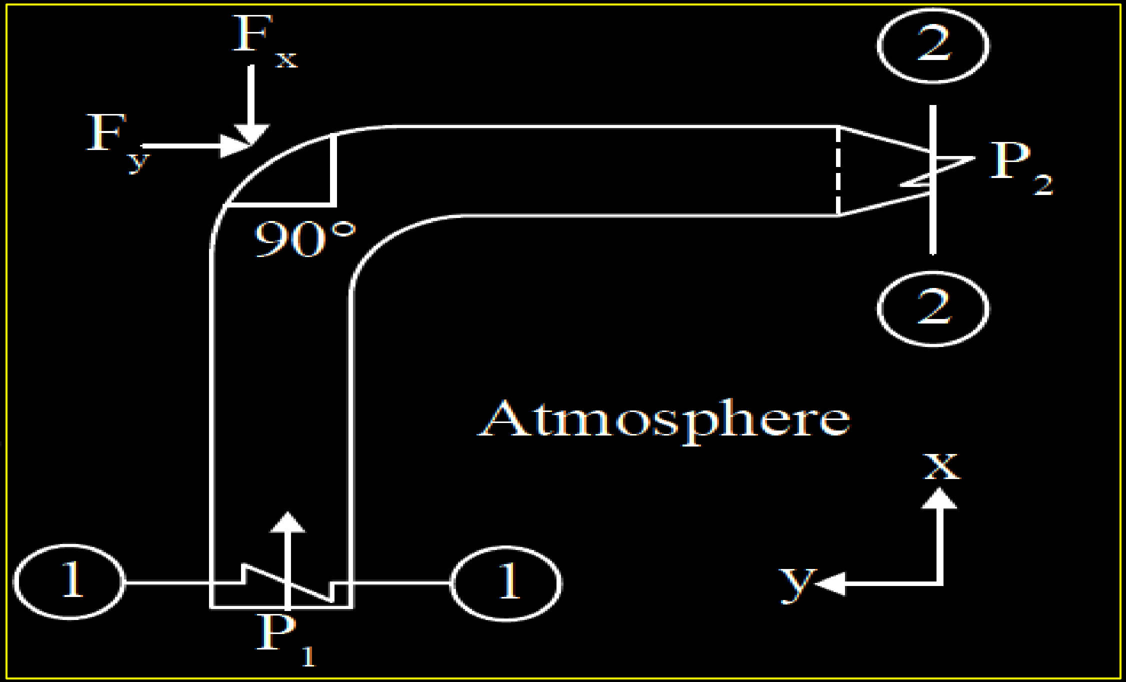
$$R_y = P_2A_2\sin\theta + \dot{m}V_2\sin\theta$$

$$R = \sqrt{R_x^2 + R_y^2}$$

$$\alpha = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$



Q. Water flows through a 90º bend in a horizontal plane as depicted in the figure



A pressure of 140 kPa is measured at section 1-1. The inlet diameter marked at a section 1 – 1 is $\frac{27}{\sqrt{\pi}}$ cm While the nozzle diameter marked at section 2-2 is $\frac{14}{\sqrt{\pi}}$ cm. Assume the following.

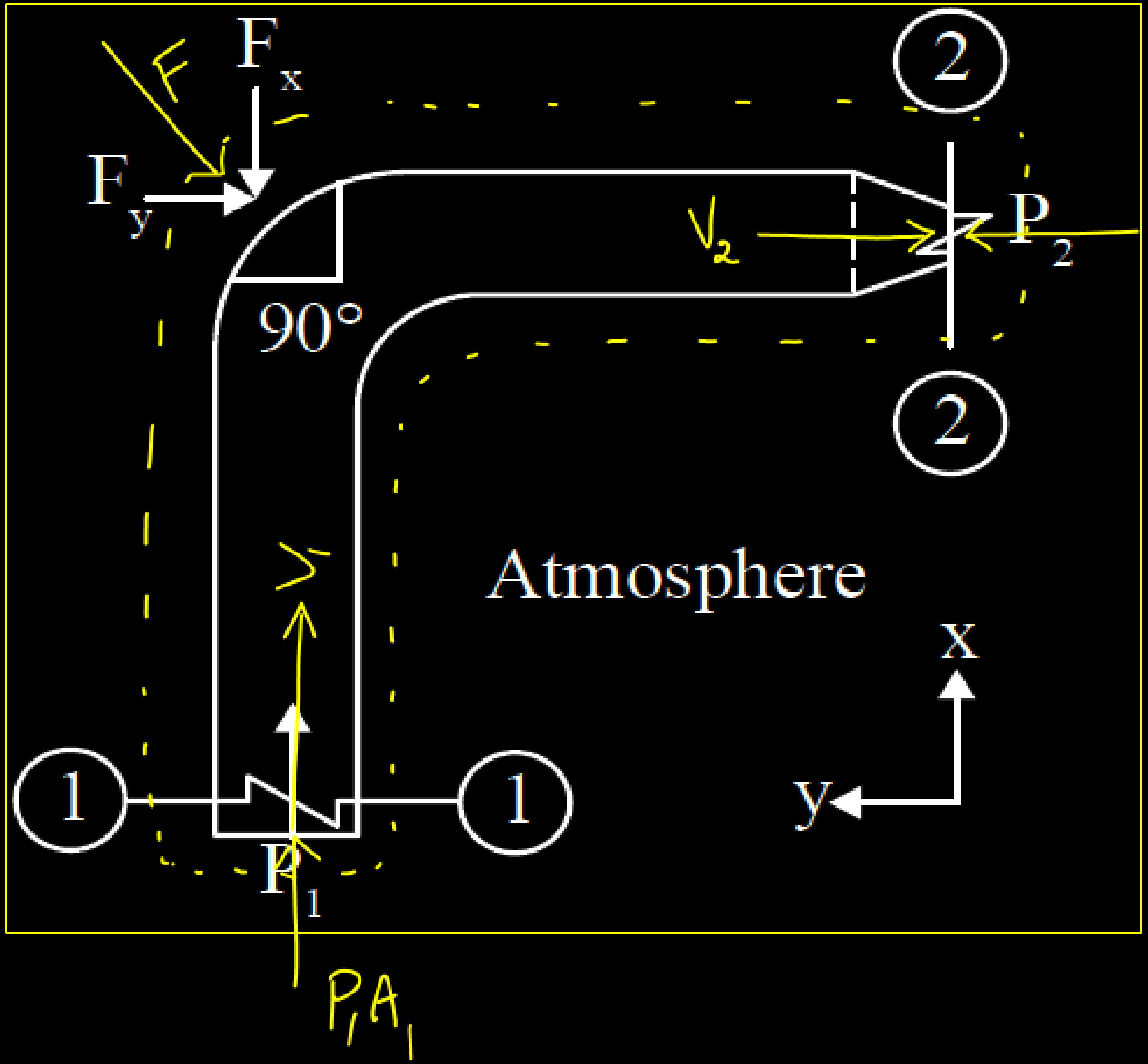
(i) Acceleration due to gravity = $10\ m/s^2$.

(ii) Weights of both the bent pipe segment as well as water are negligible.

(iii) Friction across the bend is negligible

The magnitude of the force (in kN, up to two decimal places) that would be required to hold the pipe section is ____________________

(GATE – 17 – Set 1)



$$\Sigma F_y = d(\dot{m}V_y)$$

$$= \dot{m}V_2 - \dot{m}V_1$$

$$-F_y + P_2A_2 = \dot{m}(-V_2) - \dot{m}(\cancel{0})$$

$$\boxed{F_y = P_2A_2 + \dot{m}V_2} \rightarrow (1)$$

$$P_1A_1 - F_x = \dot{m}(\cancel{0}) - \dot{m}V_1$$

$$\boxed{F_x = P_1A_1 + \dot{m}V_1} \rightarrow (2)$$

$$\frac{P_1}{\gamma}+\frac{V_1^2}{2g}+\cancel{Z_1}=\cancelto{0}{\frac{P_2}{\gamma}}+\frac{V_2^2}{2g}+\cancel{Z_2}$$

$$A_1=\frac{\pi}{4}\left(\frac{.27}{\sqrt{\pi}}\right)^2=\frac{0.27^2}{4}$$

$$A_2=\frac{\pi}{4}\left(\frac{.14}{\sqrt{\pi}}\right)^2=\frac{0.14^2}{4}$$

$$\frac{140\times10^3}{10^4}+\frac{V_1^2}{2g}=\frac{V_2^2}{2g}$$

$$\frac{V_2^2-V_1^2}{2g}=14 \Rightarrow \frac{Q^2}{A_2^2}-\frac{Q^2}{A_1^2}=280 \Rightarrow Q^2\left[\frac{1}{A_2^2}-\frac{1}{A_1^2}\right]=280$$

$$Q=1.3698\ m^3/s$$

$$V_1=\frac{Q}{A_1}=\frac{1.3698}{0.0182}=75.2637\ m/s$$

$$V_2=\frac{Q}{A_2}=\frac{1.3698}{4.9\times10^{-3}}=279.551\ m/s$$

$$F_x = P_1 A_1 + \dot{m} V_1$$

$$= 140 \times 10^3 \times 0.0182 + 10^3 \times 1.3698 \times 75.2637$$

$$F_x = 105.644 \text{ KN}$$

$$F_y = \cancel{P_2 A_2}^{\,0} + \dot{m} V_2 = \rho Q V_2 = 10^3 \times 1.3698 \times 279.551$$

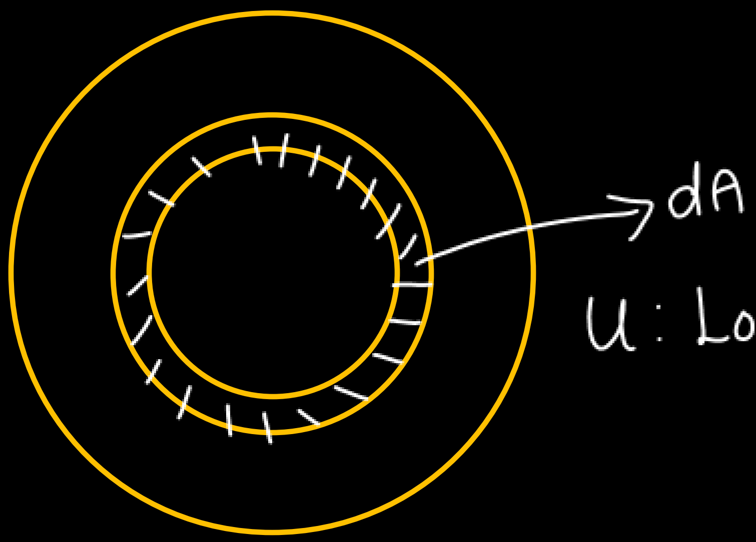
$$F_y = 382.928 \text{ KN}$$

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{105.644^2 + 382.928^2}$$

$$= \underline{\underline{397.233 \text{ KN}}}$$

Momentum correction factor:(β)

$$\beta \dot{m} V$$



u : Local velocity

Actual momentum flux $= \beta \dot{m} V \longrightarrow (1)$

Differential momentum flux $= d\dot{m}\, u$

$= [\rho\, dA\, u]\, u$

$= \rho u^2 dA$

Momentum flux $= \rho \int_A u^2 dA \longrightarrow (2)$

$(1) = (2)$

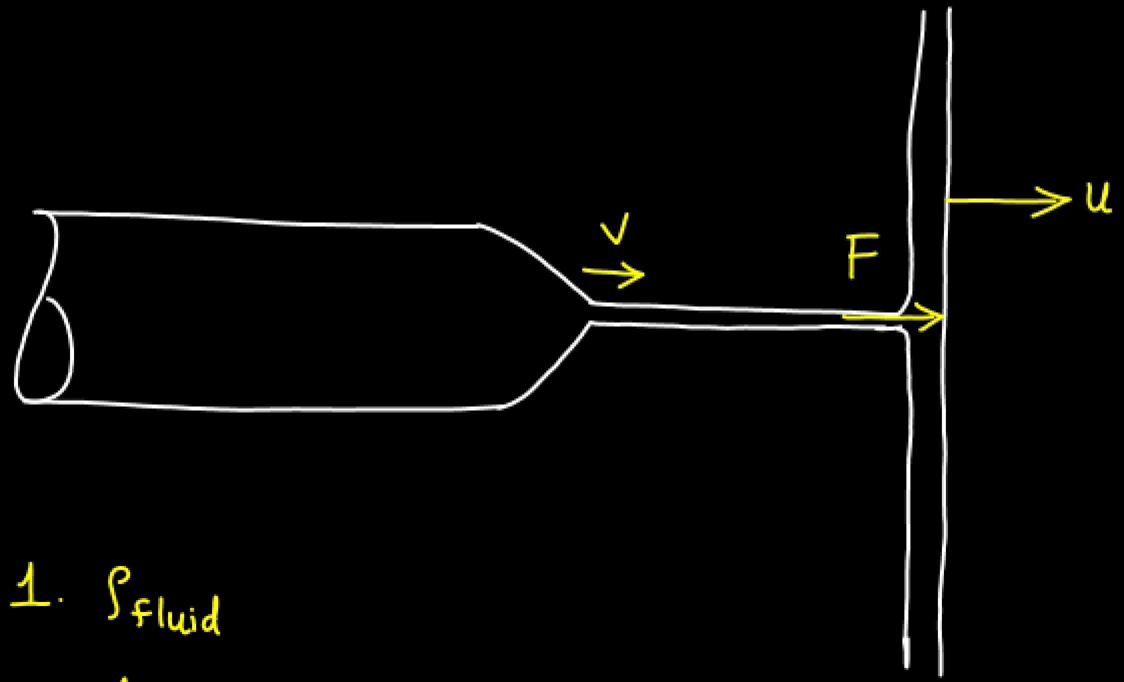
$$\beta \dot{m} V = \rho \int_A u^2 dA$$

$$\beta \cancel{\rho} A V^2 = \cancel{\rho} \int_A u^2 dA$$

$$\left\{ \beta = \frac{1}{AV^2} \int_A u^2 \, dA \right\}$$

$$\alpha > \beta$$

Impact of jets:



1. ρ_{fluid}
2. Area of jet
3. Velocity of jet

To determine:

① Force (F)

② Workdone / unit time

$= \text{Power} = F.u$

[output]

③ K.E / unit time

$= \frac{1}{2}\dot{m}v^2$

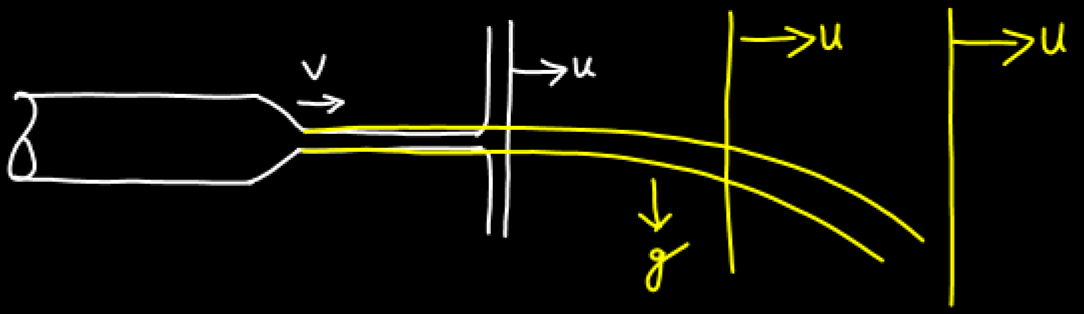
[Input]

(4) $\eta = \dfrac{\text{output}}{\text{Input}}$

$$= \frac{F \cdot u}{\frac{1}{2} \dot{m} v^2} \times 100\%$$

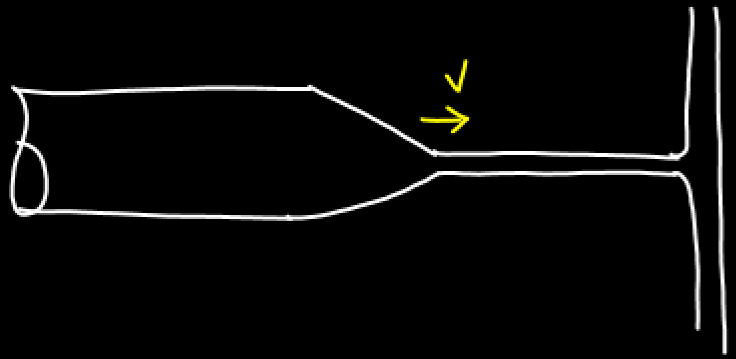
Assumptions:

1. Neglect gravity in Horizontal jets.



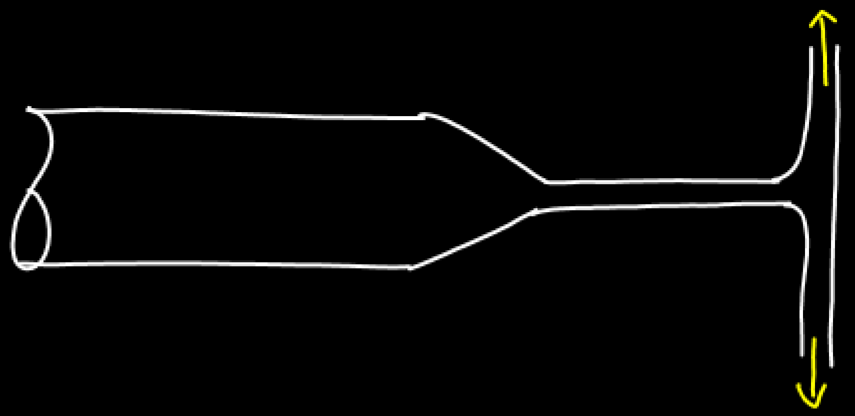
[Loses contact]

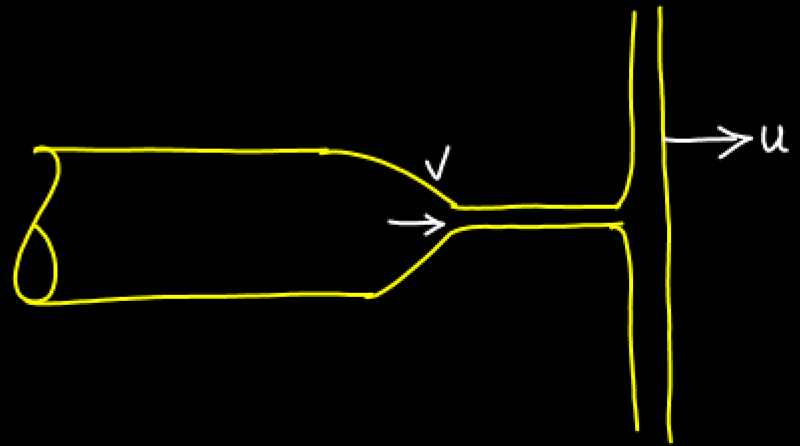
2. jet Leaving the plate tangentially after striking.



[No splashing of water]

③ Neglect friction at the surface of the plate.



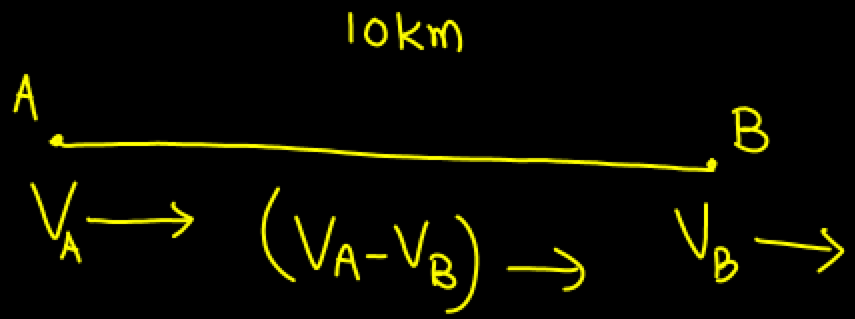


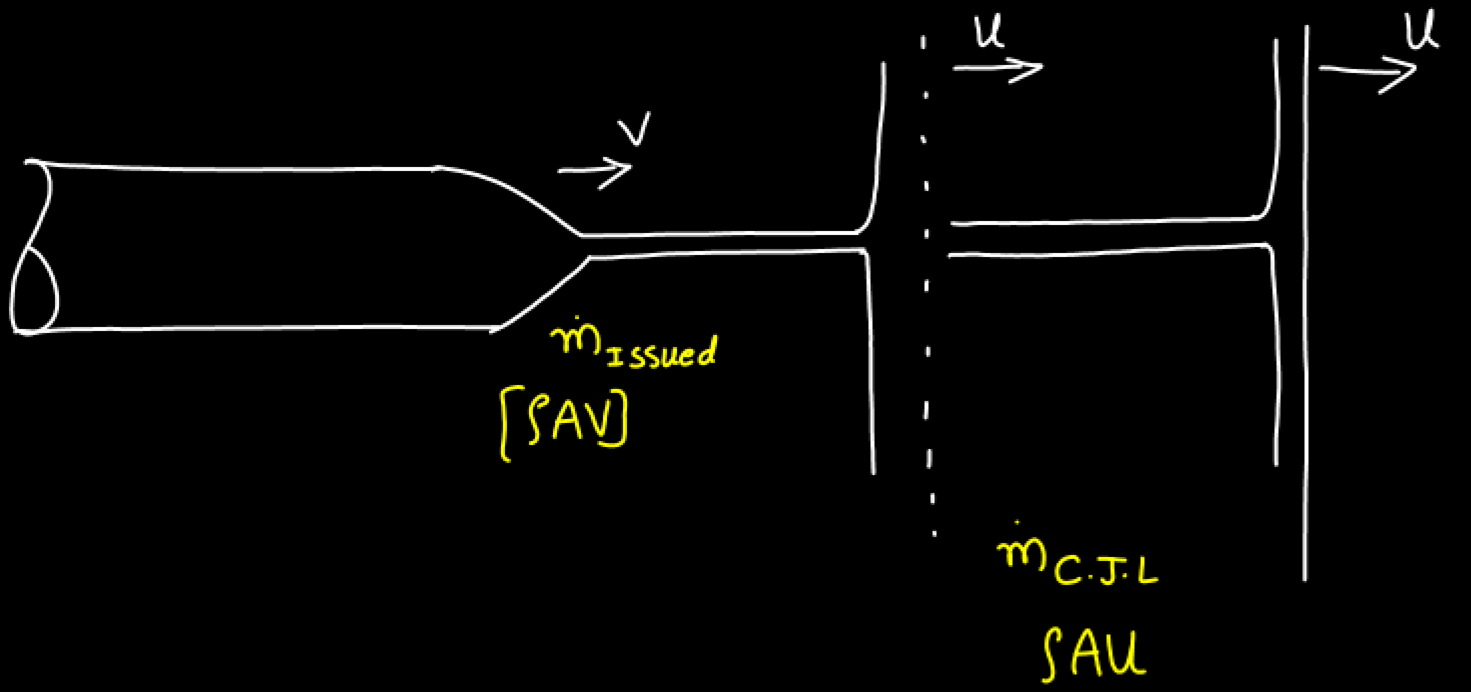
$$V_{jet} = V$$

$$\dot{m}_{jet} = \rho A V$$
[Issued]

$$V_{striking} = V - u$$

$$\dot{m}_{striking} = \rho A (V - u)$$

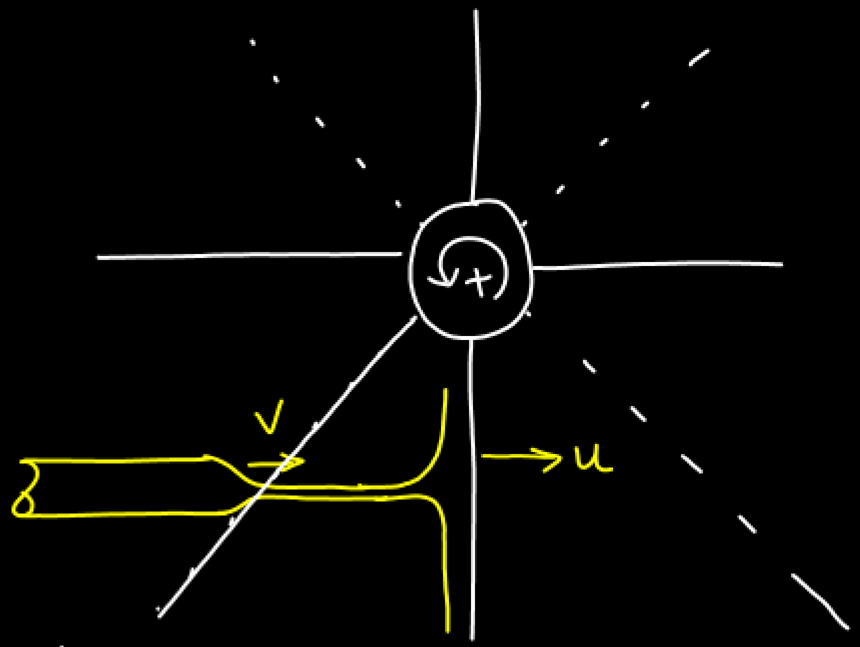




$$\dot{m}_{striking} = \dot{m}_{Issued} - \dot{m}_{\text{Change in jet Length}}$$

$$= \rho AV - \rho AU$$

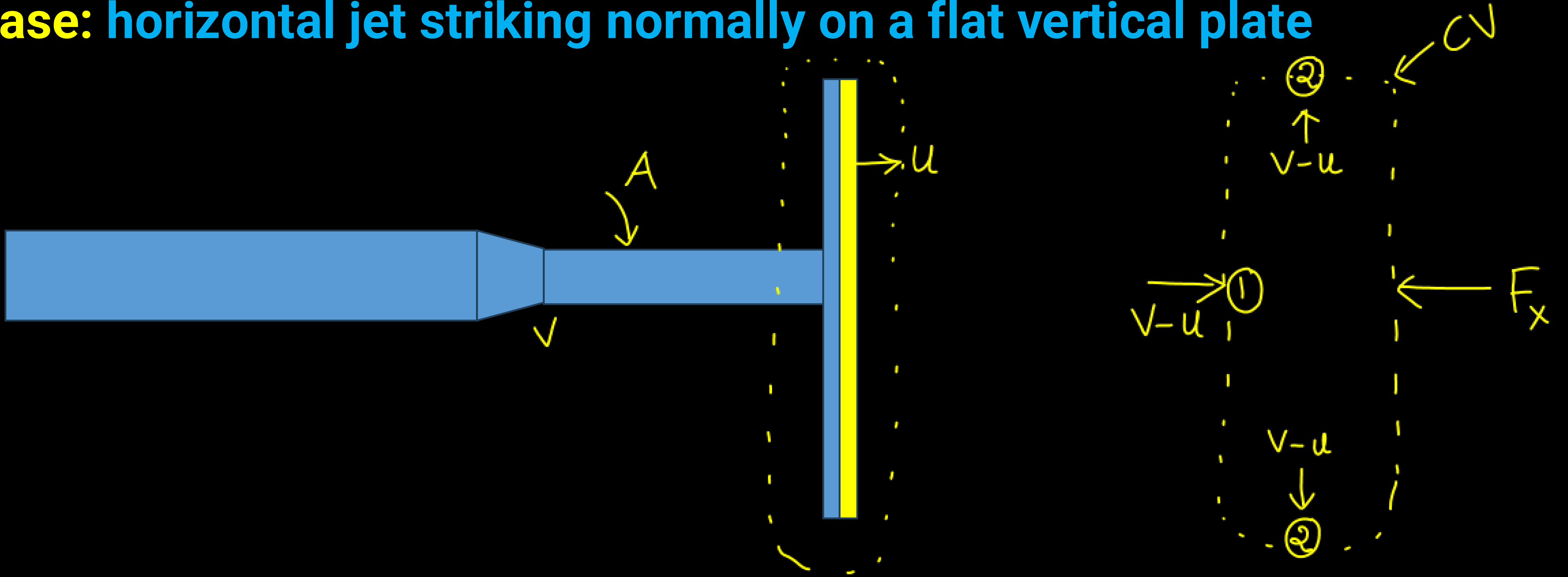
$$= \rho A(V-U)$$

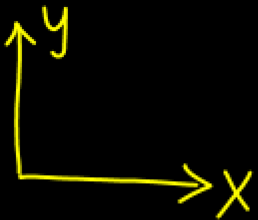


$$V_{jet} = V \quad ; \quad V_{striking} = V - u$$

$$\dot{m}_{jet} = \rho A V \quad ; \quad \dot{m}_{striking} = \rho A V$$

Case: horizontal jet striking normally on a flat vertical plate





$$\sum F_x = \Delta \dot{m} V_x$$

$$-F_x = \left(\dot{m} V_2 - \dot{m} V_1\right)_x$$

$$= \dot{m}\left(0 - (V-u)\right)$$

$$F_x = \dot{m}(V-u)$$

$$= \{\rho A (V-u)\}(V-u)$$

$$\boxed{F_x = \rho A (v-u)^2}$$

Power, $P = F_x . u$

$$\boxed{P = \rho A (v-u)^2 . u}$$

$$K.E = \frac{1}{2} \dot{m} v^2$$

$$= \frac{1}{2} [\rho A v] v^2 \Rightarrow \boxed{K.E = \frac{1}{2} \rho A v^3}$$

$$\eta = \frac{\text{Power}}{K.E}$$

$$= \frac{\rho A(V-u)^2 \cdot u}{\frac{1}{2}\rho A V^3}$$

$$\boxed{\eta = \frac{2u(V-u)^2}{V^3}}$$

Vortex motion

1. $V_r = 0$, V_θ = Exists
2. Curved streamlines
3. Types: (i) Free vortex motion

   (ii) Forced vortex motion

Ex: Tornados, whirlpool

Free V.M

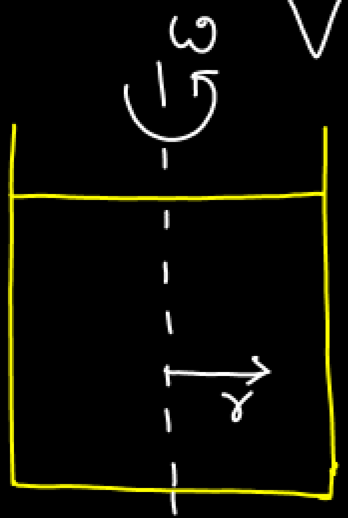
1. External torque is zero

$$T = 0 \Rightarrow \frac{d}{dt}(mvr) = 0$$

$$mvr = C$$

$$\boxed{Vr = C}$$

$$V \propto \frac{1}{r}$$



@ $r = 0$; V = not defined

"Singular Point"

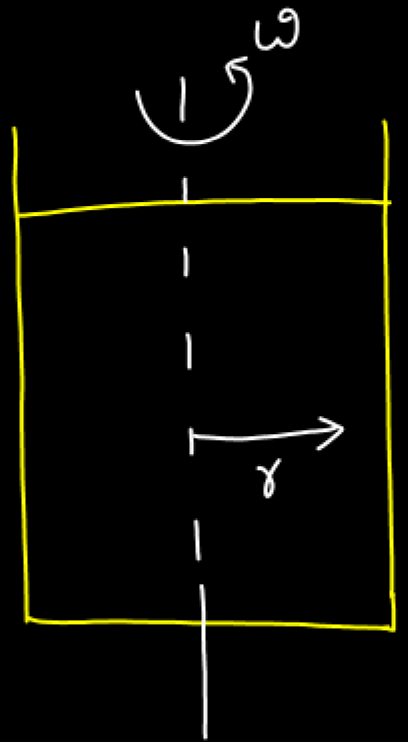
Forced V.M

Constant external torque $[T = C]$

Constant Angular velocity at every point of fluid.

$$\omega = C$$

$$\boxed{\frac{V}{r} = C \qquad V \propto r}$$



2. Irrotational	Rotational
Bernoulli's equation can be Applied on the entire flow field	B. Eq^n can be applied along a streamline.

Chapter 3
Fluid kinematics

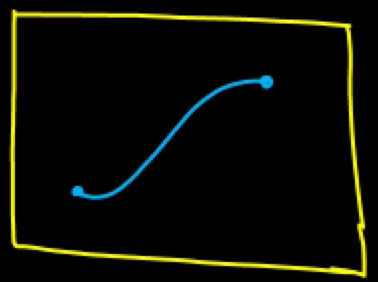
Fluid Kinematics

Def: The study of fluid without reference to the force causing it.

Approaches:

Lagrangian Approach

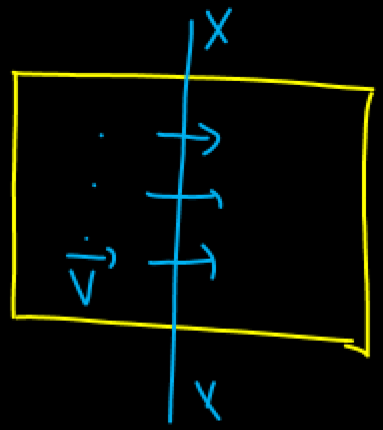
(1) Particle oriented Approach



$$\vec{V} = f(\text{Space}, \text{time})$$

Eulerian Approach

Position oriented Approach



$$\vec{V} = f(\text{Space}, t)$$

→ Eulerian Approach is a bulk mass Approach.

The classicial fluid Mechanics follows Eulerian Approach.

Velocity: The rate of change of displacement w.r.t time

$$\vec{V} = \frac{d\vec{s}}{dt}$$

(1) Cartesian Coordinate system $[x, y, z]$

(2) Polar coordinate system $[r, \theta, z, t]$

(1) Cartisian coordinate system:

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\left.\begin{aligned} u &= \frac{dx}{dt} \\ v &= \frac{dy}{dt} \\ w &= \frac{dz}{dt} \end{aligned}\right\} f(x, y, z, t)$$

(2) Polar coordinate system:

$$\vec{V} = V_r \hat{r} + V_\theta \hat{\theta} + w \hat{k}$$

V_r = Radial velocity

V_θ = Tangential velocity : $V_\theta = r\omega$: $\omega = \dfrac{d\theta}{dt}$

$$\left.\begin{aligned} V_r &= \frac{dr}{dt} \; ; \; w = \frac{dz}{dt} \\ V_\theta &= r\frac{d\theta}{dt} \end{aligned}\right\} f(r,\theta,z,t)$$

Classification of flows:

(1) Steady – unsteady flow:

The characteristics of a fluid does not change w.r.t time

$\vec{V} \neq f(t) \qquad \dfrac{dv}{dt} = \vec{a} = 0 \qquad$ Steady flow.

$\vec{V} = f(t) \qquad \dfrac{dv}{dt} = \vec{a} \neq 0 \qquad$ Unsteady flow.

(2) Uniform - Non uniform flow:

The characteristics of fluid does not change with space.

$$\vec{V} \neq f(\text{space}) \; : \; \frac{dv}{ds} = 0 \Rightarrow \text{uniform flow}$$

$$\vec{V} = f(\text{space}) \; : \; \frac{dv}{ds} \neq 0 \Rightarrow \text{Non-uniform flow}$$

Ex: Viscous flow $\Rightarrow \mu \neq 0 \; : \; \tau = \mu \frac{du}{dy} \Rightarrow \frac{du}{dy} \neq 0 \Rightarrow u = f(y)$

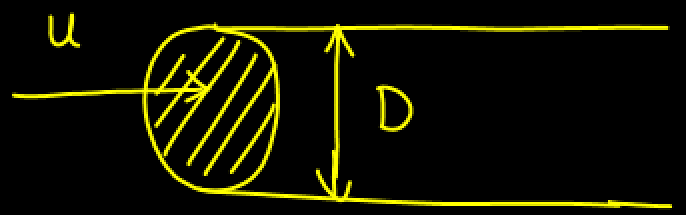
Non-uniform flow.

Continuity equation:

The result of Law of conservation of mass of a fluids is continuity equation.

1. Mass flow rate: ($\dot{m}$)

The quantity of mass of a fluid passes through a certain section per unit time.



$$\boxed{\dot{m} = \rho A V}$$

V: Avg. Velocity of flow at a section.

$$\dot{m} = \rho\left[\frac{\pi D^2}{4}\right] u$$

S.I : kg/sec

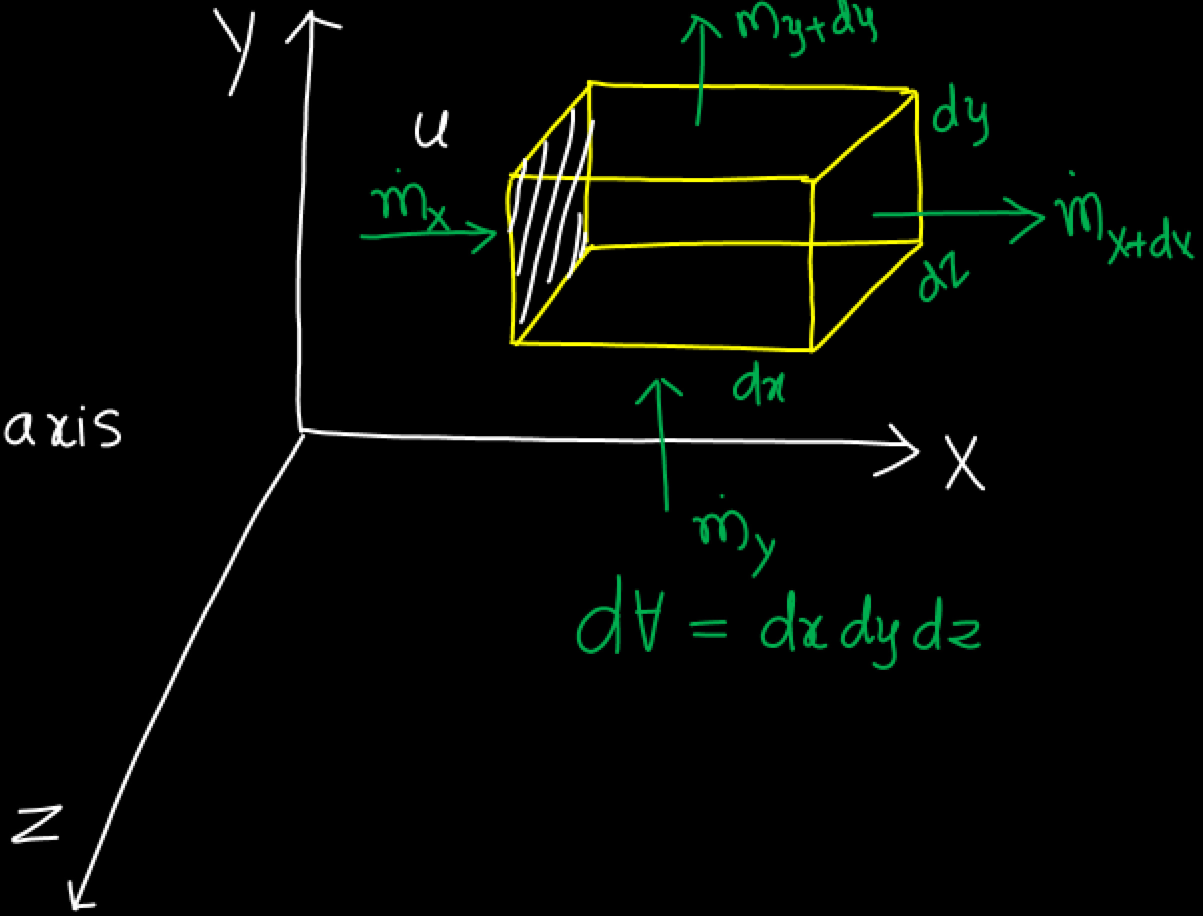
(2) Volume flow rate / Discharge : [Q]

The quantity of Volume of a fluid flows through certain section per unit time.

$$\boxed{\begin{aligned} Q &= A \cdot V \\ \dot{m} &= \rho A V = \rho Q \end{aligned}}$$

Units : m^3/sec.

Derivation:



$$d\forall = dx\,dy\,dz$$

$$\dot{m}_x = \rho A V$$

$$= \rho\, dz\, dy\, u \longrightarrow \text{Influx in x-axis}$$

$$\dot{m}_{x+dx} = \dot{m}_x + \frac{\partial \dot{m}_x}{\partial x} \cdot dx$$

$$= \dot{m}_x + \frac{\partial}{\partial x}\left[\rho\, \underbrace{dz\, dy\, dx}_{d\forall}\, u\right]$$

$$\dot{m}_{x+dx} = \rho\, dz\, dy\, u + d\forall \frac{\partial}{\partial x}\left[\rho u\right] \longrightarrow \text{Efflux in x-axis}$$

$$\left.\begin{aligned}
\dot{m}_x - \dot{m}_{x+dx} &= -d\forall \frac{\partial}{\partial x}[\rho u] \\
&+ \\
\dot{m}_y - \dot{m}_{y+dy} &= -d\forall \frac{\partial}{\partial y}[\rho v] \\
&+ \\
\dot{m}_z - \dot{m}_{z+dz} &= -d\forall \frac{\partial}{\partial z}[\rho w]
\end{aligned}\right\}$$

Accumulation of massflow rate in $d\forall$

$\longrightarrow$ 1

The rate of change of mass per unit time in $d\forall$

$$= \frac{\partial m}{\partial t} = \frac{\partial}{\partial t}[\rho \, d\forall] \qquad \left\{ \rho = \frac{m}{V} \right\}$$

$$= d\forall \frac{\partial \rho}{\partial t} \longrightarrow 2$$

① = ②

$$-\cancel{d\forall}\left[\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right]=\cancel{d\forall}\,\frac{\partial \rho}{\partial t}$$

$$\boxed{\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0}$$

Continuity equation in Cartesian coordinates for 3D

"Compressible flow"

$$\nabla \text{ (Divergence)} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\nabla \cdot \vec{V} = \left(\frac{\partial}{\partial x}\bar{i} + \frac{\partial}{\partial y}\bar{j} + \frac{\partial}{\partial z}\bar{k}\right) \cdot \left(u\bar{i} + v\bar{j} + w\bar{k}\right)$$

$$\nabla \cdot \vec{V} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} \quad \left[\text{Divergence of a vector } \vec{V}\right]$$

$$\left\{ \begin{array}{l} \longrightarrow \bar{A} \\ \longrightarrow \bar{B} \quad \theta = 0^\circ \\ \bar{A}\cdot\bar{B} = |A||B|\cos\theta \\ \quad = |A||B| \end{array} \right.$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0$$

Continuity equation in Divergence form (or) Vectorial form.

If flow is Steady: $\left[\frac{\partial \rho}{\partial t} = 0\right]$

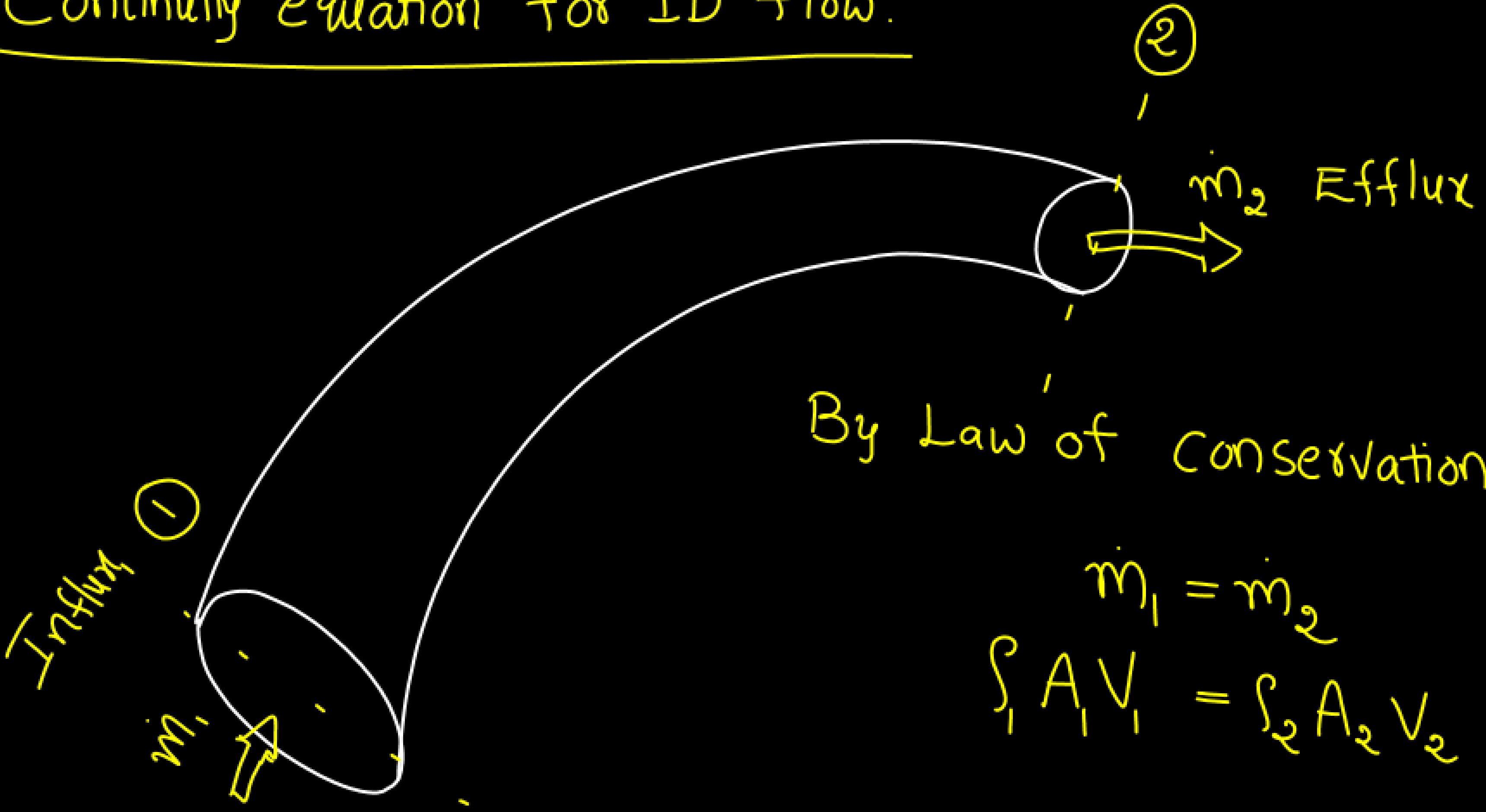
$$\frac{\partial (\rho u)}{\partial x} + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

If flow is Incompressible: $[\rho = C]$

$$\left\{ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \right\} \text{ for 3D}$$

$$\left\{ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \right\} \quad \text{2D flow.}$$

Continuity equation for 1D flow:



By Law of Conservation of Mass

$$\dot{m}_1 = \dot{m}_2$$

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2$$

for Incompressible flow: $[\rho = c]$

$$\boxed{A_1 V_1 = A_2 V_2}$$

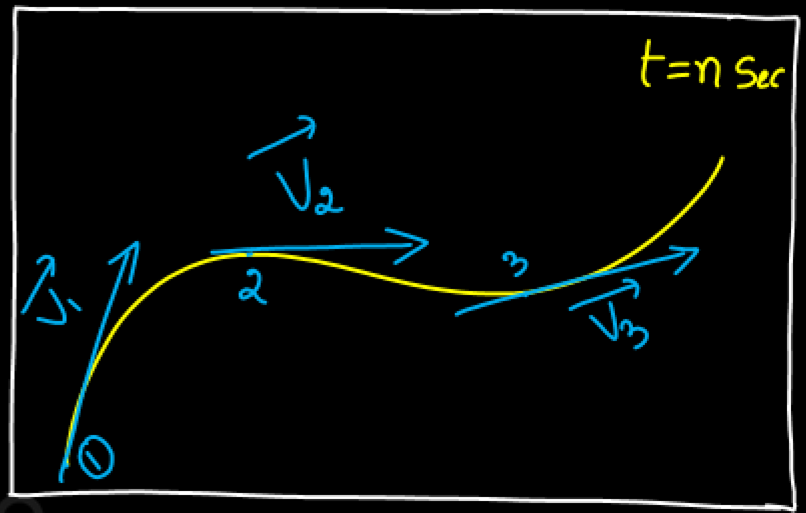
$$Q_1 = Q_2$$

V_1 , V_2 : Avg. velocities at corresponding sections.

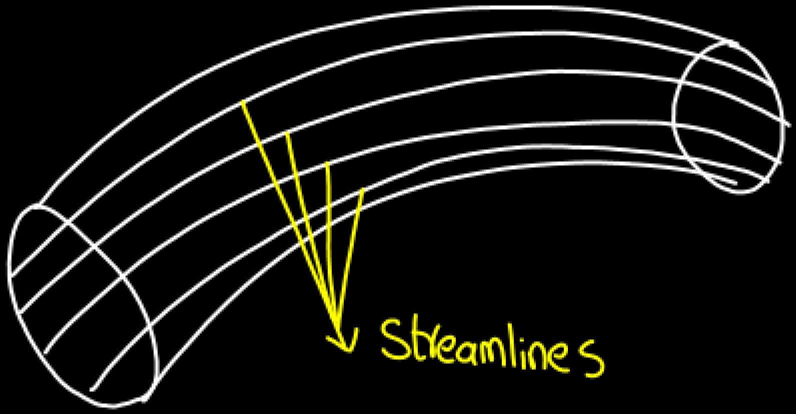
Flow Visualisation:

① Streamlines:

Def: A set of imaginary curves, drawn in a flow field at a particular instant of time, such that the tangent represents the direction of Velocity $\vec{V}$ for the corresponding positions.



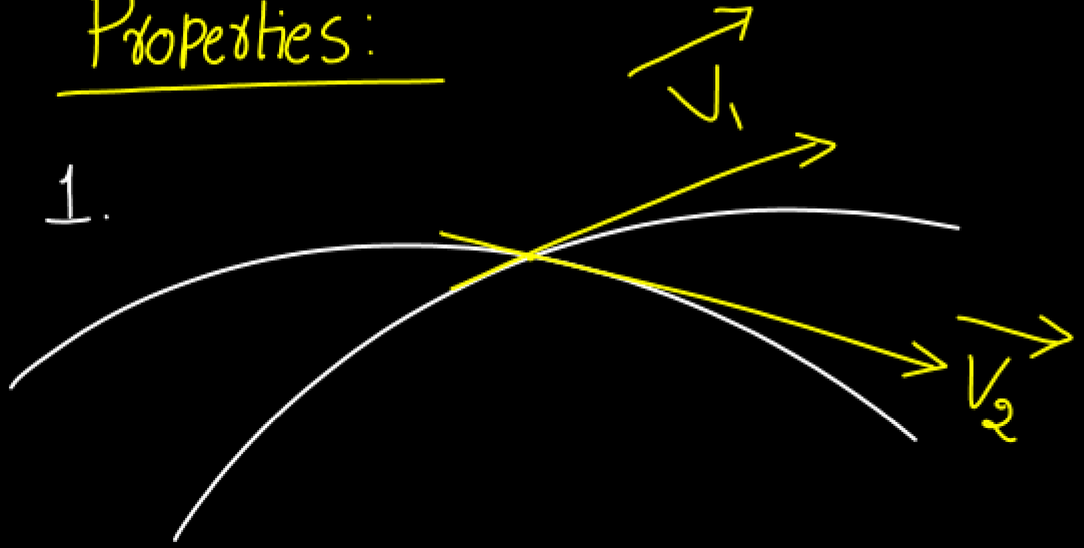
A bundle of streamlines forming a passage through which flow can be visualized.

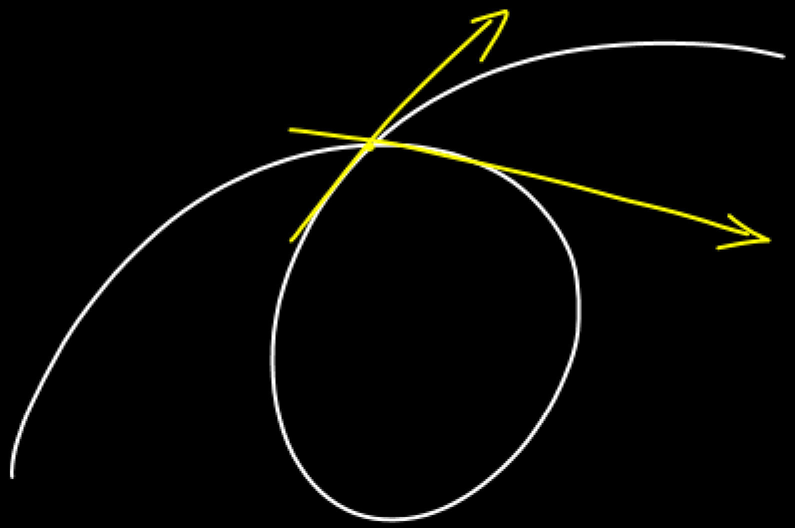


"Stream tube"

Properties:

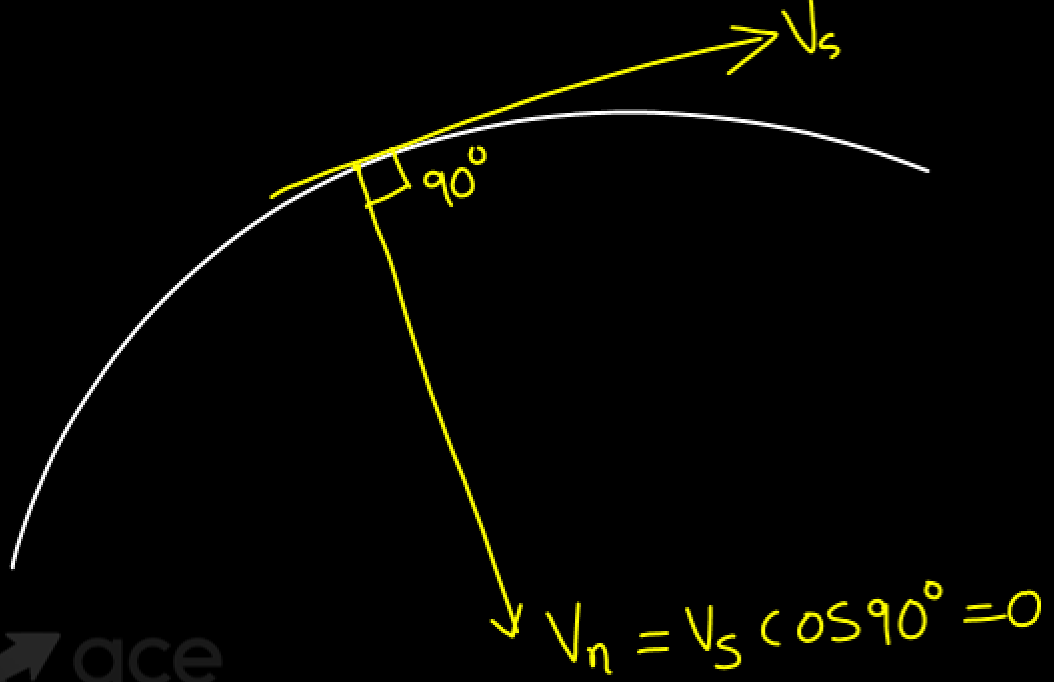
1.





Streamlines can not intersect each other nor streamline intersect itself.

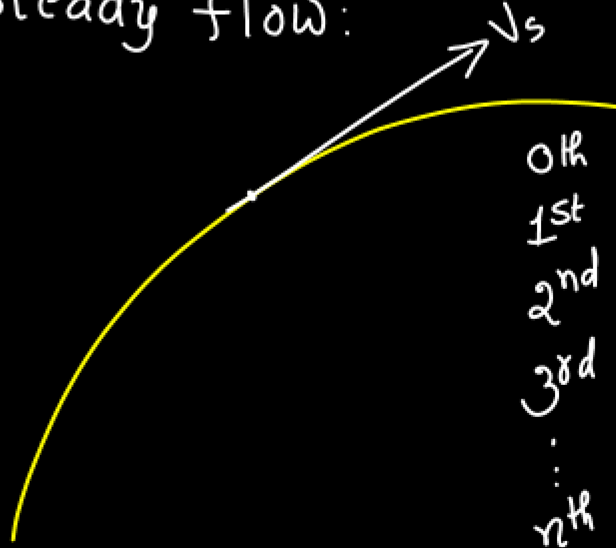
(2)



flow is possible only along a streamline.

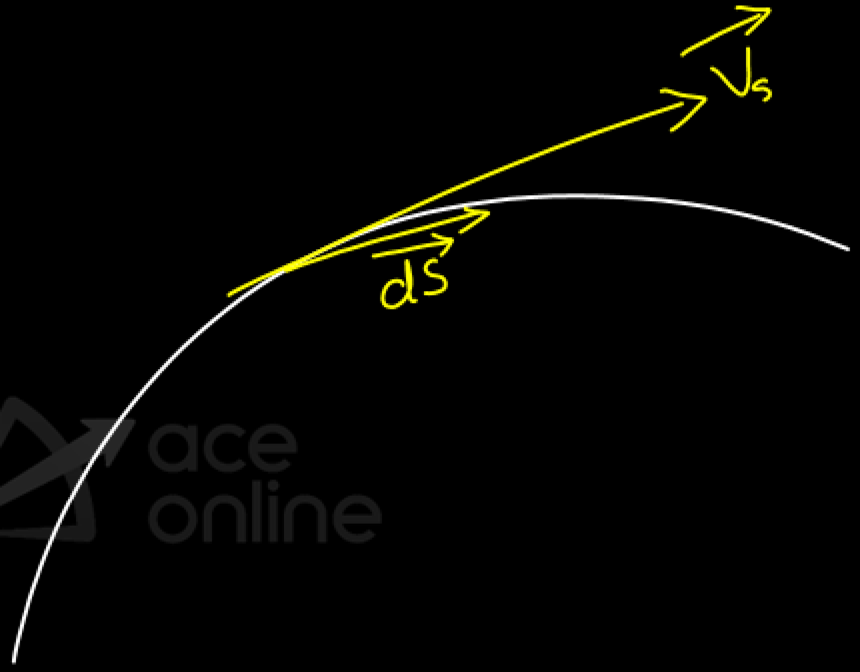
flow is impossible across a streamline.

(3) Steady flow:



In a steady flow streamlines does not fluctuates.

Eq^n of a Stream line:



$$\vec{V_s} \times \vec{dS} = 0$$

$$\vec{V} = u\hat{i} + v\hat{j} + w\hat{k}$$

$$\vec{dS} = dx\,\hat{i} + dy\,\hat{j} + dz\,\hat{k}$$

$$\begin{array}{ccc} + & - & + \end{array}$$

$$\begin{vmatrix} i & j & k \\ u & v & w \\ dx & dy & dz \end{vmatrix} = 0$$

$$\bar{i}(v\,dz - w\,dy) - \bar{j}(u\,dz - w\,dx) + \bar{k}(u\,dy - v\,dx) = 0$$

$$v\,dz - w\,dy = 0 \Rightarrow \frac{dy}{v} = \frac{dz}{w}$$

$$\left.\begin{array}{l}\dfrac{dx}{u} = \dfrac{dz}{w} \\[2ex] \dfrac{dx}{u} = \dfrac{dy}{v}\end{array}\right\} \quad \boxed{\dfrac{dx}{u} = \dfrac{dy}{v} = \dfrac{dz}{w}}$$

Eqn of a stream line for 3D

$$\boxed{\dfrac{dx}{u} = \dfrac{dy}{v}}$$

for 2D

$$\frac{dy}{dx} = \frac{v}{u}$$

→ Slope of a velocity vector.

Slope of a stream line

Acceleration:

$$\vec{a} = \frac{d\vec{V}}{dt} \qquad V = f(t)$$

$$\vec{V} = f(\text{Space}, \text{time})$$

Cartesian coordinate system: (x, y, z, t)

$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

$$a_x = \frac{du}{dt} \qquad [u = x, y, z, t]$$

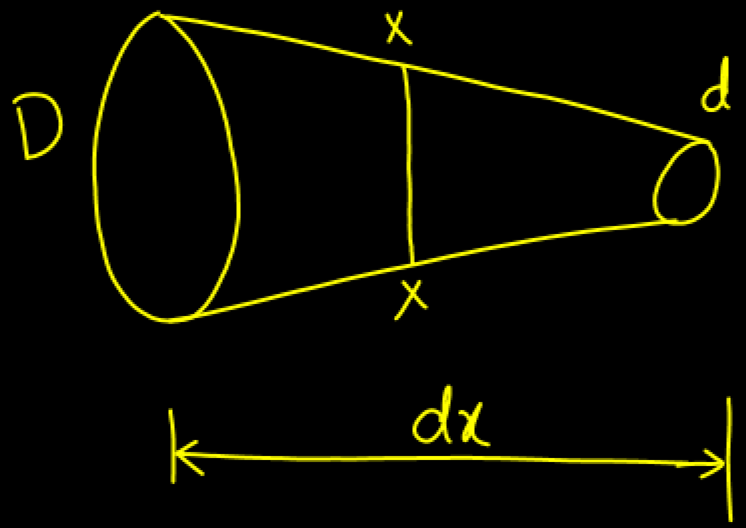
$$du = (\delta u)_x + (\delta u)_y + (\delta u)_z + (\delta u)_t$$

$$du = \frac{\partial u}{\partial x} \cdot dx + \frac{\partial u}{\partial y} \cdot dy + \frac{\partial u}{\partial z} \cdot dz + \frac{\partial u}{\partial t} \cdot dt$$

$$a_x = \frac{du}{dt}$$

$$= \frac{\partial u}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt} + \frac{\partial u}{\partial z} \cdot \frac{dz}{dt} + \frac{\partial u}{\partial t} \cdot \frac{dt}{dt}$$

$$a_x = u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} + \frac{\partial u}{\partial t}.$$



$\longrightarrow x$

$$\frac{\partial (D-d)}{\partial x} \cdot dx$$

$$a_x = \boxed{\frac{\partial u}{\partial t}} + \boxed{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z}}$$

$$a_y = \frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} + w\frac{\partial v}{\partial z}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}$$

Local (or) Temporal acceleration

= 0 (Steady flow)

Convective acceleration.

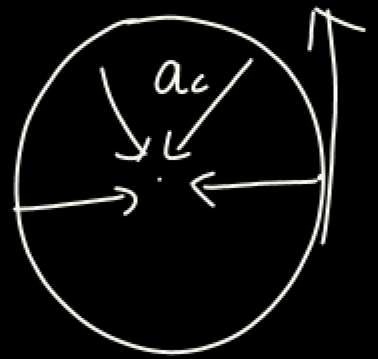
= 0 [uniform flow]

acc. in Polar coordinate system:

$$\vec{V} = f(r,\theta,z,t)$$

$$\vec{a} = a_r\hat{r} + a_\theta\hat{\theta} + a_z\hat{k}$$

$$a_r = \frac{\partial V_r}{\partial t} + V_r\frac{\partial V_r}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_r}{\partial \theta} + V_z\frac{\partial V_r}{\partial z} - \frac{V_\theta^2}{r}$$

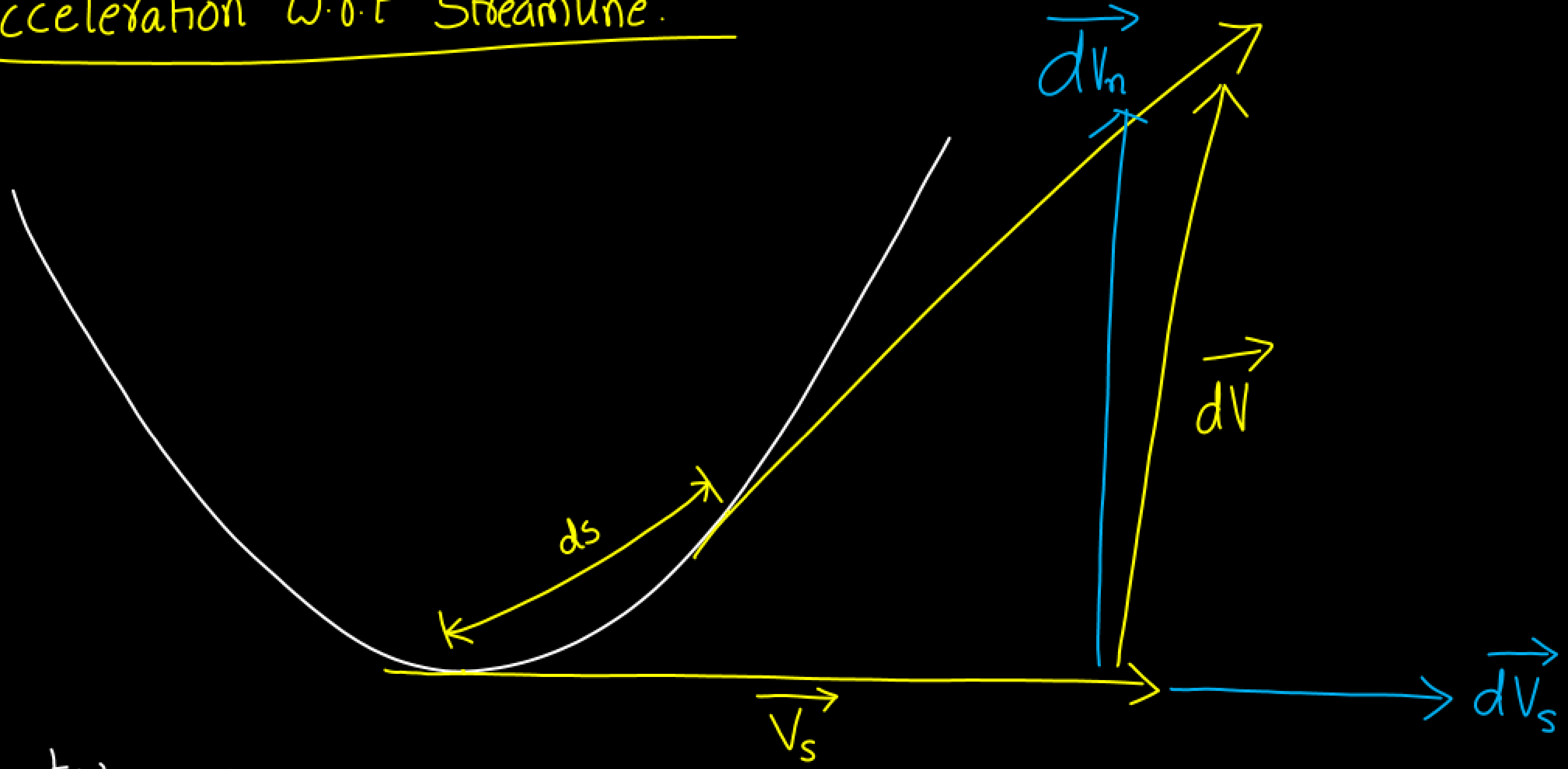


$$a_\theta = \frac{\partial V_\theta}{\partial t} + V_r\frac{\partial V_\theta}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_\theta}{\partial \theta} + V_z\frac{\partial V_\theta}{\partial z} + \frac{V_\theta V_r}{r}$$

$$a_z = \frac{\partial V_z}{\partial t} + V_r\frac{\partial V_z}{\partial r} + \frac{V_\theta}{r}\frac{\partial V_z}{\partial \theta} + V_z\frac{\partial V_z}{\partial z}$$

$$\left\{\begin{aligned} V &= r\omega \\ a_c &= \frac{dV}{dt} = \omega\frac{dr}{dt} \\ a_c &= \omega V \\ &= \frac{V.V}{r} = \frac{V^2}{r} \end{aligned}\right.$$

Acceleration w.r.t Streamline:



two mutually $\perp$r accelerations are possible in a flow field.

(1) Tangential acc. (a_s)

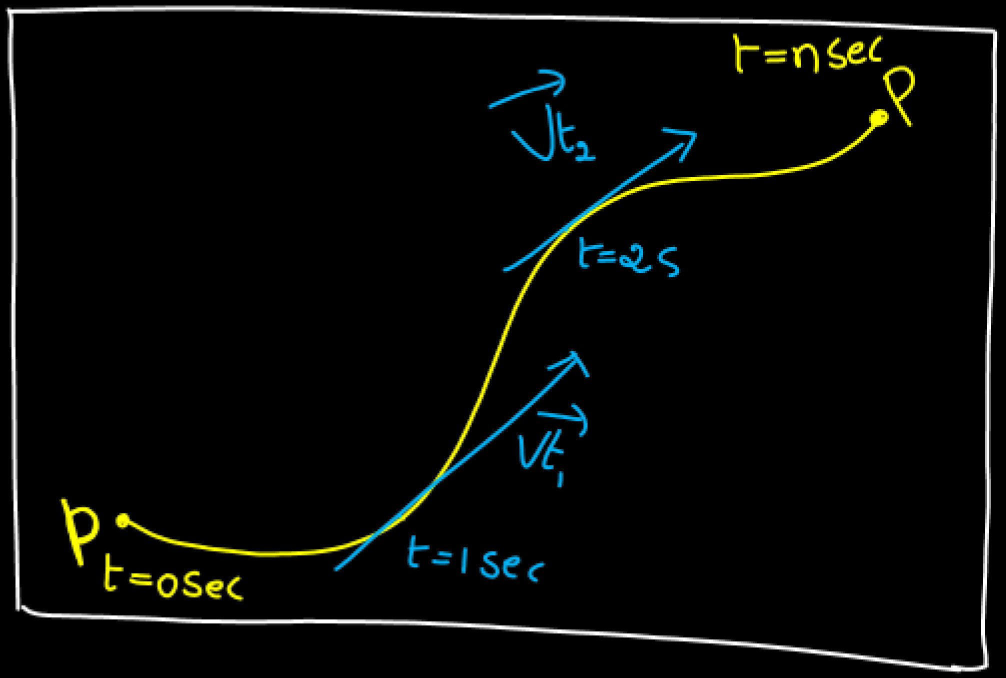
(2) Normal acc (a_n)

$$a = \sqrt{a_s^2 + a_n^2}$$

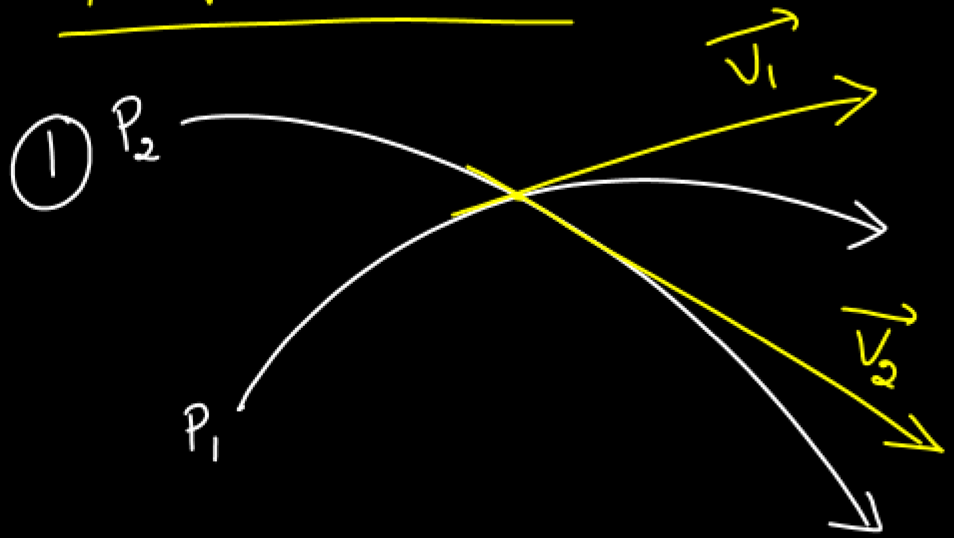
$$\left\{ \begin{aligned} a_s &= \frac{\partial V_s}{\partial t} + V_s \frac{\partial V_s}{\partial s} \\ a_n &= \frac{\partial V_n}{\partial t} + \frac{V_0^2}{r} \end{aligned} \right\}$$

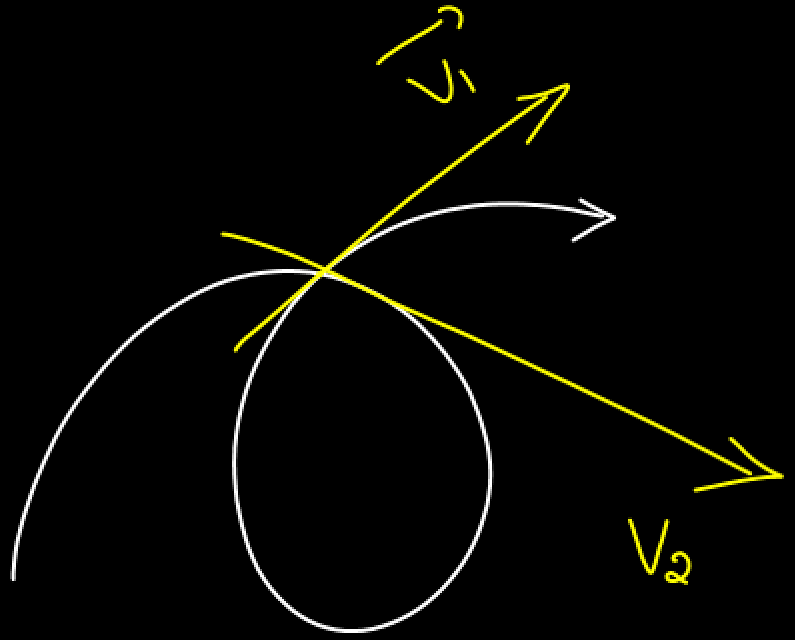
② Pathlines: [Lagrangian]

Def: The Path travelled by an individual Particle in a flow field over a Period of time.



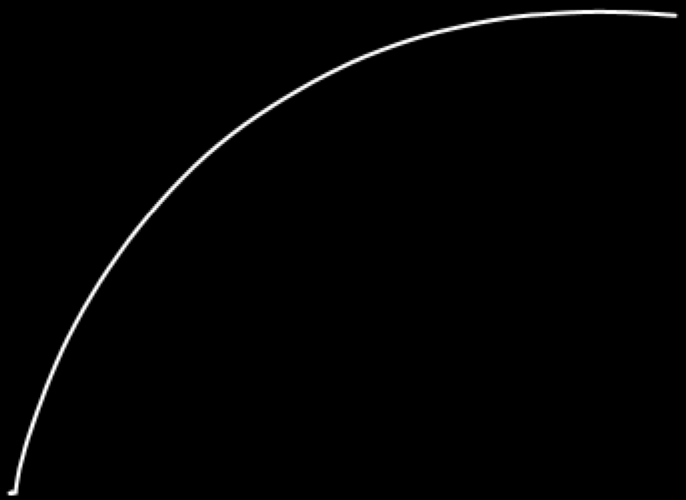
Properties:





Pathlines can intersect each other and Pathline can intersect itself.

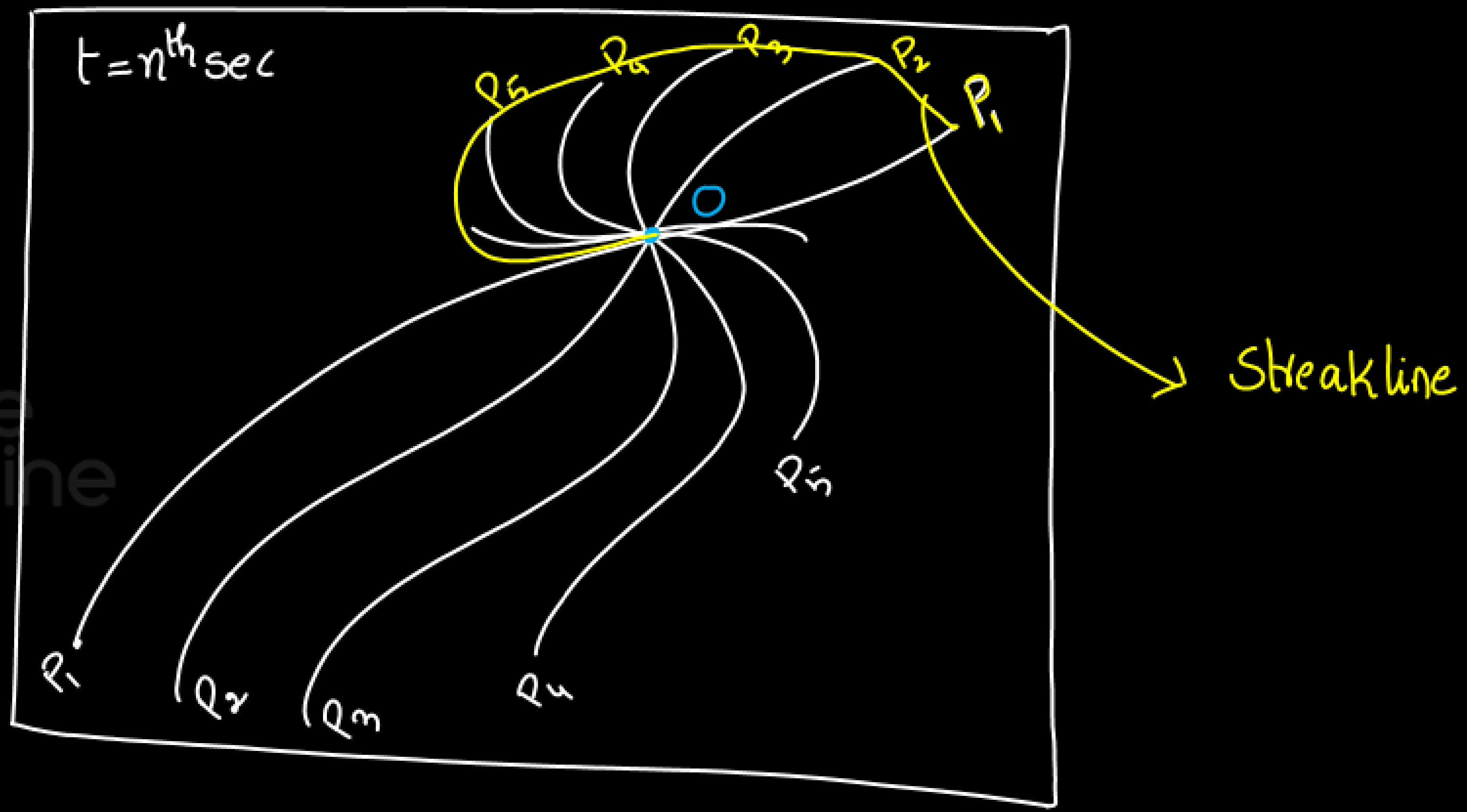
2



Streamlines = Pathlines

Identical in steady flow.

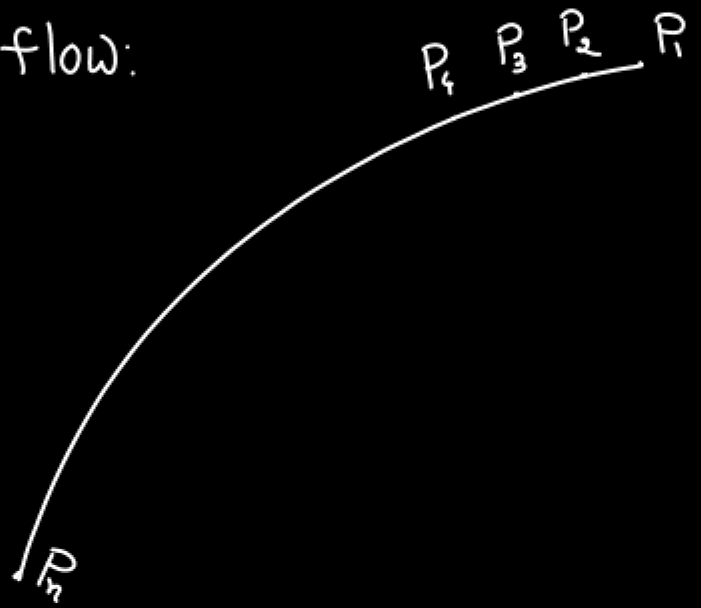
Streak lines:



Def:

A curve is drawn in a flow field at an instant of time, which represents the positions of all Particles which have crossed a common point is known as Streak line.

1. Family of streaklines is known as Rake of a streaklines.
2. Steady flow:

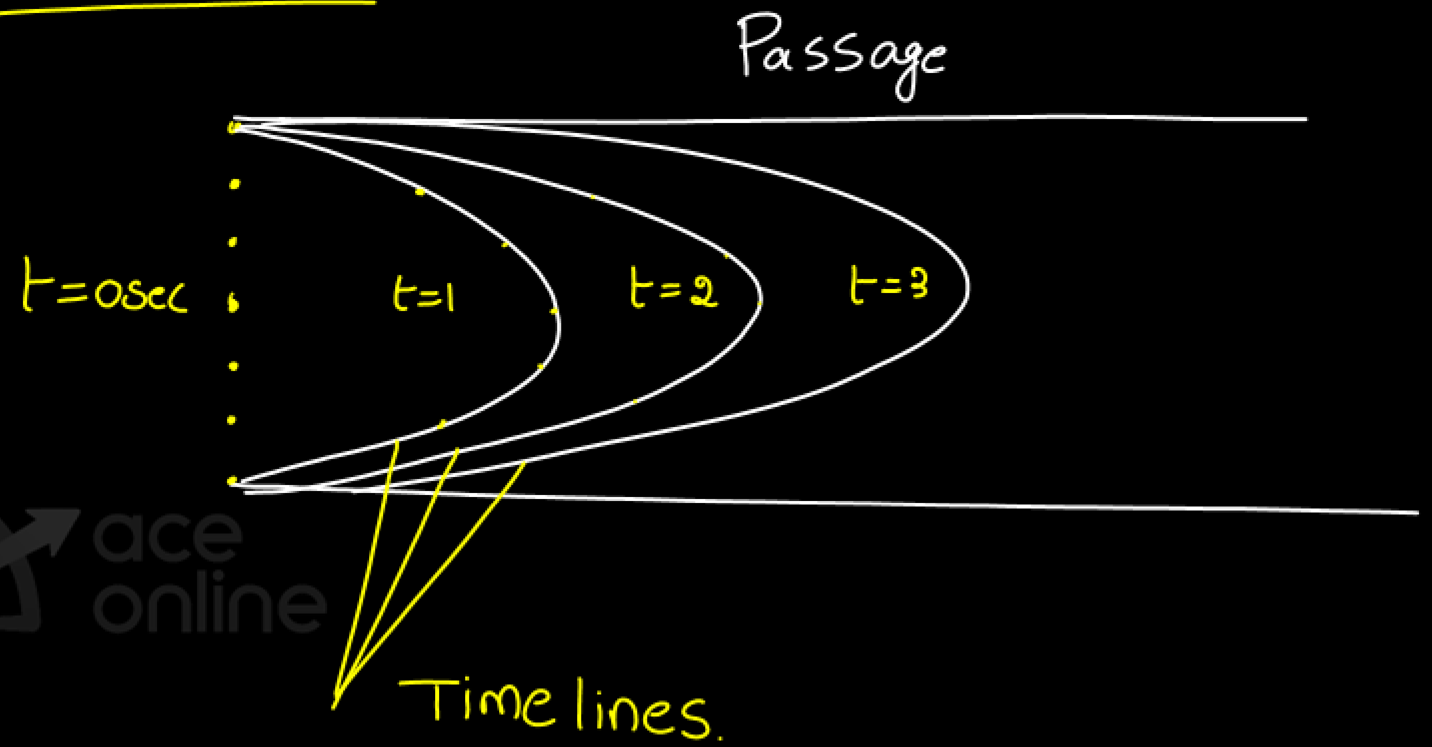


For a steady flow

Streamlines = Pathlines = Streak lines

are identical.

Time lines:



Def: A set of imaginary curves drawn in a flowfield, which represents Positions of all neighbouring Particles at Various instants of time.

- Time lines helps to understand uniformity of a flow.

Estimate the given expressions represents a flow or not?

1. $u = x^2+y^2+5$; $v = y^2+z$; $w = 3x^2yz$
2. $u = 2x^2$; $v = 2xyz$; $w = -4xz-xz^2+5y^2$

1. Sol:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

$$2x+2y+3x^2y \neq 0$$

$\nabla \cdot \vec{V} = 0$

Does not represents a flow.

2. Sol:

$$4x+2xz+(-4x-2xz)$$

$$=0$$

It represents a flow.

What is the condition for possible fluid flow?

u = ax+by; v = cx+dy

Sol:

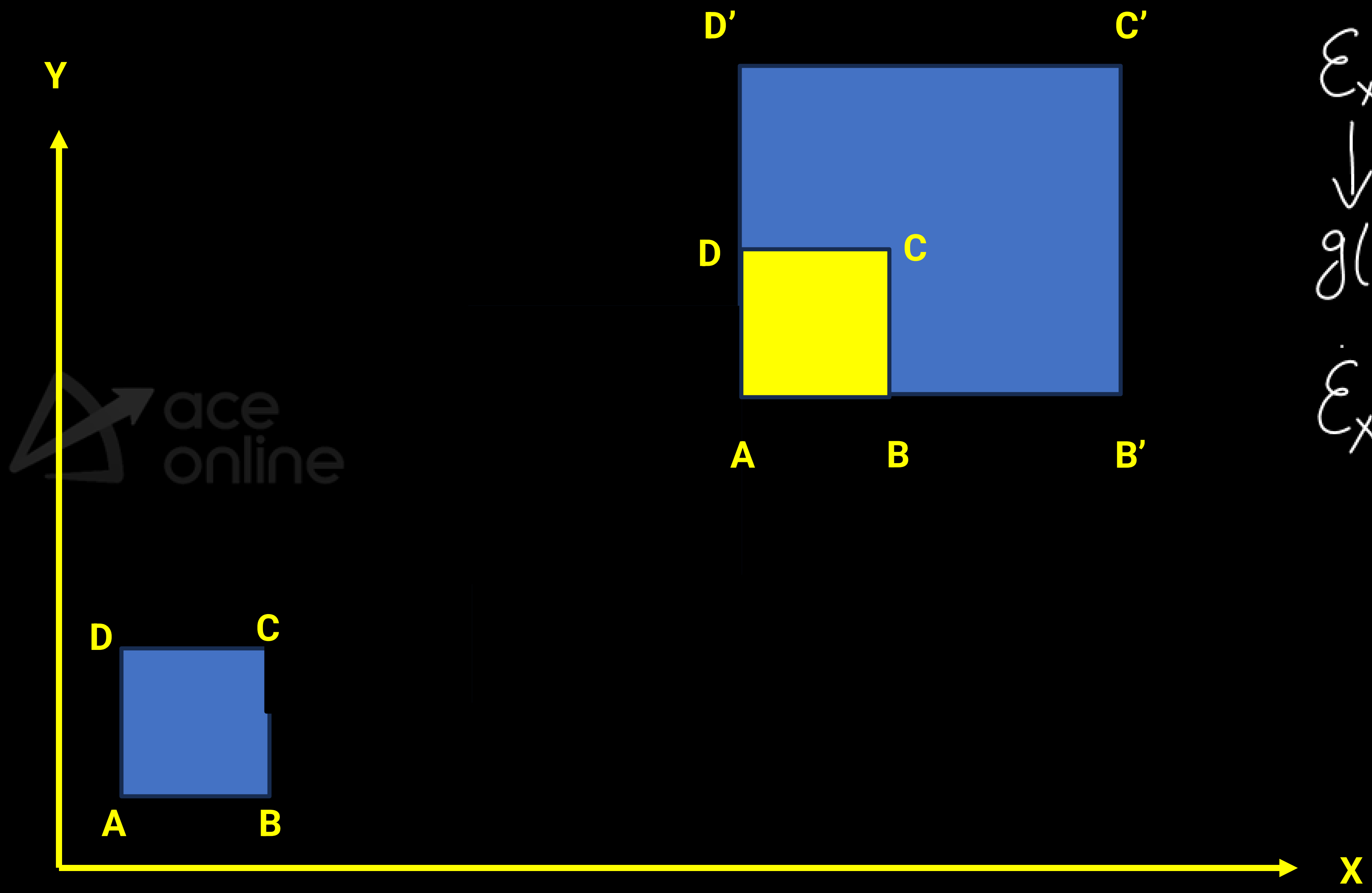
$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\boxed{a+d = 0}$$

Q. If $u = 2y^2 + 6xy$. calculate the component of velocity in y direction ?
Assume $v = 0$ at $y = 0$

Deformation & Rotation :

- **Deformation**
  - Linear deformation
  - Shear deformation
- **Rotation**
  - Vorticity
  - Circulation



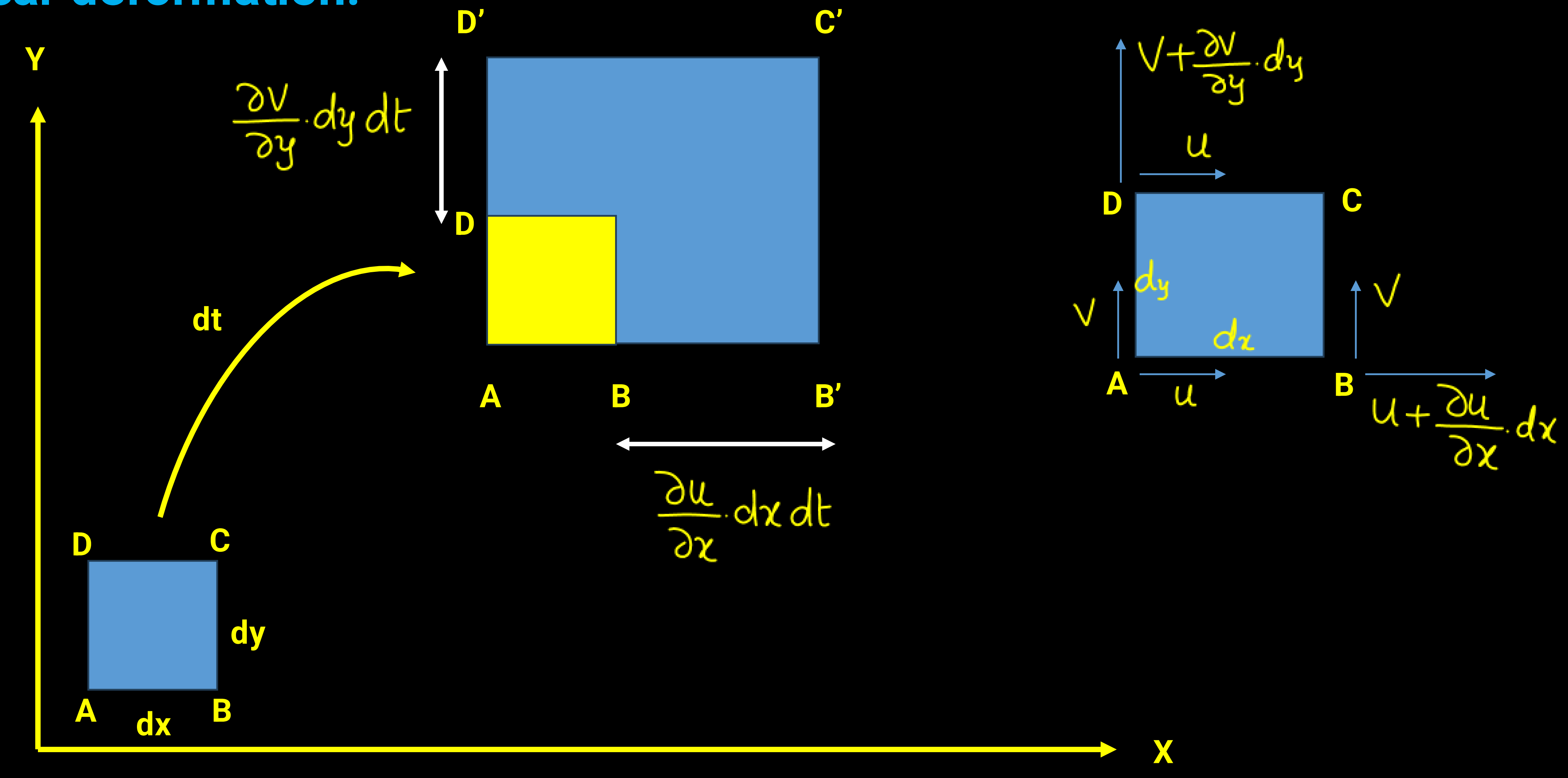
$$\varepsilon_x = \frac{BB'}{AB} \quad f(t)$$

$$g(t)$$

$$\dot{\varepsilon}_x = \text{Linear strain rate}$$

$$= \frac{\varepsilon_x}{dt}$$

Linear deformation:

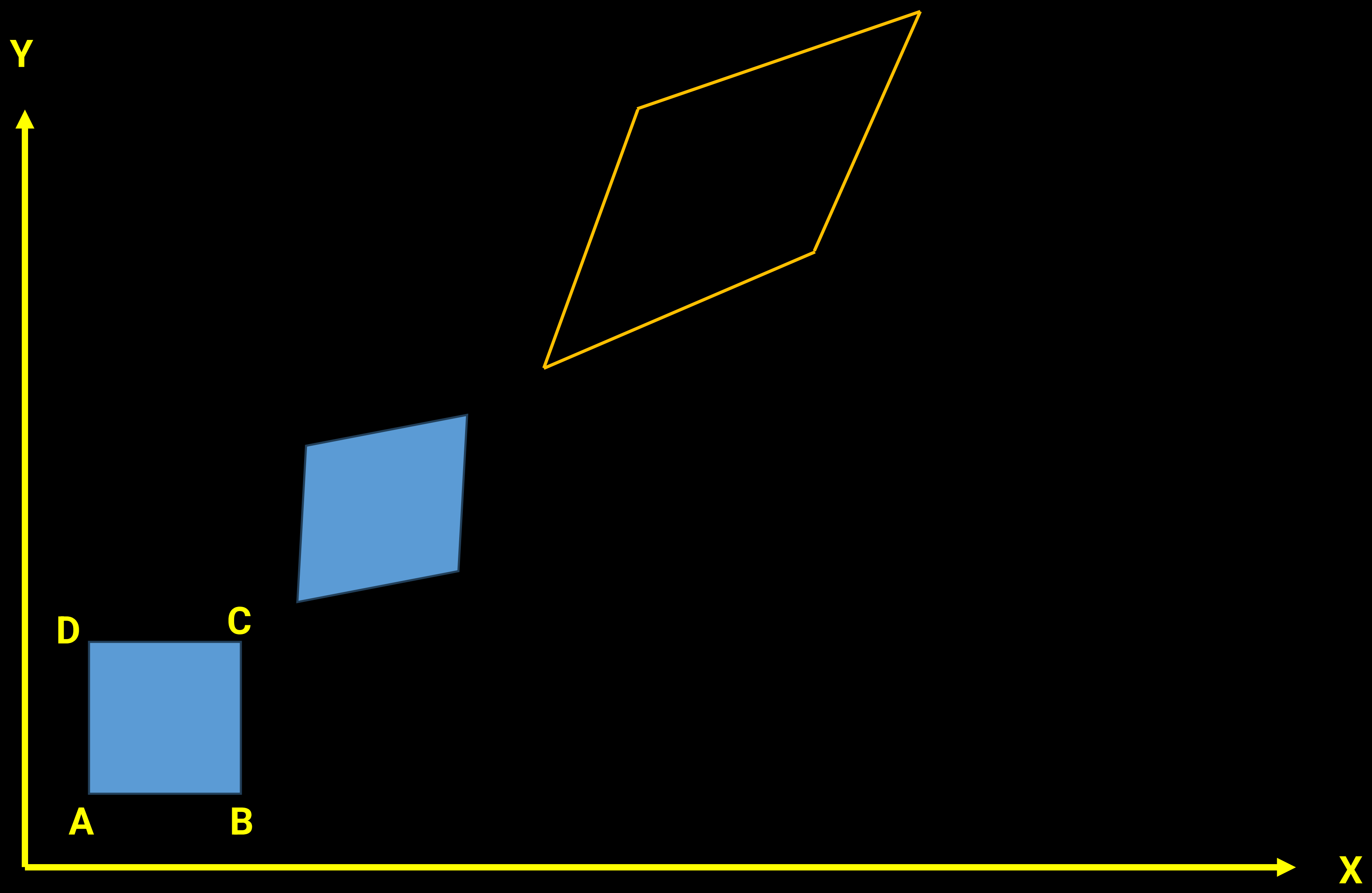


Linear strain, $\varepsilon_x = \frac{BB'}{AB} = \frac{\frac{\partial u}{\partial x} \cdot dx \, dt}{dx} = \frac{\partial u}{\partial x} \cdot dt$

Linear strain rate, $\dot{\varepsilon}_x = \frac{\varepsilon_x}{dt} = \frac{\partial u}{\partial x}$

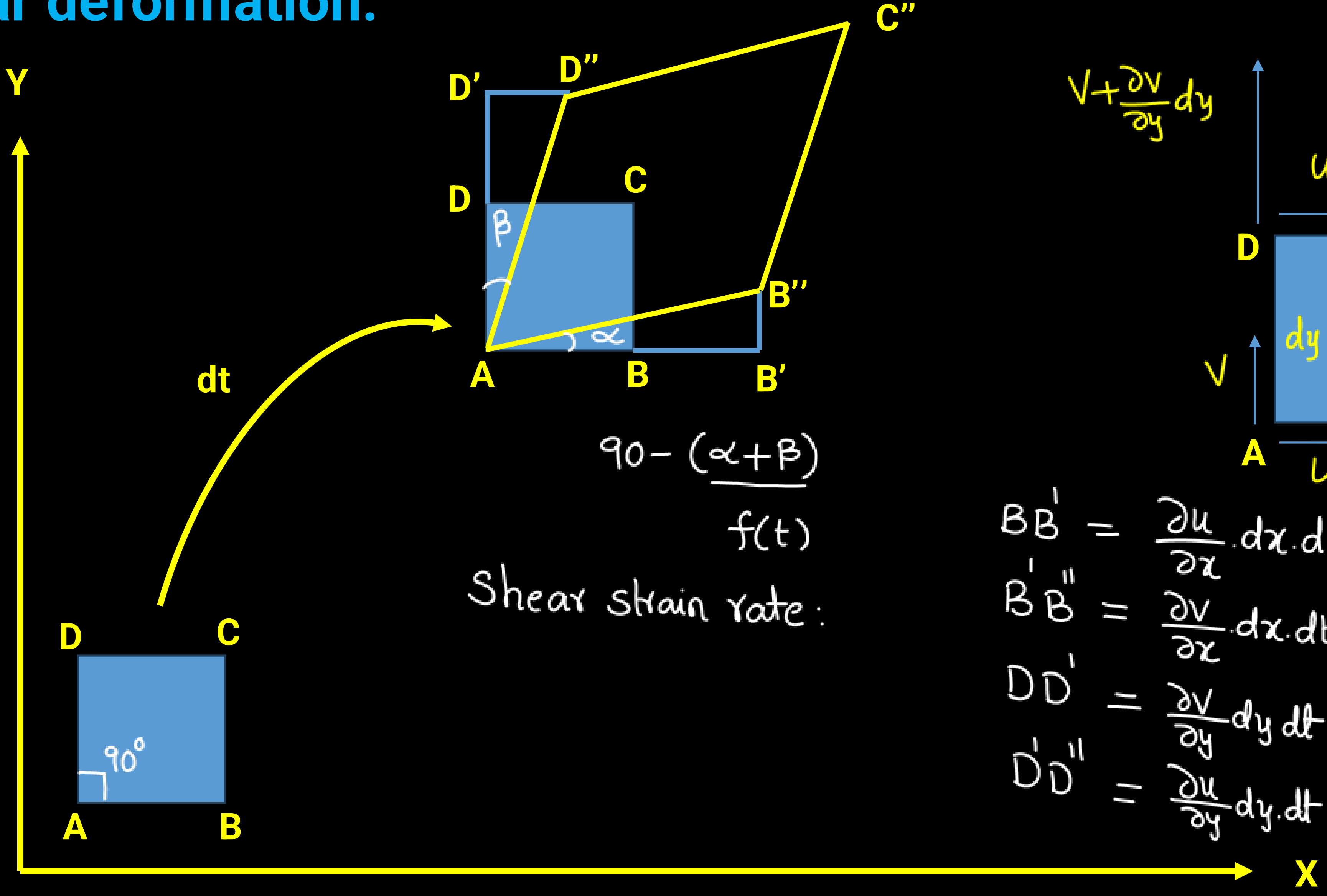
$''_y$, $\dot{\varepsilon}_y = \frac{\varepsilon_y}{dt} = \frac{\partial v}{\partial y}$

$\dot{\varepsilon}_z = \frac{\varepsilon_z}{dt} = \frac{\partial w}{\partial z}$



Y
D
C
A
B
X

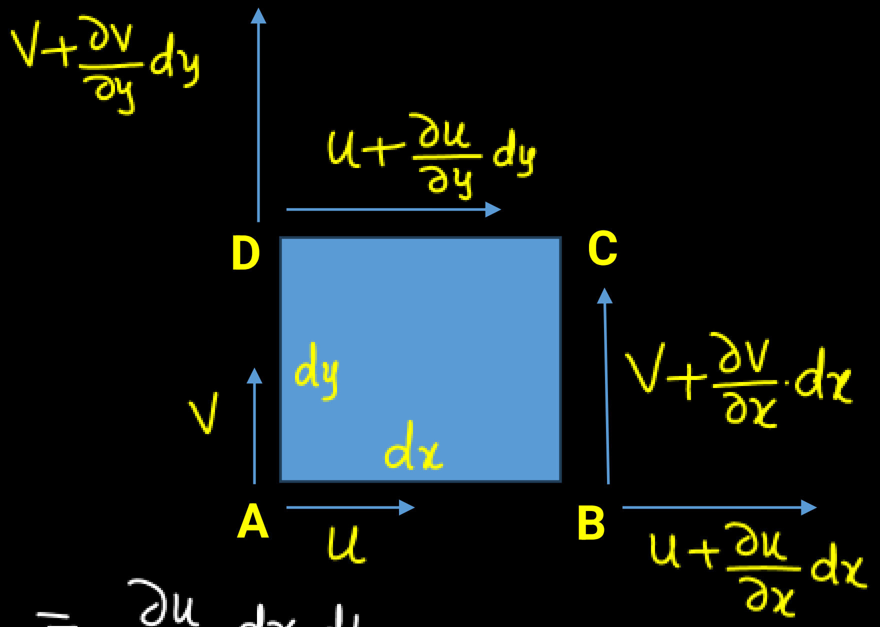
Shear deformation:



$90 - (\alpha + \beta)$

$f(t)$

Shear strain rate:



$$BB' = \frac{\partial u}{\partial x} \cdot dx \cdot dt$$

$$B'B'' = \frac{\partial v}{\partial x} \cdot dx \cdot dt$$

$$DD' = \frac{\partial v}{\partial y} dy\, dt$$

$$D'D'' = \frac{\partial u}{\partial y} dy \cdot dt$$

$$\alpha = \frac{B'B''}{AB+BB'} = \frac{\frac{\partial v}{\partial x}dx.dt}{dx+\frac{\partial u}{\partial x}.dx\,dt} = \frac{\frac{\partial v}{\partial x}.dt}{1+\cancelto{0}{\frac{\partial u}{\partial x}dt}}$$

$\frac{\partial u}{\partial x}dt$ & $\frac{\partial v}{\partial x}dt$ are very small

$$\alpha = \frac{\partial v}{\partial x}.dt$$

$$\dot{\alpha} = \frac{\alpha}{dt} = \boxed{\frac{\partial v}{\partial x}} \quad ; \quad ||_y \quad \dot{\beta} = \boxed{\frac{\partial u}{\partial y}}$$

Total shear strain rate

$$\dot{\varepsilon}_{xy} = (\dot{\alpha} + \dot{\beta})$$
$$= \left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$$

$$\dot{\varepsilon}_{PQ} = \left[\frac{\partial V_Q}{\partial P} + \frac{\partial V_P}{\partial Q}\right]$$

$$\dot{\varepsilon}_{yz} = \left[\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right]$$

$$\dot{\varepsilon}_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}$$

Avg. shear strain rate

$$\dot{\gamma}_{xy} = \frac{1}{2}\left[\dot{\alpha} + \dot{\beta}\right]$$
$$= \frac{1}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$$

$$\dot{\gamma}_{PQ} = \frac{1}{2}\left[\frac{\partial V_Q}{\partial P} + \frac{\partial V_P}{\partial Q}\right]$$

$$\dot{\gamma}_{yz} = \frac{1}{2}\left[\frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}\right]$$

$$\dot{\gamma}_{zx} = \frac{1}{2}\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right]$$

Shear stress:

$$\tau_{xy} = \mu \dot{\varepsilon}_{xy}$$

$$= \mu \left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right]$$

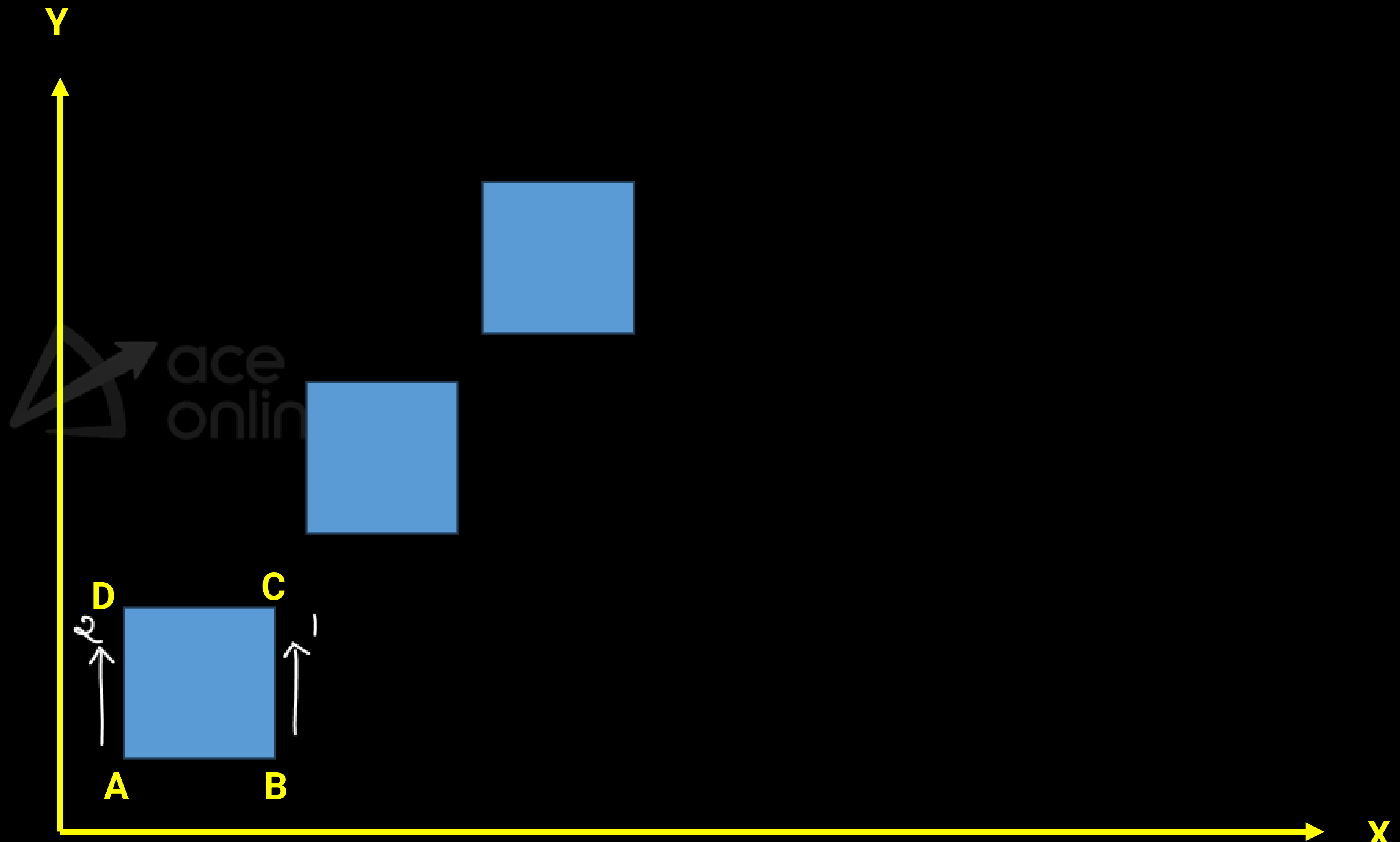
1D

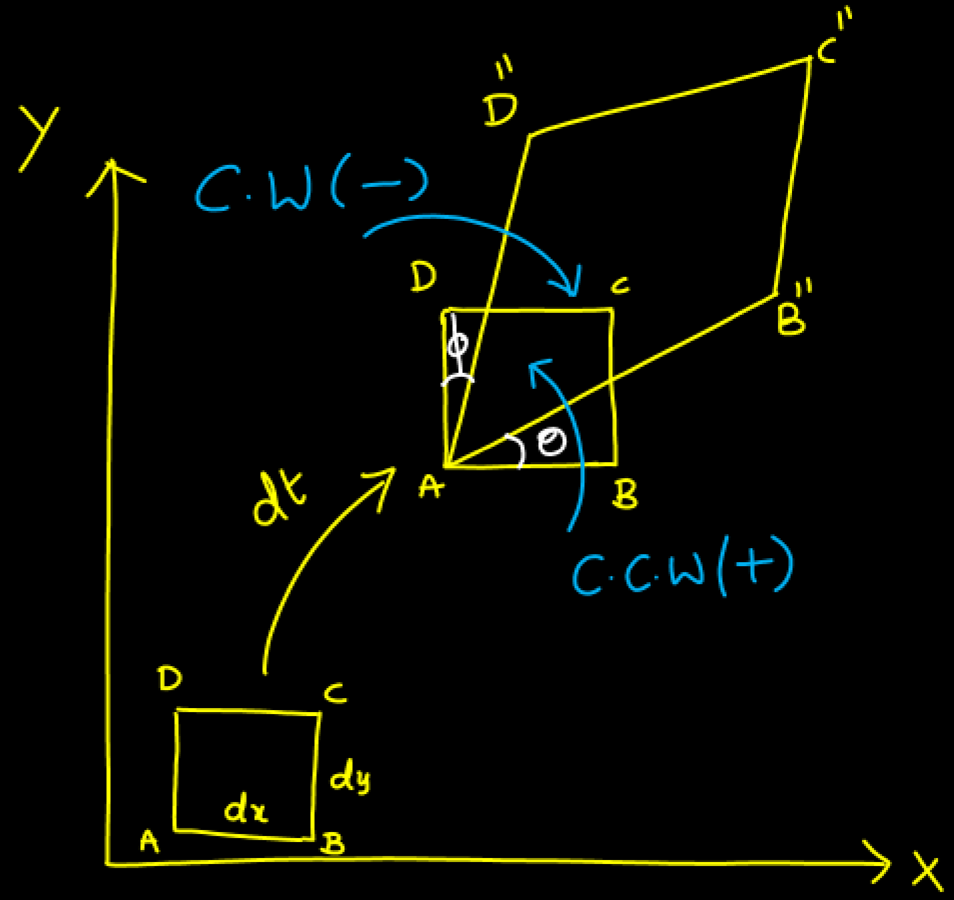
flow is 1D

$v \to 0$

$$\tau_{xy} = \mu \left[\frac{\partial u}{\partial y} \right] \longrightarrow \text{Newton's Law of viscosity.}$$

Rotation:





$$\theta = \frac{\partial v}{\partial x} dt$$

↑

Angular displacement

Angular velocity, $\dot{\theta} = \frac{\theta}{dt} = \frac{\partial v}{\partial x}$

lly, $\dot{\phi} = -\frac{\partial u}{\partial y}$

Def: The Arithmetic mean of angular velocities of two mutually $\perp$r Line segments in a fluid element.

$$\omega = \frac{1}{2}\left[\dot{\theta}+\dot{\phi}\right] = \frac{1}{2}\left[\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}\right]$$

Rotation is half of curl of velocity vector.

$$\omega = \frac{1}{2}\,\text{curl}\,\vec{V}$$

$$= \frac{1}{2}\,\vec{\nabla}\times\vec{V}$$

$$\omega = \frac{1}{2}\begin{array}{c} \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & v & w \end{vmatrix} \\ \quad + \quad\;\; - \quad\;\; + \end{array}$$

$$\omega_x = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

$$\omega_y = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

$$\omega_z = \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

Vorticity: ξ

The curl of velocity vector

$$\xi = \text{curl } \vec{V}$$

$$= 2\omega$$

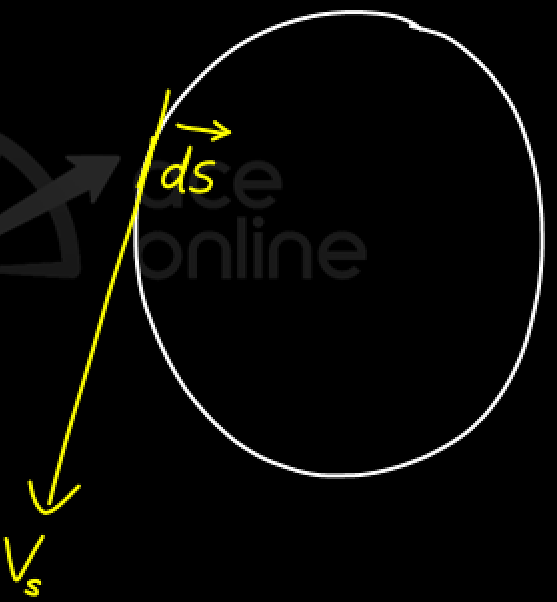
$$\xi_x = \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right)$$

$$\xi_y = \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right)$$

$$\xi_z = \left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

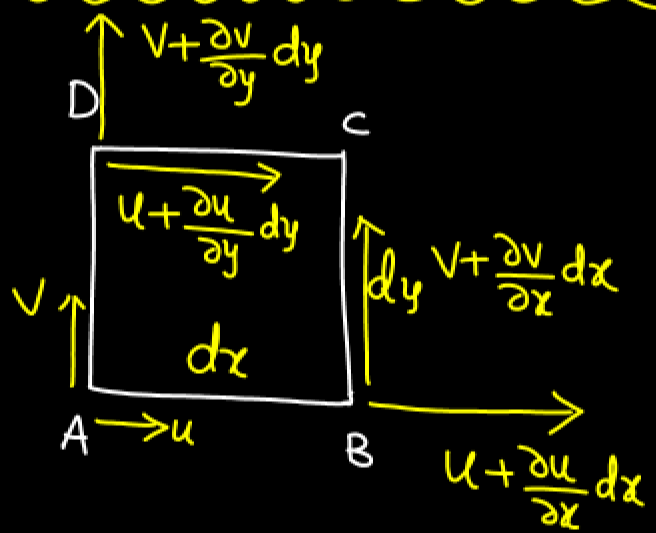
Circulation: (Γ)

Def: The line integral of Tangential component of velocity along a closed Loop.



$$\Gamma = \oint V_s \cdot ds$$

Relationship b/w Γ & ξ_z:



Differential area = $dx\,dy$

Circulation per unit area:

$$d\Gamma = u\,dx + \left[v + \frac{\partial v}{\partial x}dx\right]dy - \left[u + \frac{\partial u}{\partial y}\cdot dy\right]dx - v\,dy$$

$$= \frac{\partial v}{\partial x}dx\,dy - \frac{\partial u}{\partial y}dx\,dy$$

$$= dx\,dy\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$d\Gamma = dA\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right]$$

$$d\Gamma = dA \times \text{Vorticity}$$

$$\left\{\frac{\Gamma}{A} = \text{Vorticity}\right\}$$

$$\text{Vorticity} = \frac{\text{Circulation}}{\text{Area}}$$

Classification of flow based on rotation:

1. Rotational flow
2. Irrotational flow

Rotational flow:

A flow in which a fluid element rotates about it's mass center.

Condition:

At least one of the rotational components should be non zero.

$\omega_x \neq 0$ (or) $\omega_y \neq 0$ (or) $\omega_z \neq 0$

Irrotational flow:

A flow in which a fluid element does not rotate about it's mass center.

Condition:

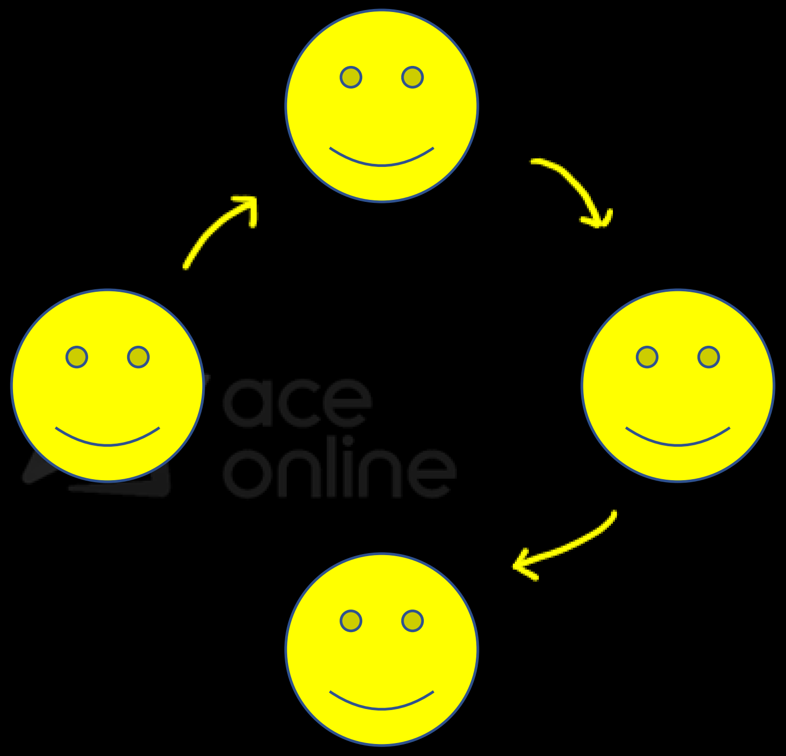
At least one of the rotational components should be zero.

$$\omega_x = 0 \text{ (or) } \omega_y = 0 \text{ (or) } \omega_z = 0$$

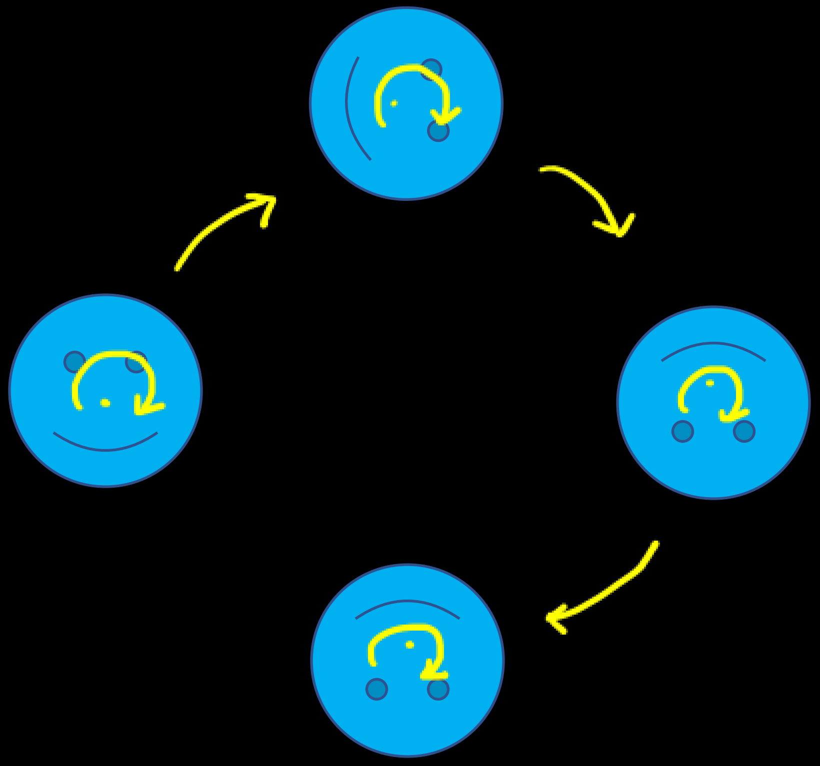
$$\omega_z = 0 \Rightarrow \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = 0$$

$$\frac{\partial v}{\partial x} = \frac{\partial u}{\partial y}$$

Cross velocity gradients are equal.



Irrotational flow



Rotational flow

Stream function: (ψ)

Definition:

a function is defined in a 2D flow field such that it takes a constant value along a particular streamline.

$d\psi = 0$ for a Particular streamline.

ψ is defined such that,

$$\frac{\partial \psi}{\partial x} = v \; ; \quad \frac{\partial \psi}{\partial y} = -u$$

(or)

$$\frac{\partial \psi}{\partial x} = -v \; ; \quad \frac{\partial \psi}{\partial y} = u$$

Stream function, $\psi = f(x, y)$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

$$d\psi = v\,dx - u\,dy \longrightarrow (1)$$

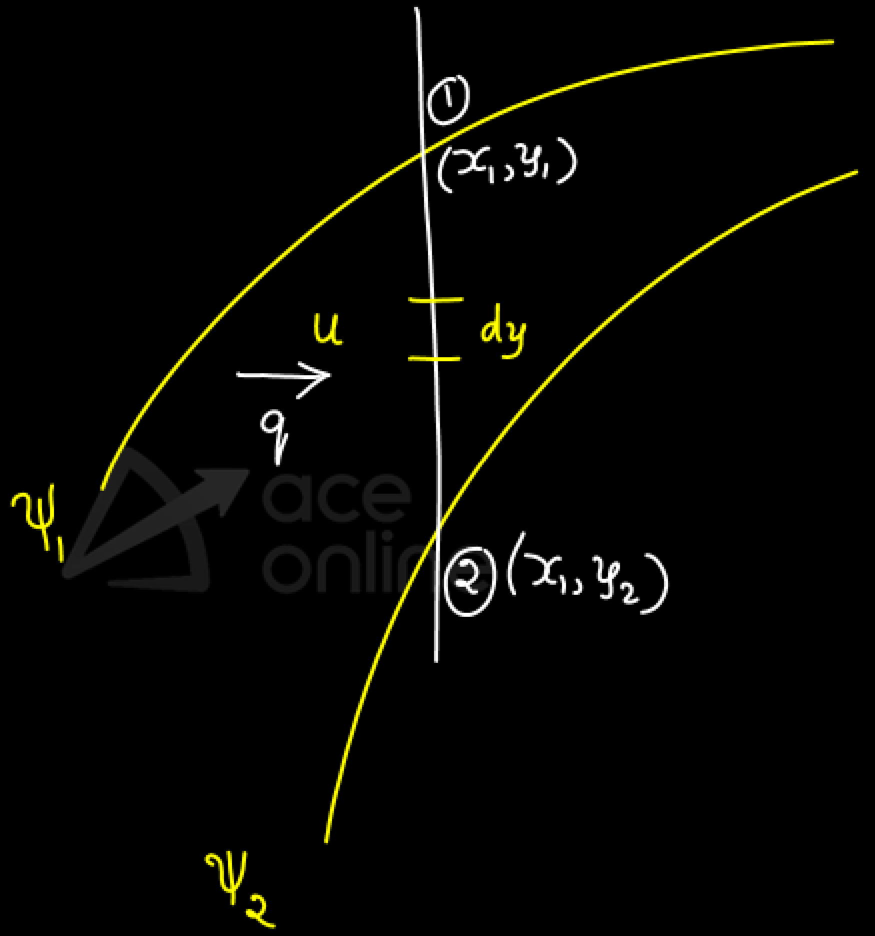
Eqn of a streamline, $\frac{dx}{u} = \frac{dy}{v} \Rightarrow v\,dx = u\,dy$

$$v\,dx - u\,dy = 0 \longrightarrow (2)$$

From (1) and (2)

$$d\psi = 0$$

Application:



q : discharge per unit width

$$dq = (dy \times 1) \cdot u = u\,dy$$

$$dq = \frac{\partial \psi}{\partial y} \cdot dy \longrightarrow ①$$

$$\psi = f(x, y)$$

$$d\psi = \frac{\partial \psi}{\partial x} dx + \frac{\partial \psi}{\partial y} dy$$

$$\boxed{d\psi = \frac{\partial \psi}{\partial y} dy}$$

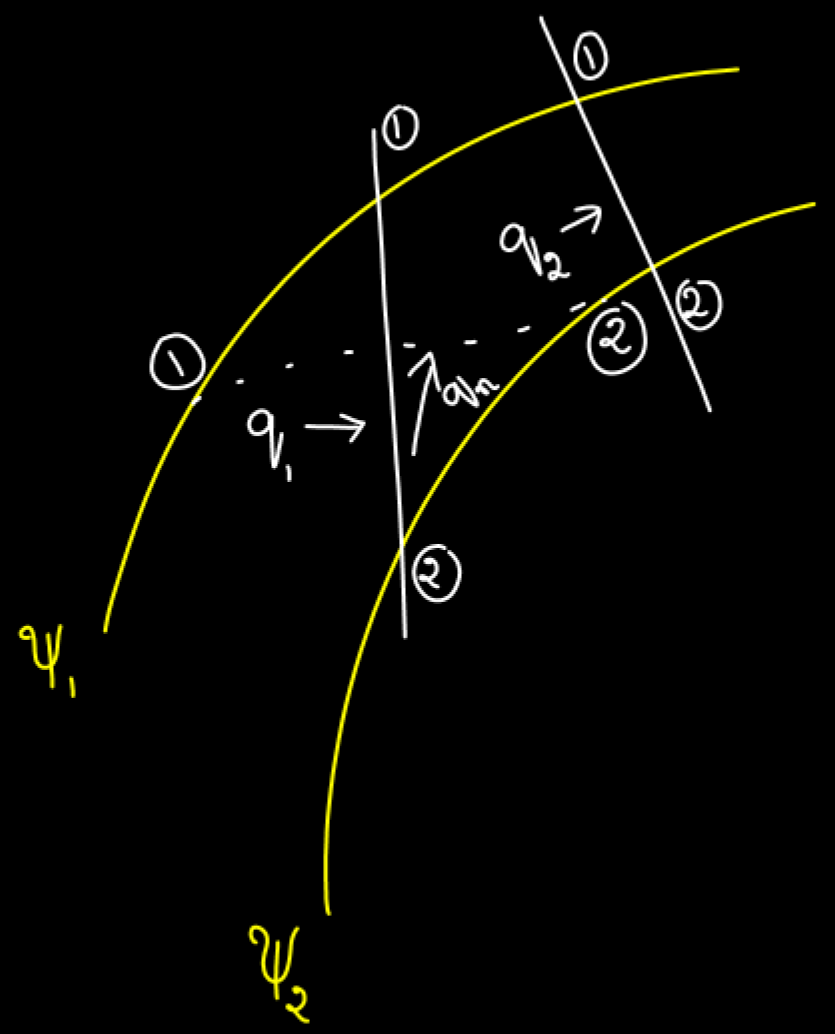
$$dq = d\psi$$

$$\int dq = \int_{\psi_1}^{\psi_2} d\psi$$

$$\boxed{q = |\psi_2 - \psi_1| = |\Delta\psi|}$$

Note: The difference in streamfunction gives discharge per unit width b/w corresponding streamlines.

Physical significance:



$$q_1 = |\psi_1 - \psi_2|$$

$$q_2 = |\psi_1 - \psi_2|$$

$$q_n = |\psi_1 - \psi_2|$$

Note: "q" is constant for any section b/w two different streamlines.

Potential function: (ϕ)

$$\vec{\nabla} \times \vec{V} = 0 \rightarrow \text{Irrotational flow}$$

$$\vec{V} = \vec{\nabla} \cdot \phi$$

Valid only for Irrotational flow.

$\downarrow$

Velocity Potential function

Irrotational flow is also known as Potential flow.

$$-\frac{\partial \phi}{\partial x} = u \; ; \; -\frac{\partial \phi}{\partial y} = V \; ; \; -\frac{\partial \phi}{\partial z} = W$$

(Or)

$$\frac{\partial \phi}{\partial x} = u \; ; \; \frac{\partial \phi}{\partial y} = V \; ; \; \frac{\partial \phi}{\partial z} = W$$

Note: -ve sign represents the flow takes place in the direction of decrease in Potential.

Equipotential line:

It is a curve along which 'ϕ' takes a constant value.

$$d\phi = 0$$

$$\phi = f(x, y)$$

$$d\phi = \frac{\partial \phi}{\partial x} dx + \frac{\partial \phi}{\partial y} dy$$

$$d\phi = u\,dx + v\,dy$$

For Equipotential line, $d\phi = 0$

$$u\,dx + v\,dy = 0$$

$$\frac{dy}{dx} = \frac{-u}{v}$$

$$\boxed{\left.\frac{dy}{dx}\right|_{Equ} = \frac{-u}{v}} \rightarrow (1)$$

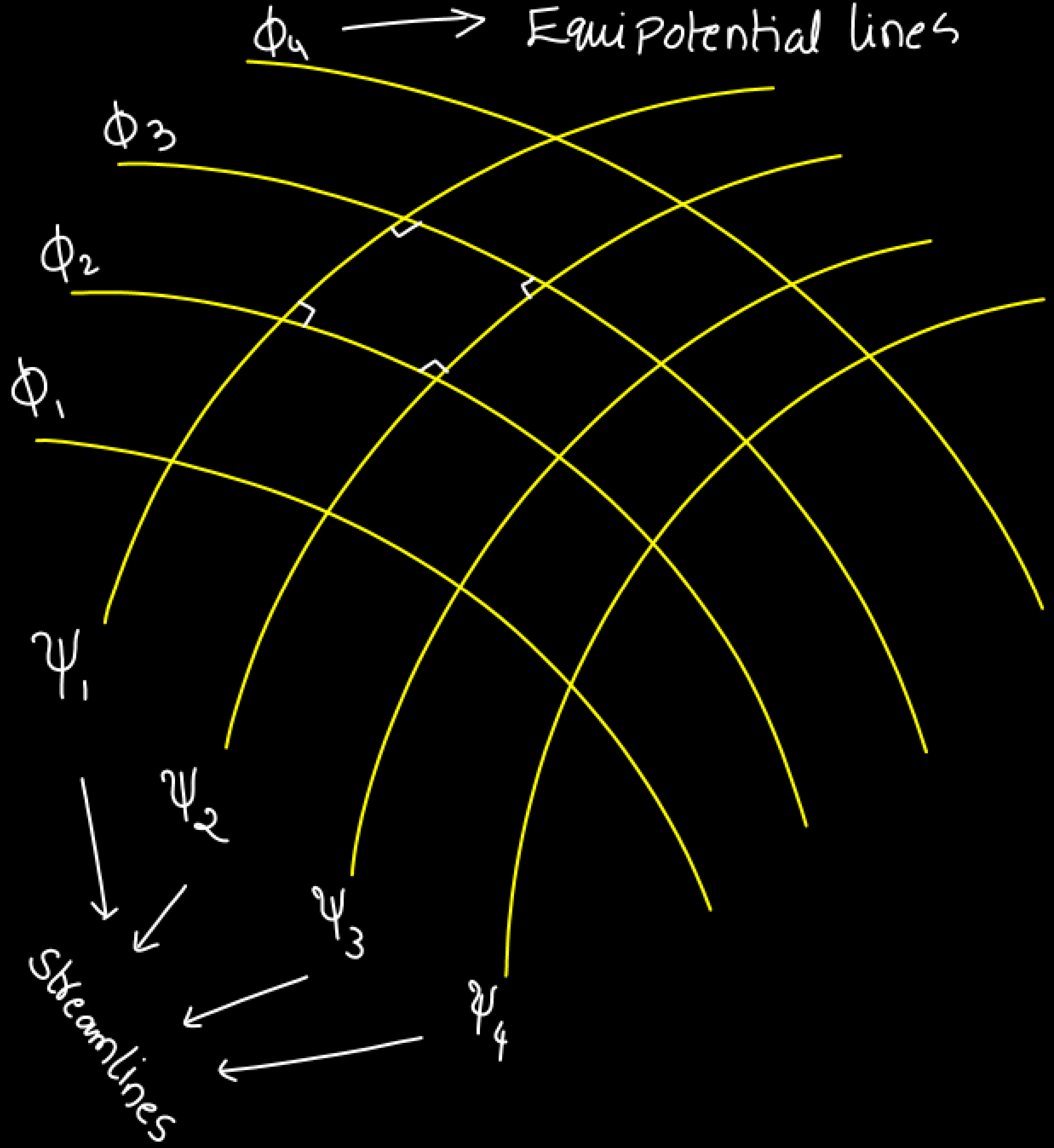
$$\frac{dy}{dx} = \frac{v}{u}$$

$$\boxed{\left.\frac{dy}{dx}\right|_{St.\,line} = \frac{v}{u}} \rightarrow (2)$$

(1) × (2)

$$\left.\frac{dy}{dx}\right|_{Equ} \times \left.\frac{dy}{dx}\right|_{St.\,line} = -1$$

Note: Equipotential lines and Streamlines are orthogonal to each other, except at Stagnation point.



→ Flow net

Def: It is the grid in a flow field by drawing Equipotential lines and Streamlines [flow lines].

Laplace equation:

Incompressible flow:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0$$

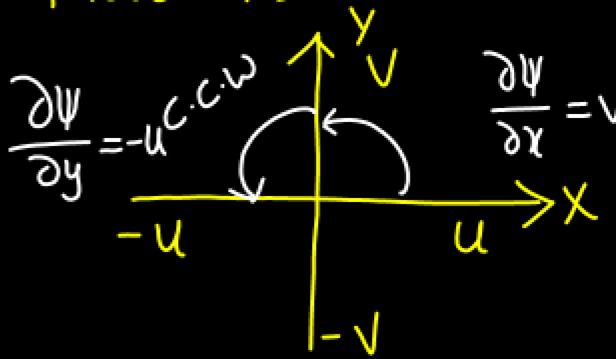
$$\frac{\partial \phi}{\partial x}=u \; ; \; \frac{\partial \phi}{\partial y}=v \; ; \; \frac{\partial \phi}{\partial z}=w$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \phi}{\partial x}\right)+\frac{\partial}{\partial y}\left(\frac{\partial \phi}{\partial y}\right)+\frac{\partial}{\partial z}\left(\frac{\partial \phi}{\partial z}\right)=0$$

Laplace equation

$$\left\{\frac{\partial^2 \phi}{\partial x^2}+\frac{\partial^2 \phi}{\partial y^2}+\frac{\partial^2 \phi}{\partial z^2}=0\right\} \quad (\text{or}) \quad \left\{\nabla^2 \phi = 0\right\}$$

If 'ϕ' Satisfies Laplace equation, it represents the flow is Incompressible & Irrotational.



Irrotational:

$$\omega_z = 0 \Rightarrow \frac{1}{2}\left[\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right] = 0 \Rightarrow \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = 0$$

$$\frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial \psi}{\partial y}\right) = 0$$

$$\left\{ \frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} = 0 \text{ (or) } \nabla^2 \psi = 0 \right\}$$

Fluid Dynamics

One Mark Questions

Q. The Pitot - static tube measures

(GATE - 89)

(a) Static pressure

(b) Dynamic pressure

(c) Difference in static and dynamic pressure

(d) Difference in total and static pressures.

Velocity measurement:

$$\frac{P}{\gamma} + \frac{V^2}{2g} + Z = H$$

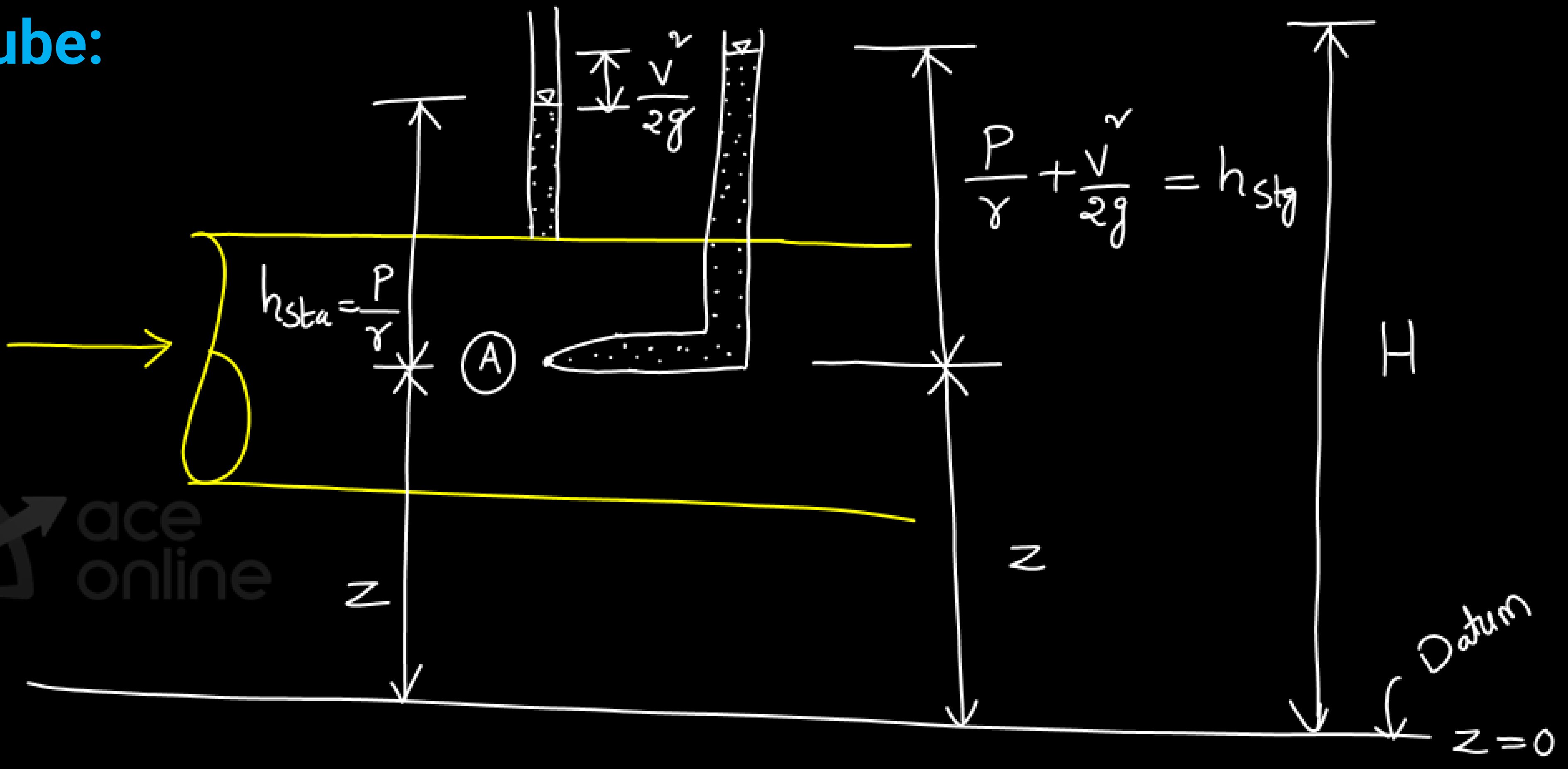
$$\frac{V^2}{2g} = H - \left(\frac{P}{\gamma} + Z\right)$$

$$= \text{Total Head} - \text{Piezometric head.}$$

Total head : Pitot tube

Piezometric head : Piezometer.

Pitot tube:



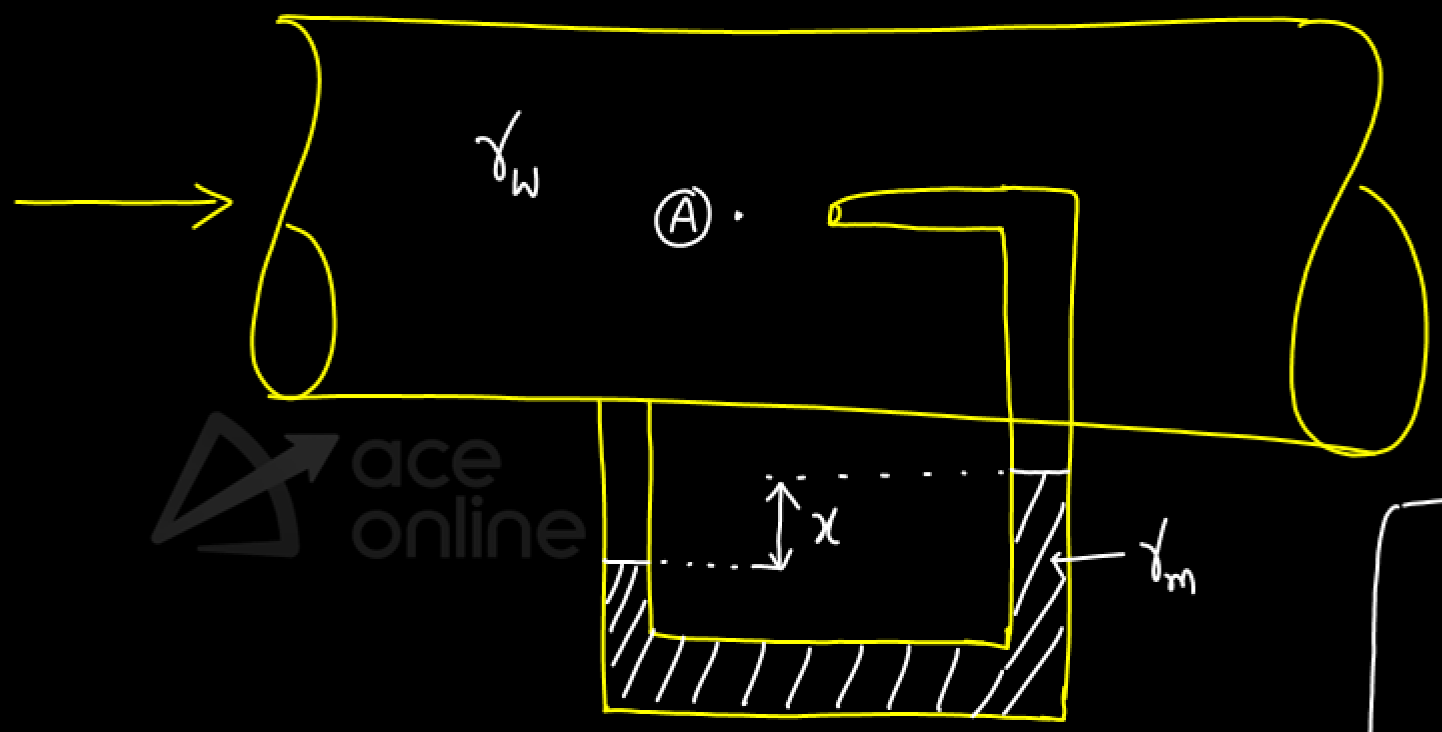
$$\frac{V^2}{2g} = \Delta h \Rightarrow V = \sqrt{2g\Delta h}$$

$$V_{th} = \sqrt{2g\,\Delta h}$$

$$V_a = C_v \sqrt{2g\Delta h}$$

$$C_v = \frac{V_a}{V_{th}}$$ Coefficient of velocity.

Pitot – static tube :



$$h = x\left[\frac{S_m}{S_w} - 1\right]$$

$$V = \sqrt{2gx\left[\frac{S_m}{S_w} - 1\right]}$$

Gate CE

① $V = \sqrt{2gh_m}$

② $V = \sqrt{2gx\left(1 - \frac{S_m}{S_w}\right)}$

③ $V = \sqrt{2gx\left(\frac{S_m}{S_w}\right)}$

④ $V = \sqrt{2gx\left(\frac{S_w}{S_m} - 1\right)}$

Q. The most appropriate governing equations of ideal fluid flow are

(GATE - 90)

(a) Euler’s equations

(b) Navier stokes equation

(c) Reynold’s equations

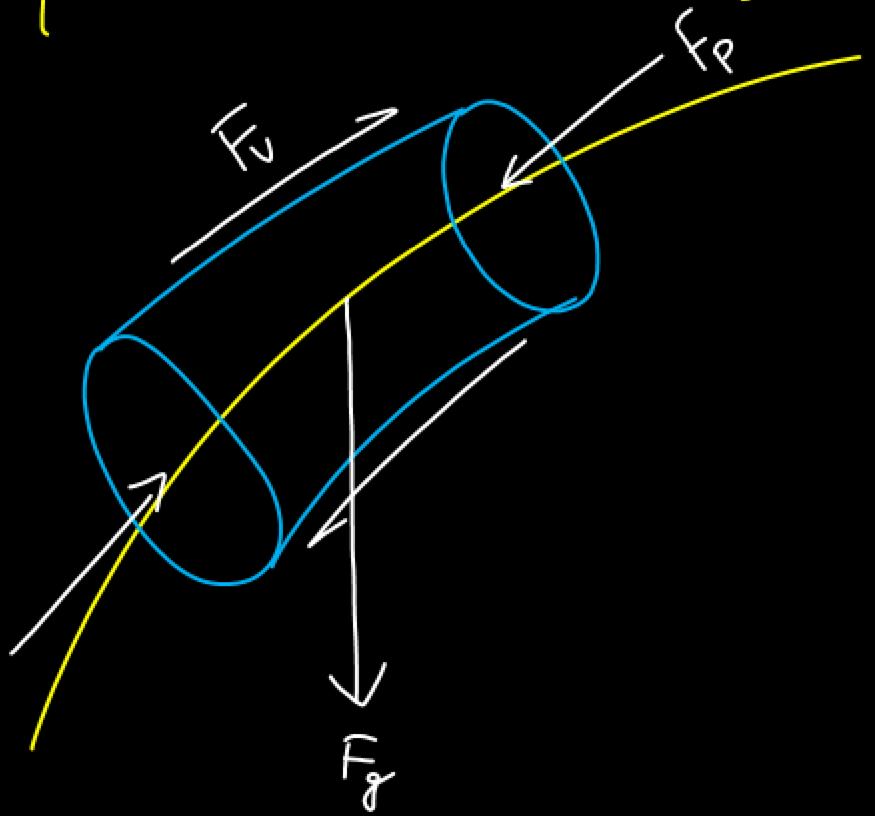
(d) Hagen-poiseuille equations

Bernoulli's Eqn:

It is based on Newton's 2^{nd} Law

$$\sum F_s = m a_s$$

[S: Streamline direction]



1. F_p = Pressure force
2. F_g = gravitational force.
3. F_v = viscous force
4. F_t = Turbulant force
5. F_s = Surface tension force.
6. F_c = Compressible force.

$$F_p + F_g + F_v + F_t + F_s + F_c = ma_s \longrightarrow \text{Newton's eqn}$$

$$F_p + F_g + F_v + F_t = ma_s \longrightarrow \text{Reynold's eqn}$$

$$F_p + F_g + F_v = ma_s \longrightarrow \text{Navier-Stokes eqn}$$

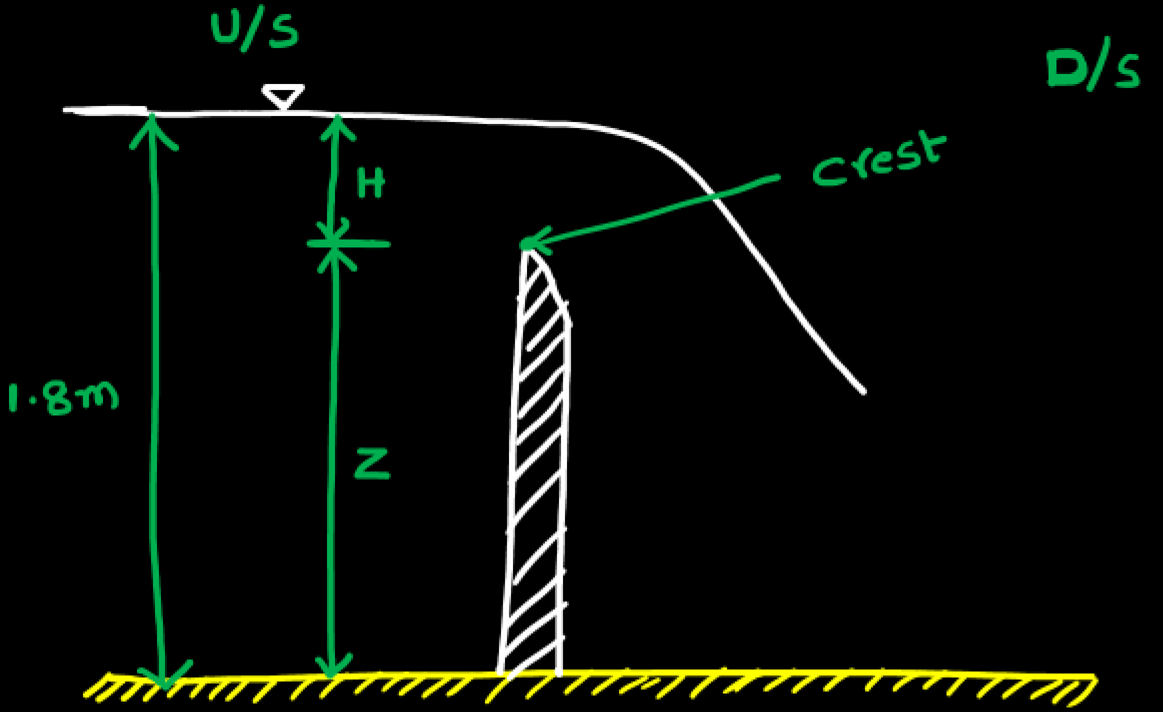
$$F_p + F_g = ma_s \longrightarrow \text{Euler's eqn}$$

Sol: given data,

Length, $L = 6m$

Discharge, $Q = 2000\ lit/s = 2\ m^3/s$

$C_d = 0.6$



Z = height of weir

H = head over the weir

Discharge through rectangular weir is given by,

$$Q = \frac{2}{3} C_d \sqrt{2g}\, L\, H^{3/2}, \quad \text{Neglect end contractions}$$

$$2 = \frac{2}{3}(0.6)\sqrt{2 \times 9.81} \times 6 \times H^{3/2}$$

$$H = 0.328\,m$$

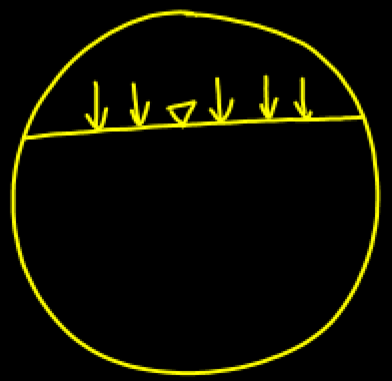
$\therefore$ the height of the weir $z = 1.8 - H$

$$= 1.8 - 0.328$$

$$\boxed{z = 1.472\,m}$$

Chapter 6
Flow through pipes

- It is known as closed conduit flow.
- It is also called as pressure flow.



NP pipes:

Non pressure pipes: the pipes which runs partially full and subjected to atmospheric pressure.

Examples : sewers, culverts.

Classification of flows:

- ✓ Based on the configuration of fluid particles while in motion pipe flow can be classified.
- ✓ Reynold's experiment is used for the classification of flows.

Reynold's number: (R_e)

Reynold's number (R_e) = $\dfrac{Inertia\ force}{Viscous\ force}$

Inertia force:

It is the force due to motion of the body corresponding to its acceleration

Viscous force:

The force of resistance against motion

Inertia force = mass x acceleration

$$= m\,a$$

$$= \frac{\rho \forall L}{T^2}$$

$$\left\{ \rho = \frac{m}{\forall} \right\}$$

$$= \frac{\rho L^3 . L}{T^2} \Rightarrow \rho \left(\frac{L}{T}\right)^2 L^2$$

$$\boxed{\text{Inertia force} = \rho V^2 L^2} \rightarrow (1)$$

Viscous force: Shear stress x Area

$$= \tau \times A$$

$$= \mu \frac{du}{dy} \times A$$

$$= \mu \frac{V}{L} \cdot L^2$$

$$\boxed{\text{Viscous force} = \mu V L} \rightarrow (2)$$

$$\text{Reynold's number} = \frac{\rho V^2 L^2}{\mu V L}$$

$$\boxed{R_e = \frac{\rho V L}{\mu}} \quad (\text{or}) \quad \boxed{R_e = \frac{VL}{[\mu/\rho]} = \frac{VL}{\gamma}}$$

ρ = Density of fluid

V = Avg. Velocity of flow

L = Characteristic length of a fluid system.

Formulae for Reynold’s number: (R_e)

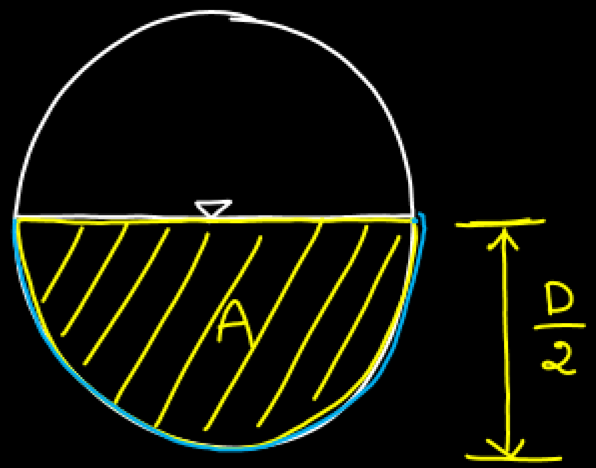
1. Circular pipe
2. Non circular pipe
3. Flow over flat plates

① $R_e = \dfrac{\rho V d}{\mu}$; $L = d$ for circular pipes

② $R_e = \dfrac{\rho V L}{\mu}$; $L = 4R$ for Non-circular pipes.

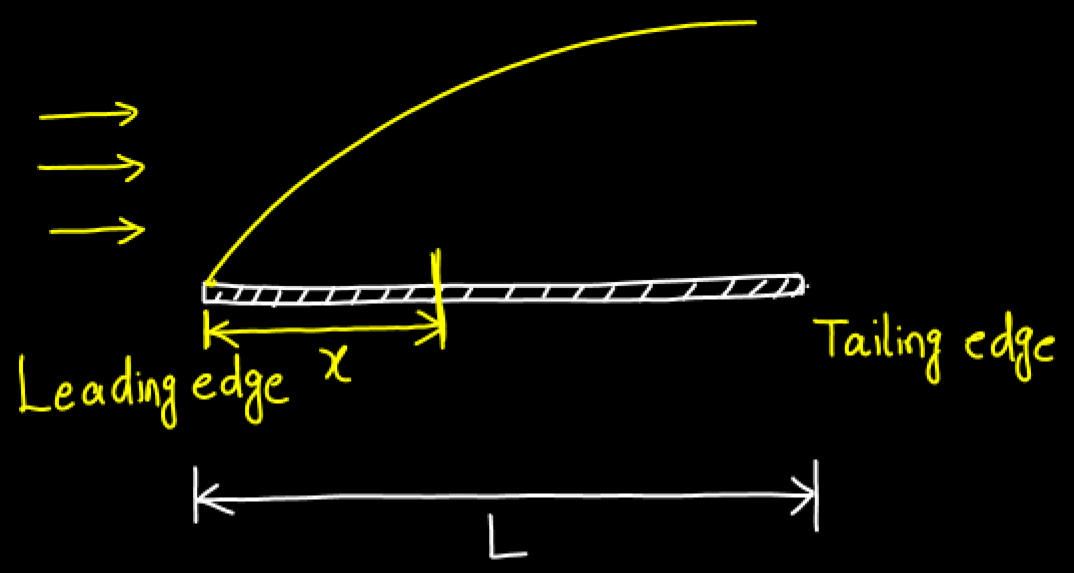
R = Hydraulic Radius (or) Hydraulic mean depth

$$= \frac{\text{Wetted area}}{\text{Wetted Perimeter}} = \frac{A}{P}$$

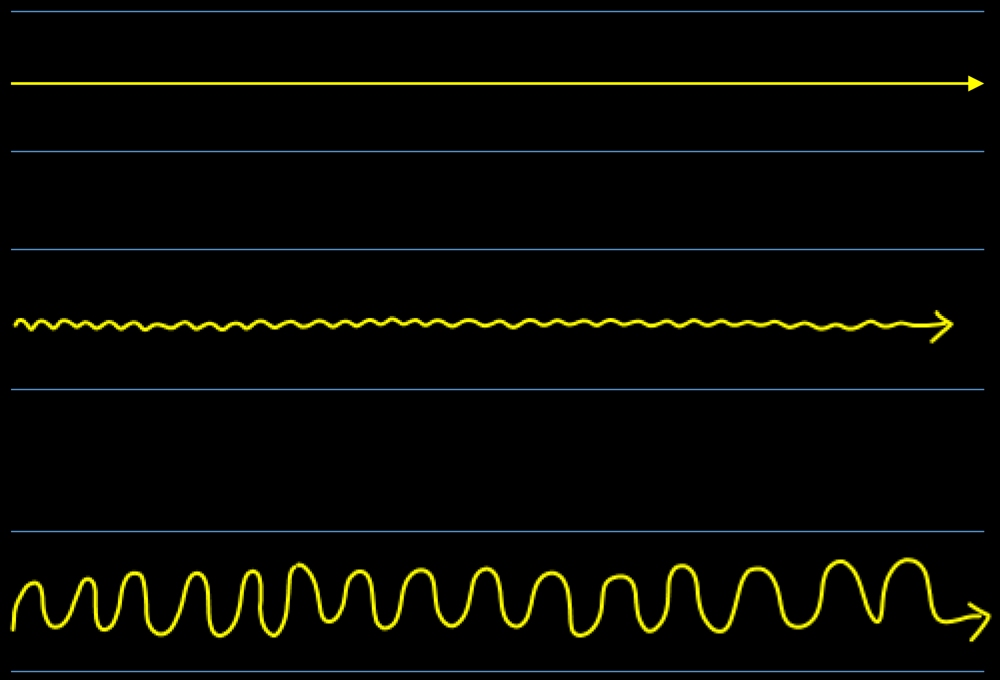


$$R = \frac{\pi D^2}{8} \Big/ \frac{\pi D}{2} = \frac{D}{4}$$

3 $$R_e = \frac{\rho V L}{\mu} \; ; \; L = x$$


Leading edge
x
Tailing edge
L

Reynold’s experiment:

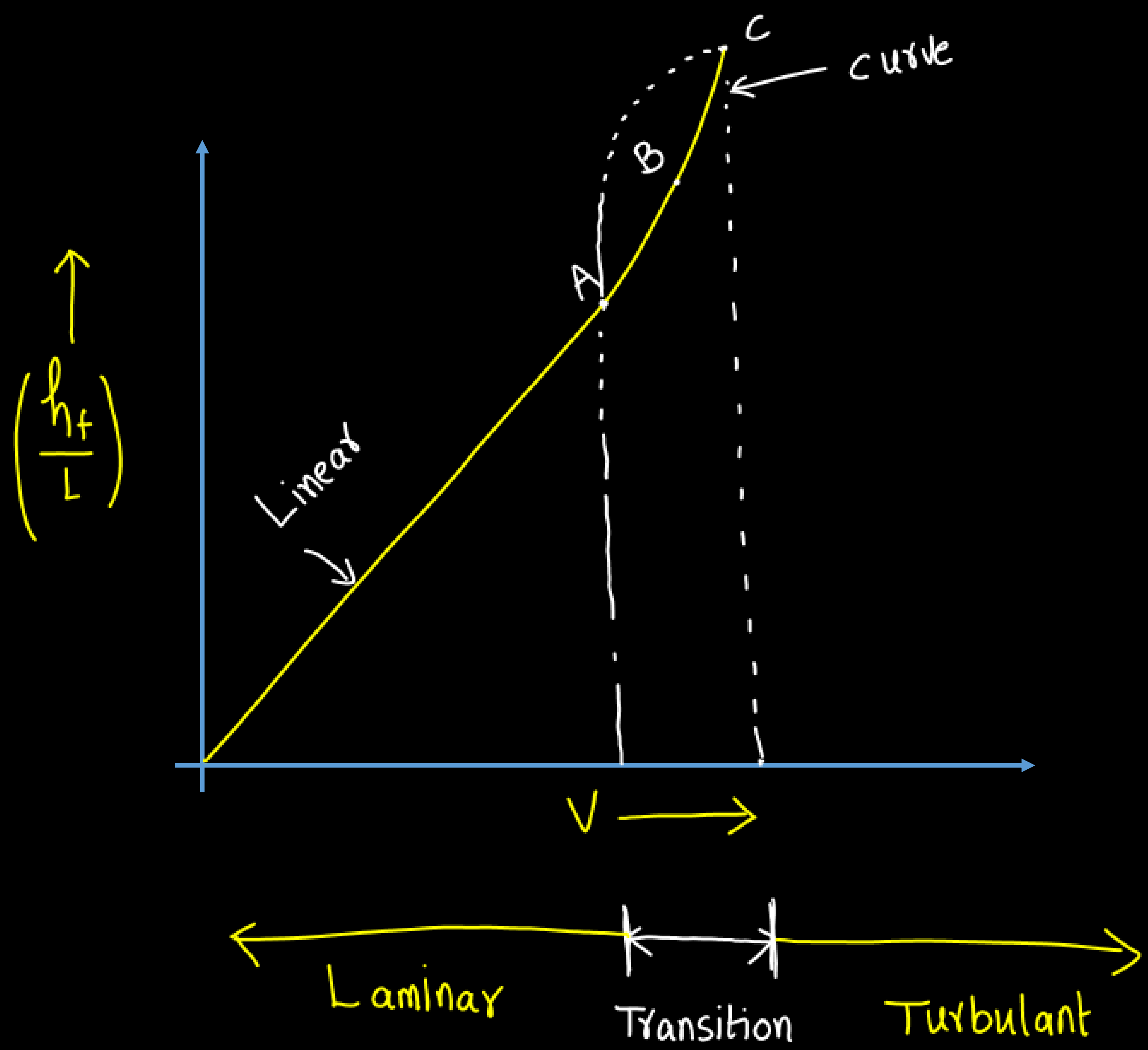


Laminar flow [$R_e < 2000$]

Transitional flow [R_e: 2000 – 4000]

Turbulent flow [$R_e > 4000$]

Reynold's chart:



A : Lower critical point (constant)

B : upper critical point
[It changes from experiment to experiment]

C : Limit of Transition

Head losses in pipe flow:

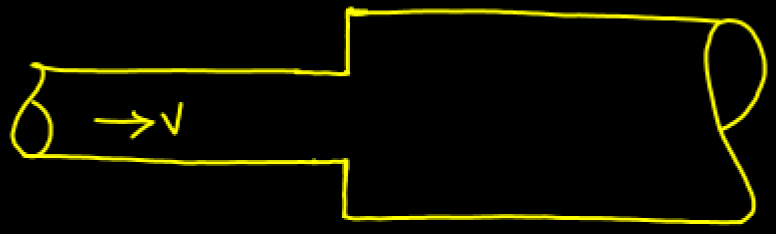
I. Major losses

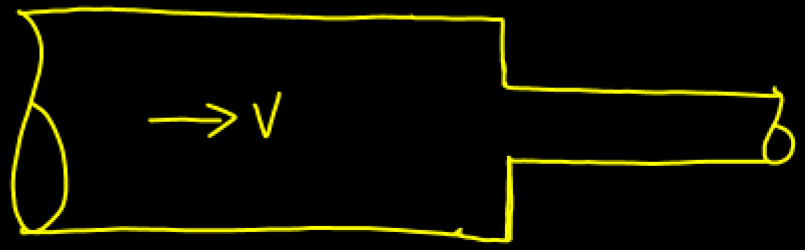
II. Minor losses

Contribution of minor losses in pipe flow are less than 5%

These losses includes:

- Loss due to sudden expansion
- Loss due to sudden contraction
- Entry loss
- Exit loss





Major losses:

- **It is due to friction in pipes**

Darcy-Weisbach equation:

$$\boxed{h_f = \frac{fLV^2}{2gd}}$$

$$= \frac{fLQ^2}{A^2 \cdot 2gd}$$

$$= \frac{16fLQ^2}{\pi^2 d^4 \cdot 2gd}$$

$$\boxed{h_f = \frac{8fLQ^2}{\pi^2 g d^5}}$$

$$\left\{ \begin{aligned} Q &= A \cdot V \\ V &= \frac{Q}{A} \\ A &= \frac{\pi d^2}{4} \end{aligned} \right.$$

$$\left\{ \frac{\pi^2 g}{8} = 12.1 \right\}$$

$$\boxed{h_f = \frac{fLQ^2}{12.1 d^5}}$$

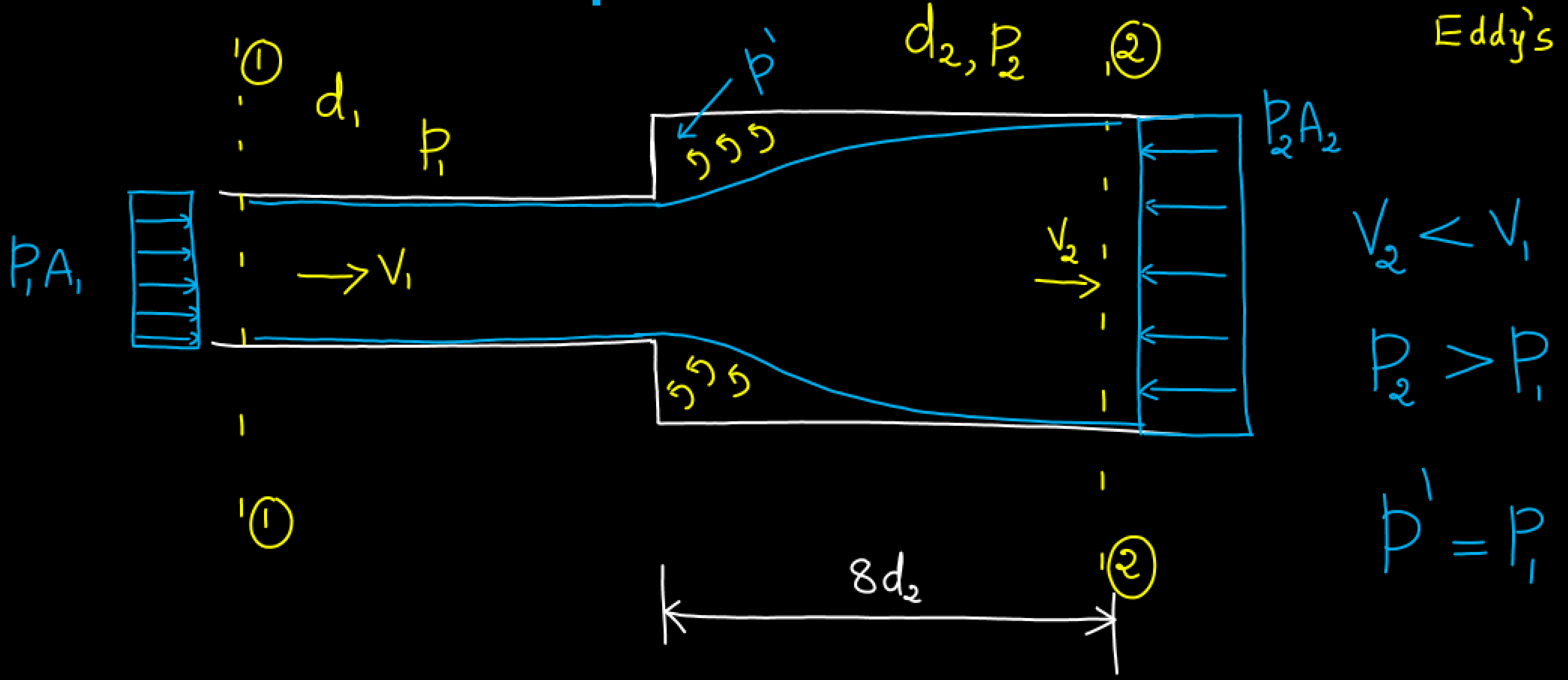
f = Darcy's friction factor

$$\boxed{f = 4f'}$$

f' = coefficient of friction (Fanny's friction coefficient)

$$\left\{ h_f \propto \frac{1}{d^5} ; \; h_f \propto V^2 \right\}$$

Loss due to sudden expansion:



$V_2 < V_1$

$p_2 > p_1$

$p' = p_1$

- Due to sudden change in cross-section(suddenly enlarged from small dia to large dia), the fluid emerging cannot follow the boundary of the pipe as the stream lines take diverging pattern as shown.

V_1 = Velocity of flow in Smaller dia Pipe

V_2 = Velocity of " " Larger dia pipe

As Per Continuity equation

$$A_1 V_1 = A_2 V_2$$

$$V_2 < V_1$$

As Per Bernoulli's equation.

If $V_2 < V_1$ then $P_2 > P_1$ [Pressure uphill]

Apply momentum eq^n in control volume.

$$\Sigma F = \dot{m}V_f - \dot{m}V_i$$

$$P_1A_1 - P_2A_2 + P'(A_2 - A_1) = \dot{m}V_2 - \dot{m}V_1 \qquad [\because P' = P_1]$$

$$P_1A_1 - P_2A_2 + P_1A_2 - P_1A_1 = \dot{m}(V_2 - V_1)$$

$$A_2[P_1 - P_2] = \rho Q(V_2 - V_1)$$

$$A_2(P_1 - P_2) = \rho A_2V_2(V_2 - V_1)$$

$$P_1 - P_2 = \rho V_2 (V_2 - V_1) \longrightarrow (1)$$

Apply Bernoulli's eqn b/w (1) & (2)

$$\frac{P_1}{\gamma} + \frac{V_1^2}{2g} + \cancel{Z_1} = \frac{P_2}{\gamma} + \frac{V_2^2}{2g} + \cancel{Z_2} + h_L \qquad [Z_1 = Z_2]$$

$$\frac{P_1}{\rho} + \frac{V_1^2}{2} = \frac{P_2}{\rho} + \frac{V_2^2}{2} + g h_L$$

$$\frac{1}{\rho}(P_1 - P_2) = \frac{V_2^2 - V_1^2}{2} + g h_L \longrightarrow (2)$$

Substitute eqn (1) in eqn (2)

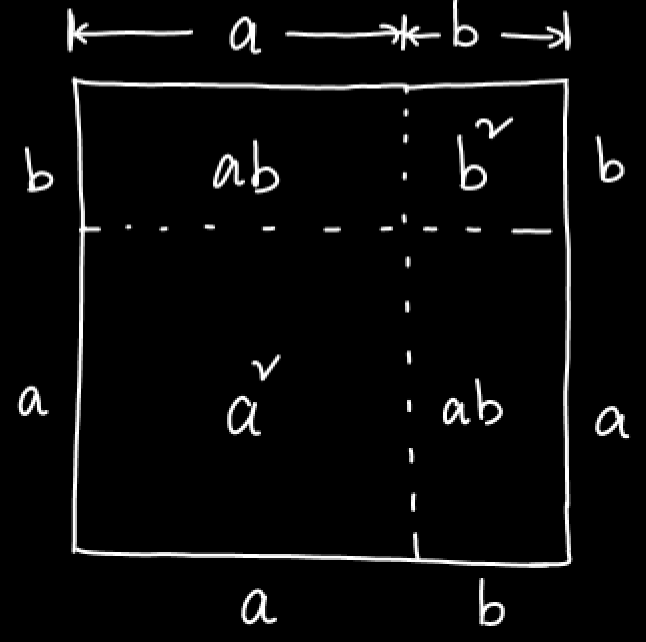
$$\frac{1}{\rho}\rho V_2(V_2 - V_1) = \frac{V_2^2 - V_1^2}{2} + g h_L$$

$$V_2^2 - V_1 V_2 = \frac{V_2^2 - V_1^2 + 2 g h_L}{2}$$

$$2V_2^2 - 2V_1V_2 + V_1^2 - V_2^2 = 2 g h_L$$

$$V_1^2 + V_2^2 - 2V_1V_2 = 2 g h_L$$

$$\boxed{h_L = \frac{(V_1 - V_2)^2}{2g}}$$



$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$h_L = \frac{(V_1 - V_2)^2}{2g} = \frac{V_1^2}{2g}\left(1 - \frac{V_2}{V_1}\right)^2 \qquad \left\{ Q = A_1 V_1 = A_2 V_2 \right\}$$

$$= \frac{V_1^2}{2g}\left(1 - \frac{A_1}{A_2}\right)^2$$

$$\boxed{h_L = \frac{V_1^2}{2g}\left[1 - \left(\frac{d_1}{d_2}\right)^2\right]^2}$$

$$h_L = \frac{(V_1 - V_2)^2}{2g} = \frac{V_2^2}{2g}\left[\frac{V_1}{V_2} - 1\right]^2$$

$$= \frac{V_2^2}{2g}\left(\frac{A_2}{A_1} - 1\right)^2$$

$$\boxed{h_L = \frac{V_2^2}{2g}\left[\left(\frac{d_2}{d_1}\right)^2 - 1\right]^2}$$

Q. What is the ratio of head loss to initial kinetic head, if the diameter of pipe is doubled suddenly?

(a) 0

(b) $\frac{9}{1}$

(c) $\frac{9}{16}$

(d) $\frac{16}{9}$

$$h_L = \frac{V_1^2}{2g}\left(\left(\frac{d_1}{d_2}\right)^2 - 1\right)^2 \qquad [d_2 = 2d_1]$$

$$\frac{h_L}{V_1^2/2g} = \frac{\frac{V_1^2}{2g}\left(\frac{3}{4}\right)^2}{\frac{V_1^2}{2g}} = \frac{9}{16}$$

Q. What is the ratio of head loss to final kinetic head, if the diameter of pipe is doubled suddenly?

(a) 0

(b) $\frac{9}{1}$

(c) $\frac{9}{16}$

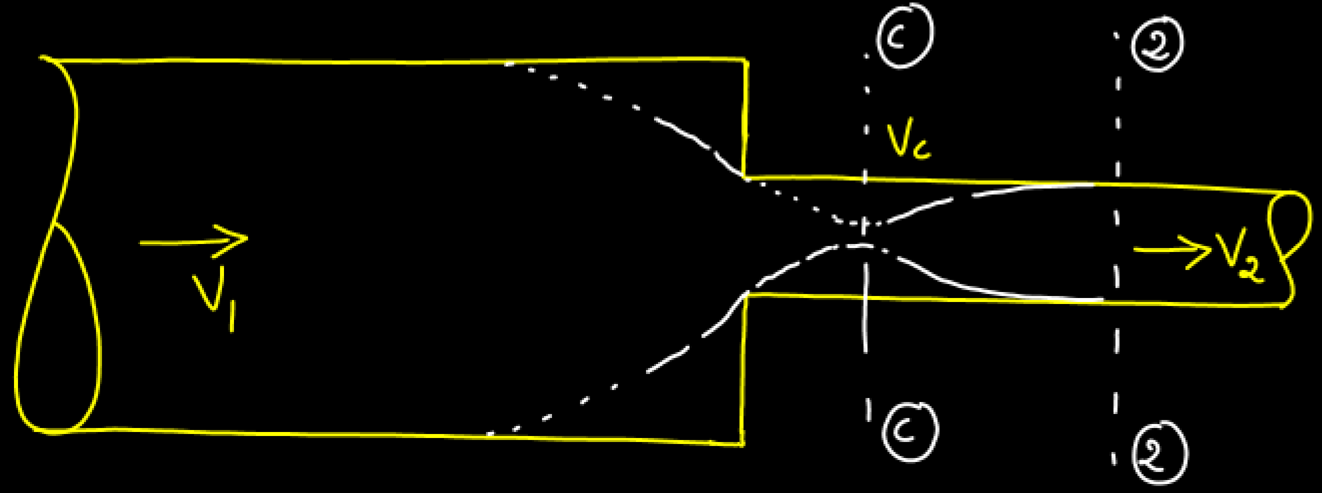
(d) $\frac{16}{9}$

$$h_L = \frac{V_2^2}{2g}\left[\left(\frac{d_2}{d_1}\right)^2 - 1\right]^2$$

$$[d_2 = 2d_1]$$

$$\frac{h_L}{\left(V_2^2/2g\right)} \Rightarrow \frac{9V_2^2}{2g} \Big/ V_2^2/2g \Rightarrow \frac{9}{1}$$

Loss due to sudden contraction:



$$h_L = \frac{(V_c - V_2)^2}{2g} = \frac{V_2^2}{2g}\left(\frac{V_c}{V_2} - 1\right)^2 \qquad \left[Q = A_c V_c = A_2 V_2\right]$$

$$= \frac{V_2^2}{2g}\left(\frac{A_2}{A_c} - 1\right)^2$$

Coefficient of contraction, $C_c = \dfrac{\text{Area of Venacontracta}}{\text{Area of Smaller dia pipe}} = \dfrac{A_c}{A_2}$

$$h_L = \frac{V_2^2}{2g}\left(\frac{1}{(A_c/A_2)} - 1\right)^2$$

$$\boxed{h_L = \frac{V_2^2}{2g}\left[\frac{1}{C_c} - 1\right]^2}$$

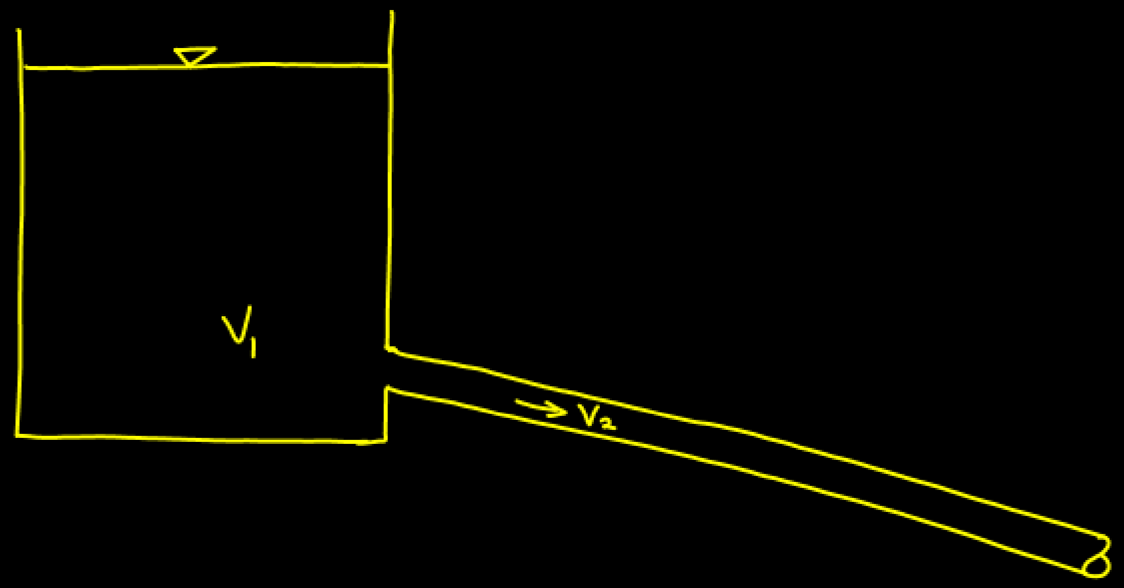
$$\left[\frac{1}{C_c} - 1\right]^2 = k$$

$= \text{Loss coefficient}$

$$\boxed{h_L = k\frac{V_2^2}{2g}}$$

V_2 = Velocity in smaller diameter pipe.

Entry loss:

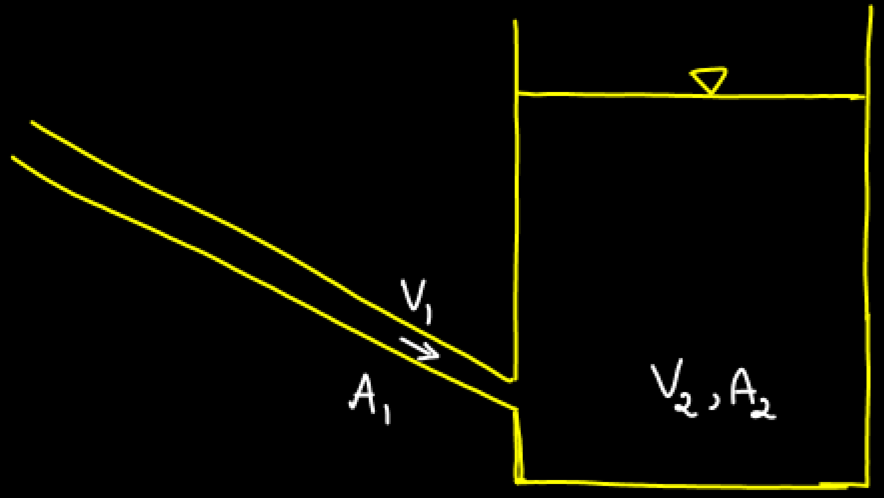


$$h_L = k\frac{V_2^2}{2g}$$

$$k = 0.5$$

$$\boxed{h_L = \frac{0.5V^2}{2g}}$$

Loss at exit:



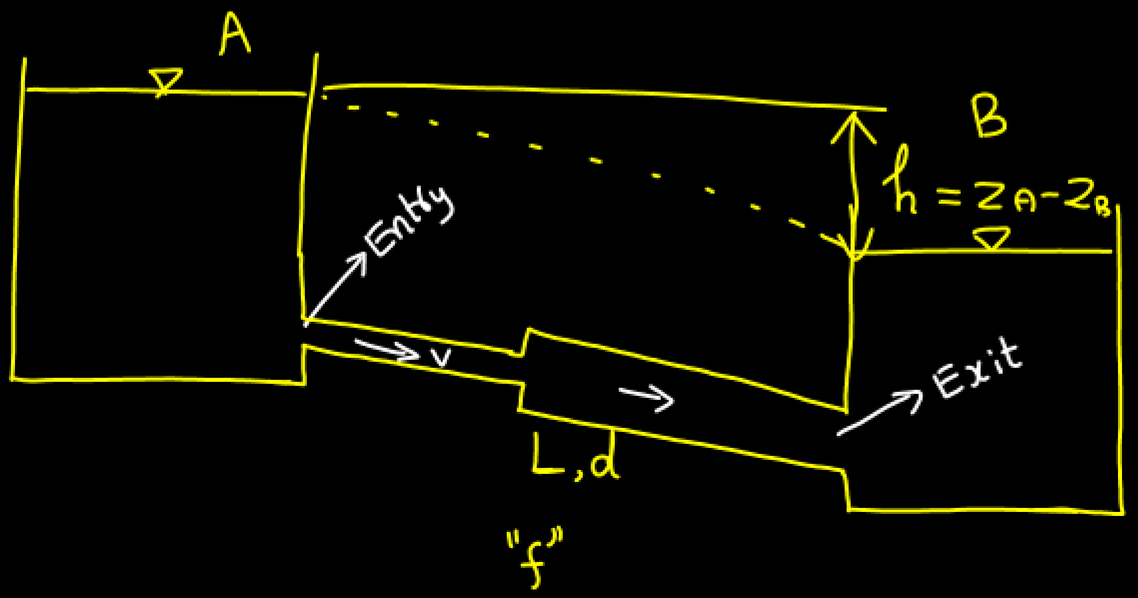
$$h_L = \frac{(V_1 - V_2)^2}{2g} = \frac{V_1^2}{2g} = \frac{V^2}{2g}$$

$$\boxed{h_L = \frac{V^2}{2g}}$$

$$h_L = \frac{V_1^2}{2g}\left[1 - \frac{A_1}{A_2}\right]^2$$

$$A_2 \to \infty \Rightarrow \frac{A_1}{A_2} \to 0$$

$$\boxed{h_L = \frac{V^2}{2g}}$$



$A \rightarrow B$

$$\cancelto{0}{\frac{p_A}{\gamma}} + \cancelto{0}{\frac{V_A^2}{2g}} + z_A = \cancelto{0}{\frac{p_B}{\gamma}} + \cancelto{0}{\frac{V_B^2}{2g}} + z_B + h$$

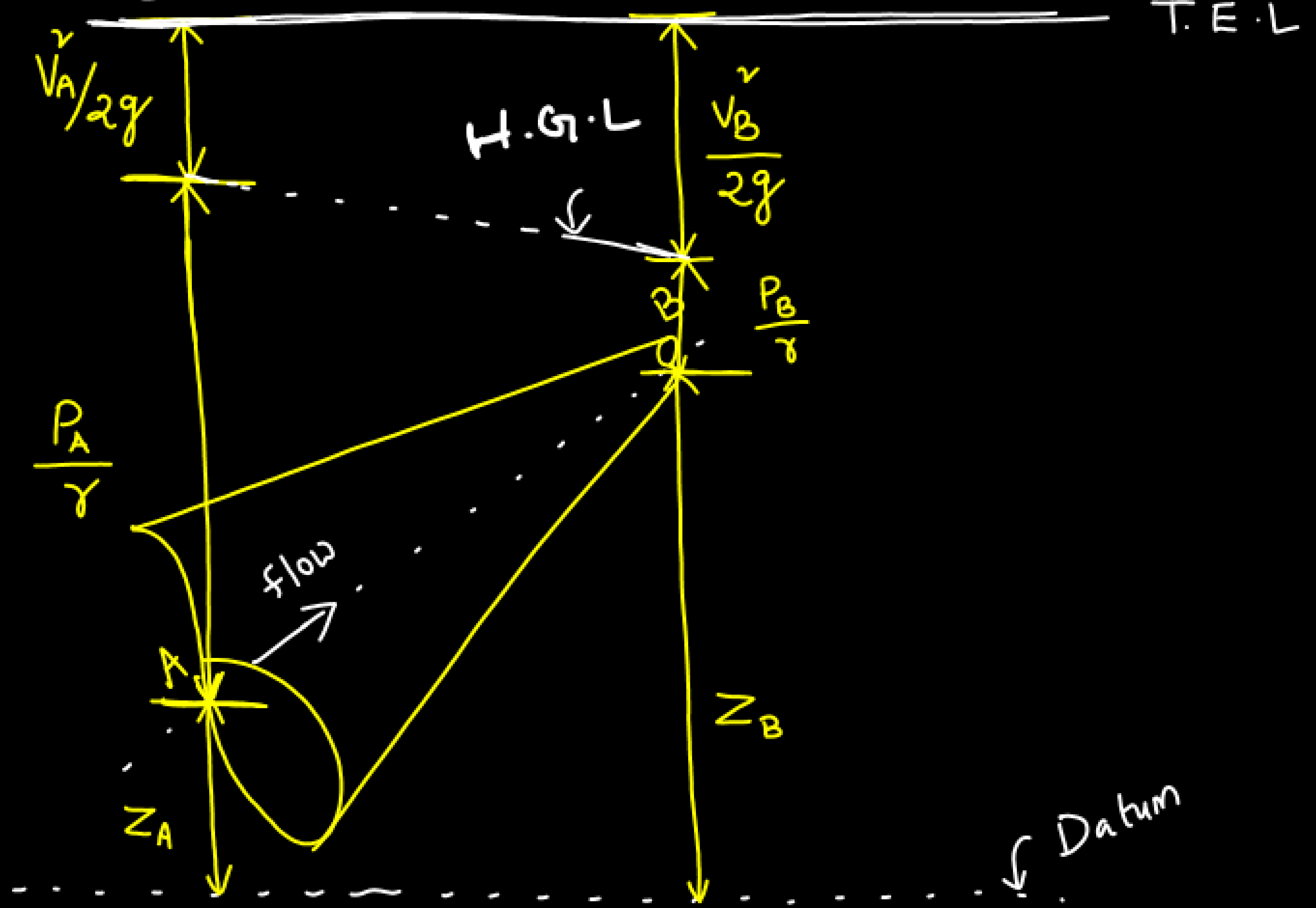
$$\left\{ z_A - z_B = h \right\}$$

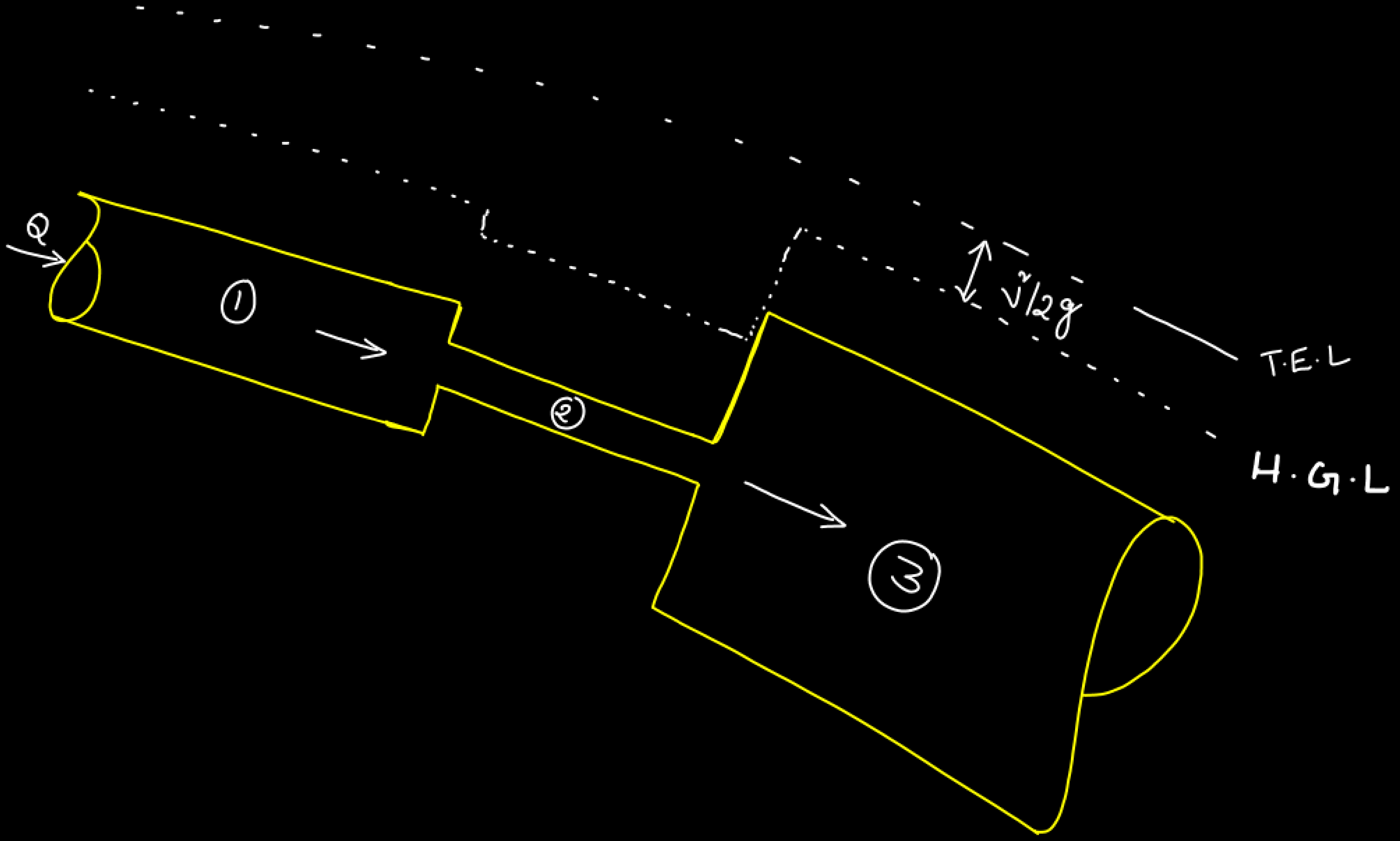
$$\left\{ h = 0.5\frac{V^2}{2g} + \frac{fLV^2}{2gd} + \frac{V^2}{2g} + \frac{(V_1 - V_2)^2}{2g} \right\}$$

Grade Lines:

① Energy grade lines / Total Energy line (T.E.L)

② Hydraulic grade Line (H.G.L)





Q
1
2
3
$V^2/2g$
T.E.L
H.G.L

1. Do they coincide with each other? (H.G.L & T.E.L) Yes
2. Do they run parallel to one another? (H.G.L & T.E.L) Yes

→ T.E.L & H.G.L coincides with each other when $V^2/2g$ will become zero.
Ex: In the case of reservoirs $V=0 \Rightarrow V^2/2g = 0$

→ T.E.L & H.G.L are Parallel to each other in the case of pipe flow through constan dia.

Connections in pipes:

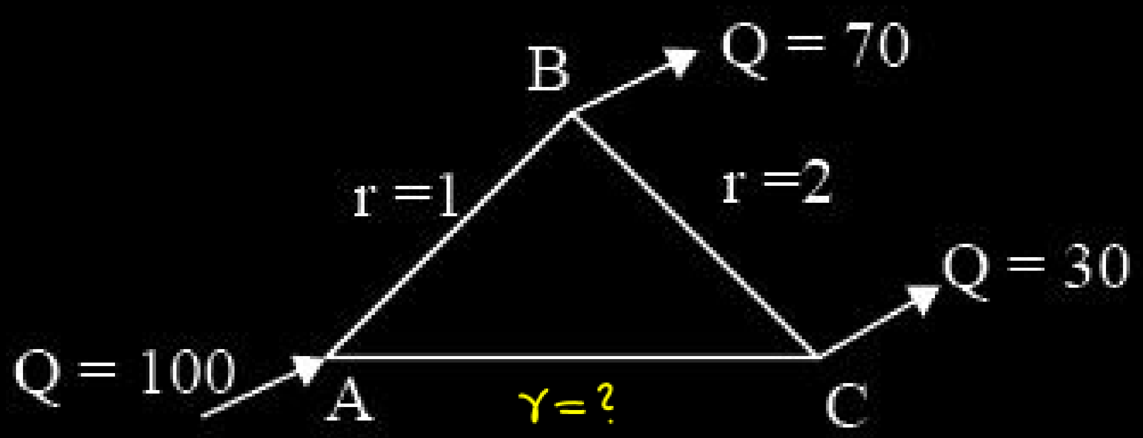
1. Pipes in series
2. Pipes in parallel

Pipes in series:

Pipes in parallel:

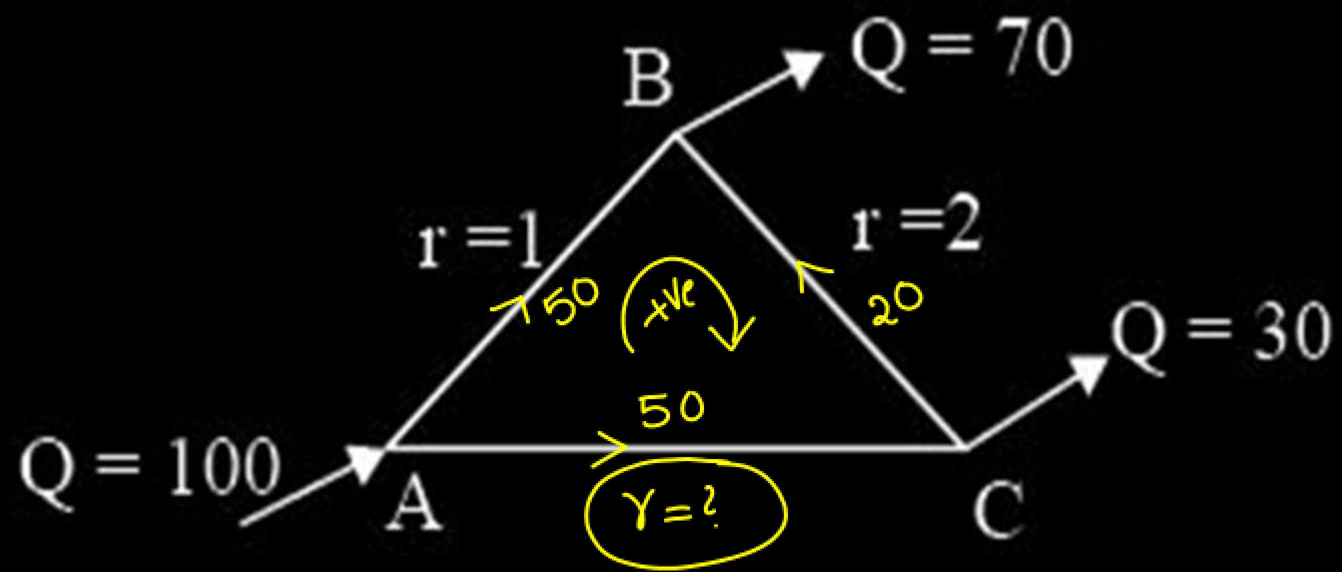
Flow through syphon:

Q. A triangular pipe network is shown in the figure.



The head loss in each pipe is given by $h_f = rQ^{1.8}$, with the variables expressed in a consistent set of units. The value of r for the pipe AB is 1 and for the pipe BC is 2. If the discharge supplied at the point A(i.e. 100) is equally divided between the pipes AB and AC, the value of r (up to two decimal places) for the pipe AC should be _____

(GATE – 17 – Set 1)



$$h_f = r Q^{1.8}$$

$$\sum h_f = 0$$

$$h_{AB} - h_{BC} - h_{CA} = 0$$

$$r_{AB} Q_{AB}^{1.8} - r_{BC} Q_{BC}^{1.8} - r_{CA} Q_{CA}^{1.8} = 0$$

$$(1)(50)^{1.8} - 2(20)^{1.8} - r_{CA}(50)^{1.8} = 0$$

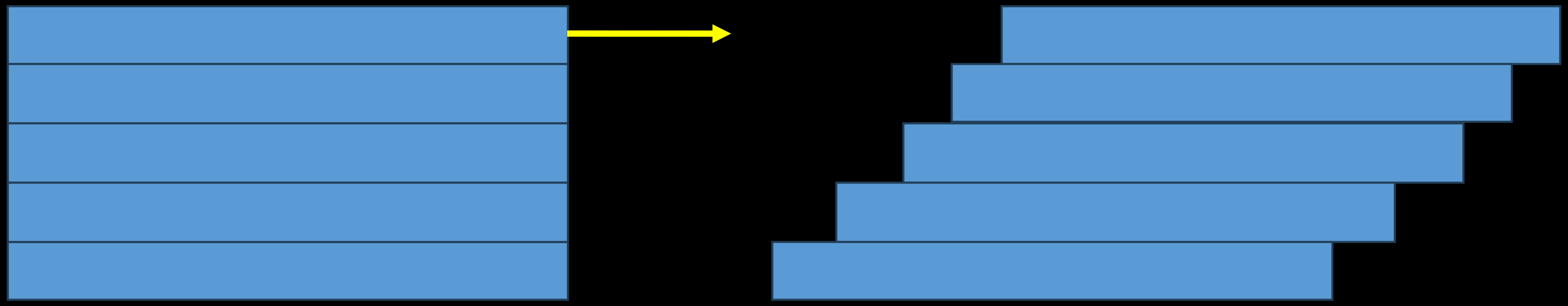
$$r_{CA} = 0.615$$

$$\simeq 0.62$$

Laminar flow:

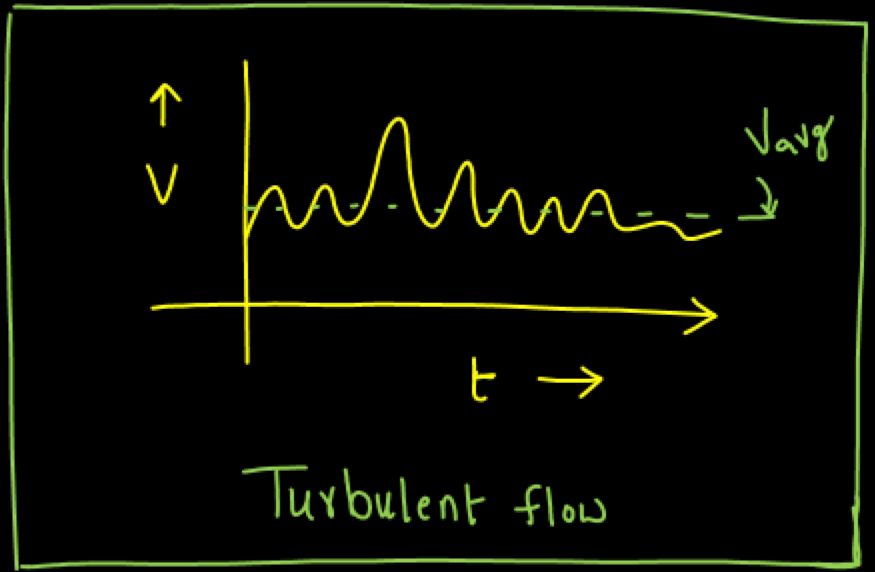
Definition :

The flow takes place in the form of layers(Laminas) by the virtue of viscous forces is known as laminar flow.



- Reynold's number and kinetic energy of flow is low
- No inter mixing of fluid particles
- Flow is steady
- Viscosity effect is dominant therefore, newton's law of viscosity is sufficient to calculate shear stress.
- Flow is rotational
- Surface roughness does not effect losses in flow

$$\left[\tau = \mu \frac{du}{dy}\right]$$



Examples :

1. Settling of impurities
2. Capillarity in soils
3. Flow of blood in veins

[Suspended solids]

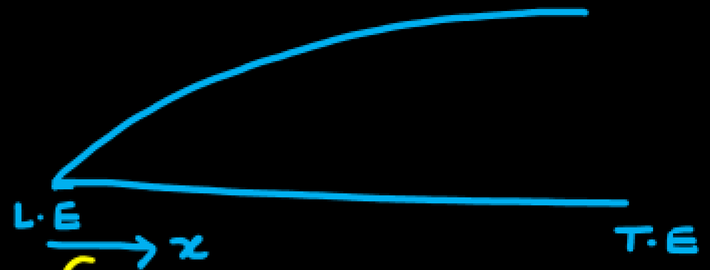
Boundary layer theory:

- When a real fluid flows past a solid boundary, the viscous region get concentrated in a very thin region adjacent to the surface.
- The flow in this region is known as boundary layer flow

- The flow beyond boundary layer is ideal, irrotational with almost uniform velocity profile.

B.L on a flat plate:

Turbulent boundary layer:(T.B.L)



$Re > 5\times10^5$

$$\boxed{\frac{\delta}{x} = \frac{0.371}{(Re_x)^{1/5}}} \Rightarrow \delta = \frac{0.371\,x}{\left(\frac{\rho U_\infty x}{\mu}\right)^{1/5}}$$

$$\left\{ \frac{\delta}{x} = \frac{k}{\sqrt{Re_x}} \quad \text{for L.B.L} \qquad k = 5 \; ; \; \delta \propto \sqrt{x} \right.$$

$$\delta \propto \frac{1}{(Re_x)^{1/5}}$$

$$\boxed{\delta \propto x^{4/5}}$$

$$\left\{ Re_x = \frac{\rho U_\infty x}{\mu} \right\}$$

L.B.L $\Rightarrow x^{0.5}$

T.B.L $\Rightarrow x^{0.8}$

Note: T.B.L grows faster along a plate compared to L.B.L

$$L.B.L \Rightarrow \delta \propto x^{1/2}$$

$$T.B.L \Rightarrow \delta \propto x^{4/5}$$

Velocity profile:

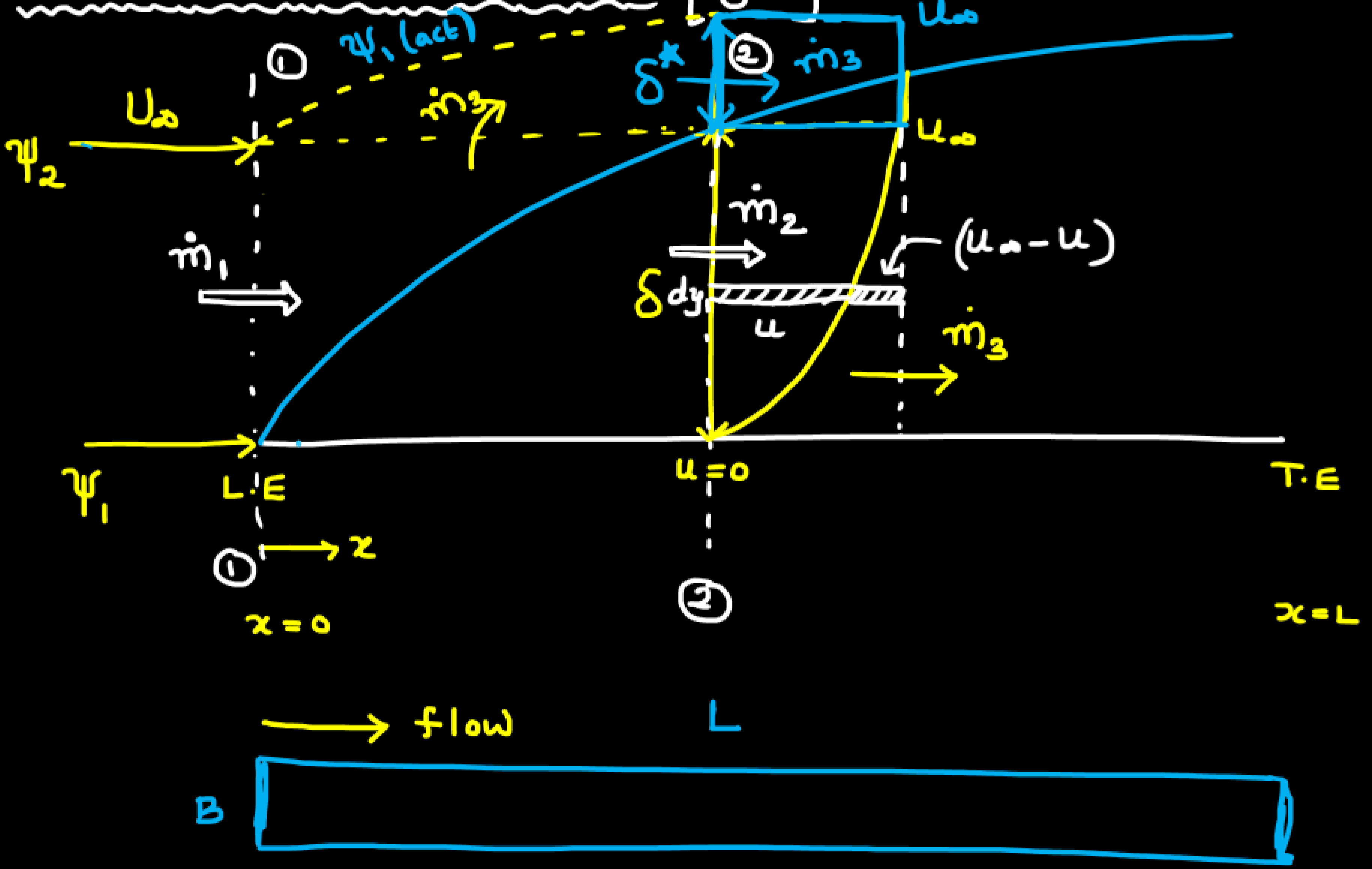
Velocity follows n^{th} power law

$$\frac{u}{u_\infty} = \left[\frac{y}{\delta}\right]^{1/n}$$

for $n = 7$

$$\frac{u}{u_\infty} = \left[\frac{y}{\delta}\right]^{1/7} \quad 7^{th} \text{ Power Law.}$$

Displacement thickness: $[\delta^*]$



$$\left\{ \begin{aligned} \dot{m} &= \text{Mass flux} \\ &= \rho A V \end{aligned} \right\}$$

$\dot{m}_1$ = Issued mass flux

$\dot{m}_2$ = mass flux allowed to flow in B.L

$\dot{m}_1 > \dot{m}_2$

$\dot{m}_3$ = mass flux reduced

$$\dot{m}_1 = \rho A_1 V_1$$

$$\left\{ \begin{array}{l} A_1 = B\delta \\ V_1 = u_\infty \end{array} \right\}$$

$$\boxed{\dot{m}_1 = \rho B \delta u_\infty} \rightarrow (1)$$

$$d\dot{m}_2 = \rho \, dA \, u$$

$$\left\{ \begin{array}{l} dA = B\,dy \\ u = u \end{array} \right\}$$

$$d\dot{m}_2 = \rho B \, dy \, u$$

$$\int d\dot{m}_2 = \rho B \int_0^\delta u \, dy$$

$$\boxed{\dot{m}_2 = \rho B \int_0^\delta u \, dy} \rightarrow (2)$$

$$d\dot{m}_3 = \rho \, dA \, (u_\infty - u)$$

$$\left[\begin{array}{l} dA = B\,dy \\ u = u_\infty - u \end{array} \right]$$

$$\dot{m}_3 = \rho B \int_0^\delta dy \; (u_\infty - u)$$

$$\dot{m}_3 = \rho B \int_0^\delta (u_\infty - u)\, dy \longrightarrow 3$$

$$\boxed{\dot{m}_3 = \rho B \delta^* u_\infty} \longrightarrow 4$$

$$\left[\begin{array}{l} A = B\delta^* \\ u = u_\infty \end{array} \right]$$

$$4 = 3$$

$$\cancel{\rho B}\int_0^\delta (u_\infty - u)\, dy = \cancel{\rho B}\,\delta^* u_\infty$$

$$\delta^* = \frac{1}{u_\infty}\int_0^\delta (u_\infty - u)\, dy$$

$$\left\{ \delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy \right\}$$

$$\left\{ \delta^* < \delta \right\}$$

Def:

The distance measured $\perp r$ to the surface, by which flow gets displaced in order to compensate reduced massflux.

$$\boxed{\frac{u}{u_\infty} = \left[\frac{y}{\delta}\right]^{1/n}}$$

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{u_\infty}\right) dy$$

$$= \int_0^\delta \left(1 - \left[\frac{y}{\delta}\right]^{1/n}\right) dy$$

$$= y\Big|_0^\delta - \frac{1}{\delta^{1/n}} \int_0^\delta y^{1/n}\, dy$$

$$= \delta - \frac{1}{\delta^{1/n}} \left.\frac{y^{1+\frac{1}{n}}}{\left(1+\frac{1}{n}\right)}\right|_0^\delta$$

$$\Rightarrow \delta - \left[\frac{1}{\delta^{1/n}}\left(\frac{n}{n+1}\right)\delta^{\frac{n+1}{n}} - 0\right]$$

$$\Rightarrow \delta - \frac{n}{n+1}(\delta)$$

$$\Rightarrow \delta\left[\frac{n+1-n}{n+1}\right]$$

$$\Rightarrow \left(\frac{1}{n+1}\right)\delta$$

$$\left\{ \delta^* = \frac{\delta}{n+1} \right\}$$

If Linear velocity profile, $\frac{u}{u_\infty} = \frac{y}{\delta}$; $n=1$

$$\delta^* = \frac{\delta}{n+1}$$

$$= \frac{\delta}{1+1}$$

$$\left\{ \delta^* = \frac{\delta}{2} \right\}$$

Boundary layer separation: ★★★

1. Inertia force
2. Viscous force
3. Pressure force

→ $\frac{dp}{dx} < 0$ [Favourable pressure Gradient]

→ It gives accelerated effect

$F_{inertia}$ & $F_{pressure}$ opposing $F_{viscous}$

No eddies formation taking place.

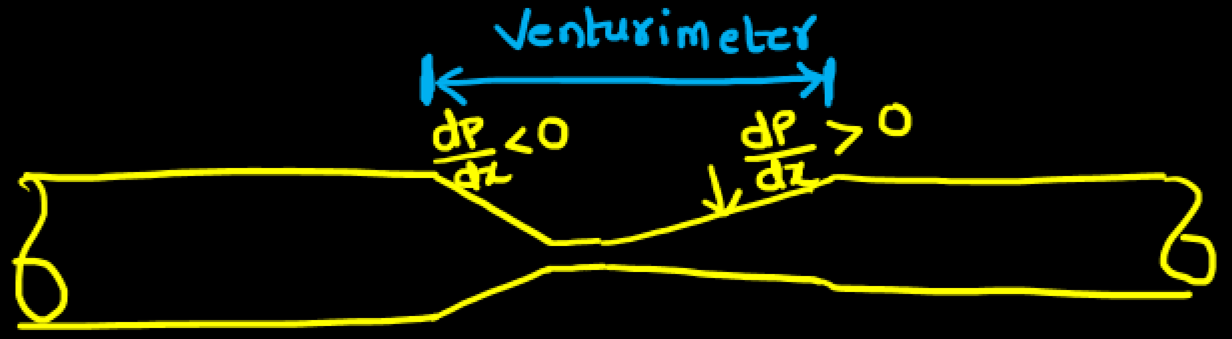
∴ Less Losses and higher efficiency

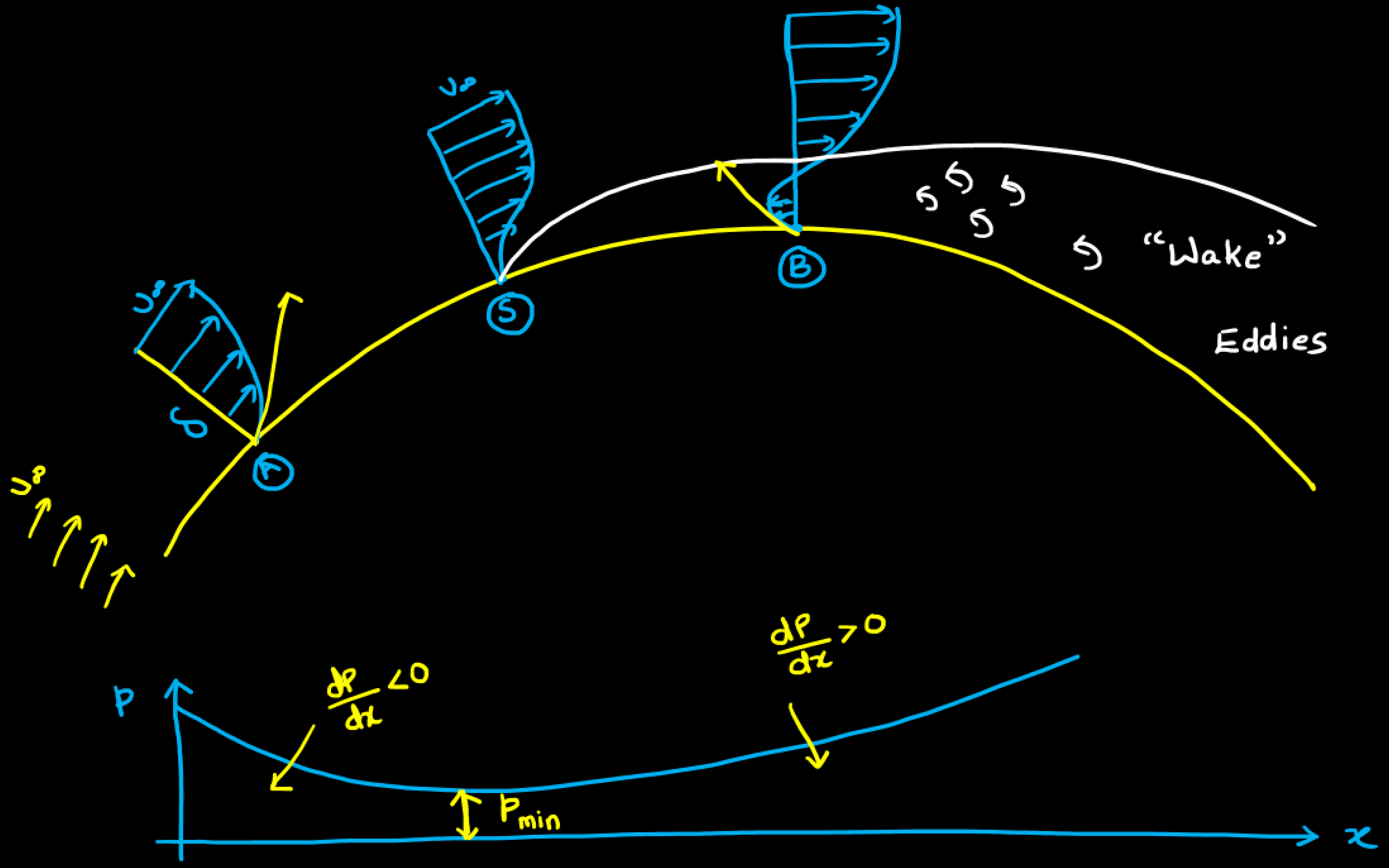
→ $\frac{dp}{dx} > 0$ [Adverse pressure Gradient]

→ decelerating flow

Pressure & viscous forces opposing inertia force

- Formation of eddies taking Place (or) Reversal of flow
- Higher losses and Lesser efficiency.





U_∞
U_∞
U_∞
δ
A
S
B
"Wake"
Eddies
P
$\frac{dP}{dx} < 0$
$\frac{dP}{dx} > 0$
P_{min}
x

@ A : $\left.\frac{\partial u}{\partial y}\right|_{y=0} > 0 \Rightarrow$ Attached flow.

@ B : $\left.\frac{\partial u}{\partial y}\right|_{y=0} < 0 \Rightarrow$ Detached flow.

@ S : $\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0 \Rightarrow$ flow is at a verge of separation.

- When a B.L encounters adverse pressure gradient, flow experiences deceleration.
- The B.L thickness drastically increases
- A portion of B.L near the surface separates from the wall
- This region is characterized by formation of eddies(Wake)
- Losses increases

$$\frac{dp}{dx} > 0$$

Factors influencing B.L separation:

- Curvature of the surface
- Reynold’s number
- Roughness or smoothness of a surface

- TBL is lesser B.L separation
- L.B.L is higher B.L separation

Drag force:

$$F_D = \frac{C_D \, A \, \rho \, U_\infty^2}{2}$$

Where, C_D = Drag coefficient

$$\text{For L.B.L,} \quad C_D = \frac{1.328}{\sqrt{Re_L}}$$

A = Projected area

ρ = density of fluid

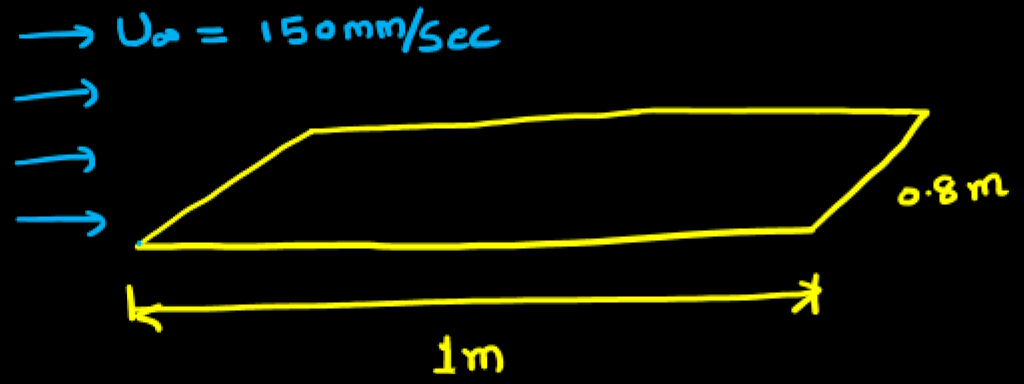
U_∞ = free stream velocity

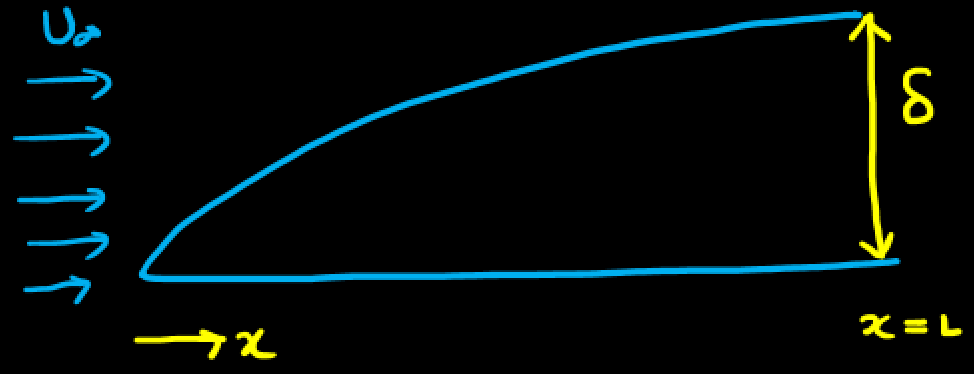
(OR)

11 For the velocity profile given in, find the thickness of boundary layer at the end of the plate and the drag force on one side of a plate 1m long and 0.8m wide when placed in water flowing with a velocity of 150 mm per second. Calculate the value of co-efficient of drag also. Take μ for water = 0.01 poise. 10

10M

Sol:





$$\mu = 0.01 \text{ Poise}$$
$$= 0.01 \times 0.1 \frac{NS}{m^2} \quad \left[\frac{1 N\text{-}S}{m^2} = 10 \text{ Poise}\right]$$
$$= 10^{-3} \text{ N-S}/m^2$$

$$\frac{\delta}{x} = \frac{K}{\sqrt{Re_x}} \quad \text{for L.B.L}$$

$$Re_L = \frac{\rho V L}{\mu} = \frac{1000 \times 150 \times 10^{-3} \times 1}{10^{-3}} = 1.5 \times 10^{5}$$

$$Re < 5 \times 10^{5} \ (\text{L.B.L})$$

$$\frac{\delta}{1} = \frac{5}{\sqrt{1.5 \times 10^{5}}} \qquad [k = 5]$$

$$\boxed{\Rightarrow \ \delta = 0.0129\,m = 12.9\,mm}$$

$$C_D = \frac{1.328}{\sqrt{Re_L}} = \frac{1.328}{\sqrt{1.5\times10^5}} = 3.428\times10^{-3} = \boxed{0.0034}$$

$$\text{Drag force, } F_D = \frac{C_D A \rho U_\infty^2}{2}$$

$$\left\{ \begin{aligned} U_\infty &= 150\,mm/s \\ &= 0.15\,m/s \end{aligned} \right\}$$

$$= \frac{1}{2} \times 0.0034 \times (1\times0.8) \times 1000 \times 0.15^2$$

$$\boxed{F_D = 0.0306\,N}$$